MAE 143B
Linear Control

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MIDTERM 2 SOLUTIONS

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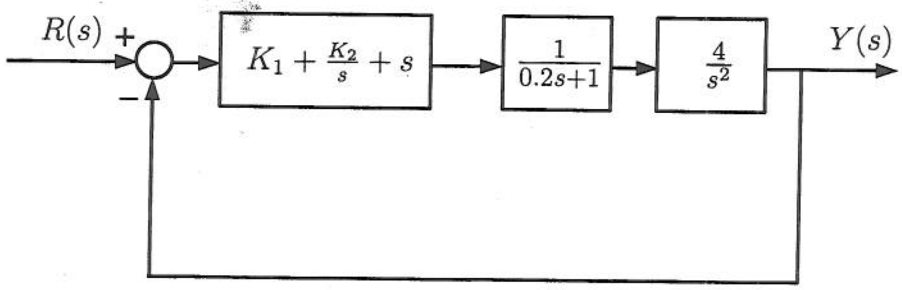
Problem 1. (5 points) Consider a laser guided missile with PID controller:



Determine the range of K_1 and K_2 to ensure stable operation.

$$1 + \left(K_1 + \frac{K_2}{s} + s\right)\frac{4}{(0.2s+1)s^2} = 0$$

$$s^4 + 5s^3 + 20s^2 + 20K_1 s + 20K_2 = 0$$

s^4	1	20	$20K_2$
s^3	5	$20K_1$	
s^2	$20-4K_1$	$20K_2$	
s^1	$20K_1 - \frac{25K_2}{5-K_1}$		
s^0	$20K_2$		

$\boxed{K_1 < 5}$ $\boxed{K_2 > 0}$ $\boxed{K_2 < \frac{4}{5}K_1(5-K_1)}$

From the 3 inequalities above we also get $K_1 > 0$

Final answer: pick $0 < K_1 < 5$,

then pick $0 < K_2 < \frac{4}{5}K_1(5-K_1)$

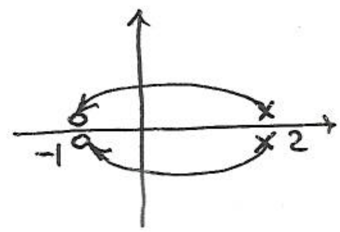
Problem 2. (6 points) Sketch the root locus with respect to K for the equation $1 + KG(s) = 0$ for the systems below. In each case, indicate what type of feedback gains K you would use to ensure system's stability (large, medium, or small)?

a) (2 points) $G(s) = \dfrac{(s+1)^2}{(s-2)^2}$

b) (2 points) $G(s) = \dfrac{1}{(s^2+2s+10)(s^2+6s+10)}$

c) (2 points) $G(s) = \dfrac{2s-1}{s^2-6s+10}$

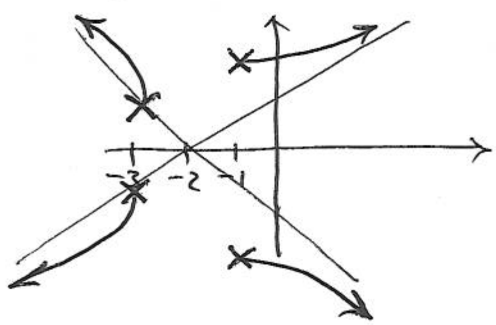
a) rel. deg = 0.



stable for sufficiently high K

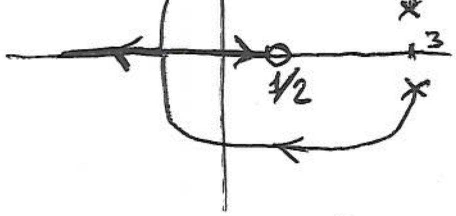
b) rel. deg = 4, $G(s) = \dfrac{1}{((s+1)^2+3^2)((s+3)^2+1^2)}$

$\alpha = \dfrac{-3-3-1-1}{4} = -2$



stable for sufficiently small K

c) rel. deg = 1, $G(s) = \dfrac{2(s-\frac{1}{2})}{(s-3)^2+1^2}$,



Stable for some range of K
(very small or very large $K \Rightarrow$ instability)
Need to use Routh's test to make sure that break-in point is on the negative real axis.

$s^2 - 6s + 10 + 2Ks - 2K = 0 \Rightarrow s^2 + (2K-6)s + 10 - K = 0$

$\boxed{3 < K < 10}$

Problem 3. (7 points) Consider the plant

$$G(s) = \frac{s-1}{s^2-2s+2}$$

in the feedback loop with compensator $KD(s)$.

a) (1 point) Using Routh's criterion, show that $D(s) = 1$ is not sufficient, i.e. the system is unstable for all $K > 0$. Sketch the root locus.

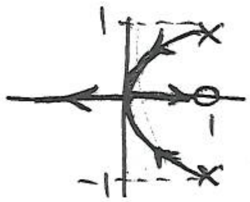
b) (2 points) Use the non-robust compensator $D(s) = \dfrac{s+z}{s-1}$ to stabilize the system. Sketch the root locus with respect to K and indicate values of z and K that give a stable system (it is OK to give a qualitative condition on K).

c) (4 points) Use the robust compensator $D(s) = \dfrac{1}{s+p}$ to stabilize the system. Sketch the root locus with respect to K (for a general p). Use Routh's criterion to determine conditions on p and K that ensure the system's stability. Give one example of a pair (p, K) that satisfies all conditions. (Note that, depending on the value of p, two different root locus curves may result. The exact one can only be determined with Matlab. You will get full credit for sketching either one of these two possible curve shapes.)

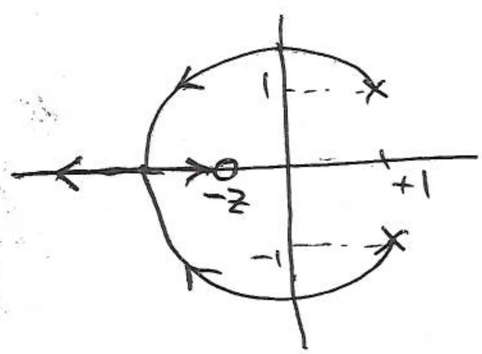
a) $1 + K\dfrac{(s-1)}{s^2-2s+2} = 0 \Rightarrow s^2 + (K-2)s + 2 - K = 0$

$K > 2$ or $K < 2 \Rightarrow$ unstable

$K = 2 \Rightarrow$ double integrator $\Rightarrow$ unstable



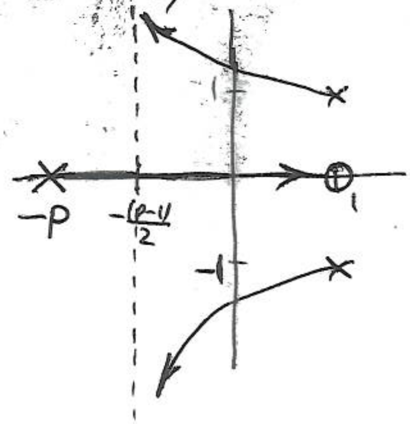
b) $1 + K\dfrac{s+z}{s^2-2s+2} = 0$ rel. deg $= 1$



Any $z > 0$ and sufficiently high K $\Big\}$ stable

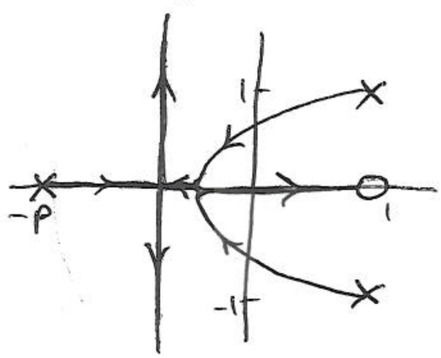
c) $1 + K\dfrac{s-1}{(s+p)(s^2-2s+2)} = 0$

rel. degree $= 2$, $\alpha = \dfrac{-p+1+1-1}{2} = -\dfrac{(p-1)}{2}$



From this plot we can see that, depending on p, there is a range of K's for which the system is stable. However, we need to use Routh's test to determine the conditions on p and K.

Note that, depending on p, root locus may look like this:



$$s^3 + (p-2)s^2 + (K+2-2p)s + 2p - K = 0$$

s^3	1	$K+2-2p$
s^2	$p-2$	$2p-K$
s^1	$K+2-2p+\dfrac{K-2p}{p-2}$	
s^0	$2p-K$	

$\boxed{p>2}$ $\boxed{K<2p}$

$$(K+2-2p)(p-2) + K - 2p > 0$$

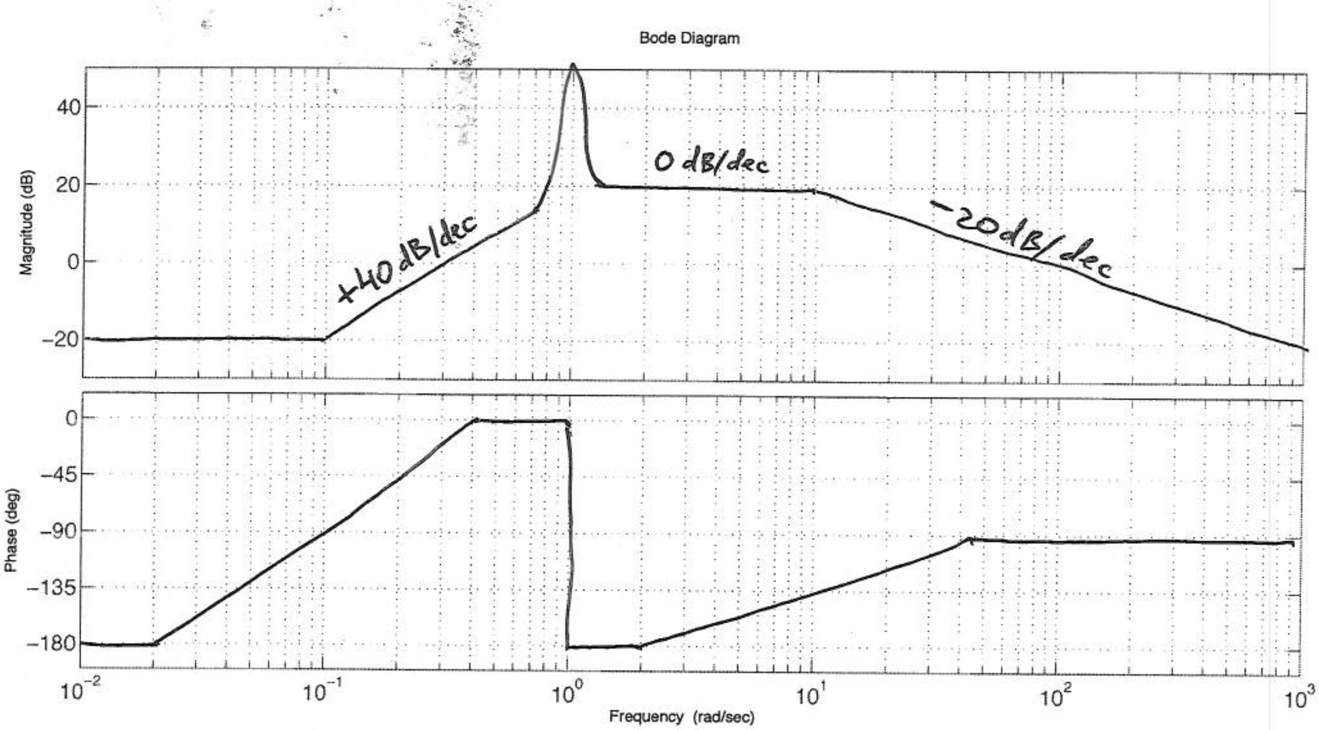
$$K > \frac{2p^2-4p+4}{p-1} = 2p - \frac{2(p-2)}{p-1}$$

Stable for $\boxed{p>2 \text{ and } 2p - \frac{2(p-2)}{p-1} < K < 2p}$

For example, one can pick $p=4$ and $K=7$

Problem 4. (6 points) Sketch the magnitude and phase Bode plots of

$$G(s) = \frac{(10s+1)^2}{(s-10)(s^2+0.1s+1)}$$



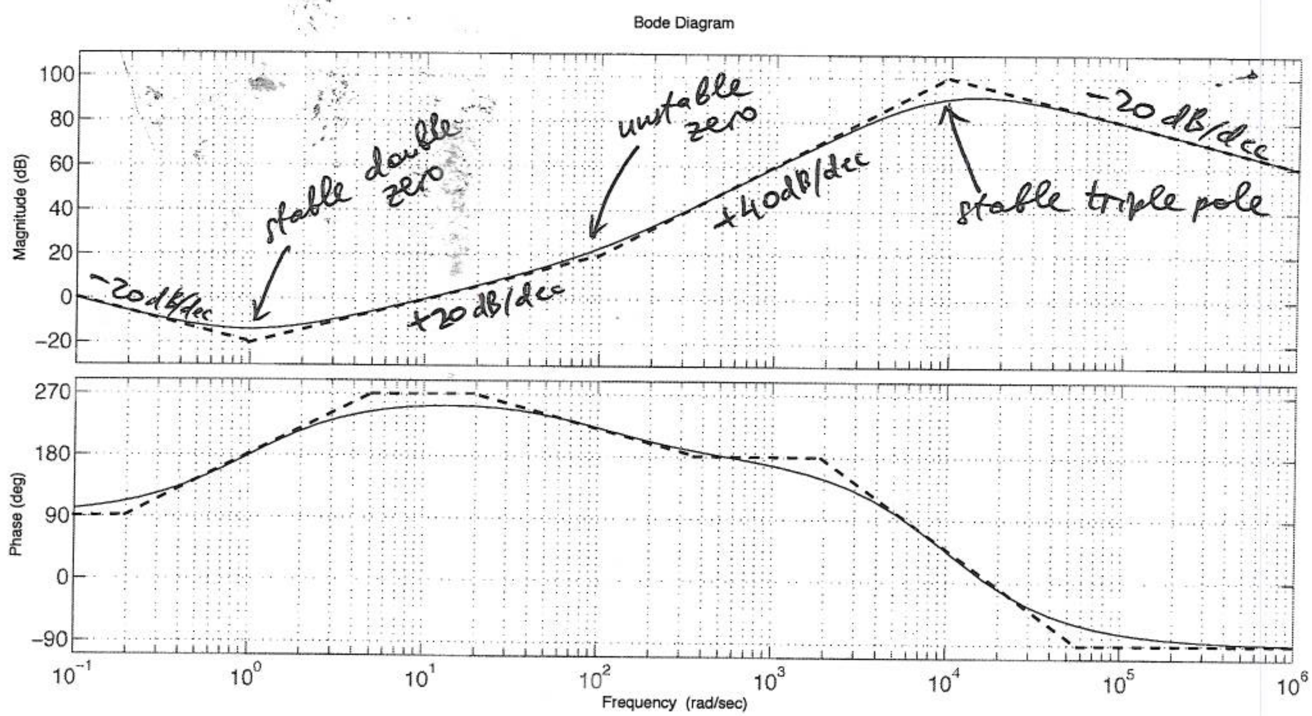
1) For $s=0$ $G(0) = -\frac{1}{10}$, so initial magnitude is $20\log\left|\frac{1}{10}\right| = -20\,dB$ and initial phase is $-180°$.

2) At $\omega = 10^{-1}$, there is a stable double zero, so $\begin{cases} 0 \to +40\ dB/dec \\ -180° \to 0° \end{cases}$

3) At $\omega = 10^0$, there is a couple of resonant poles, so $\begin{cases} +40 \to 0\ dB/dec \\ 0° \to -180°\ \text{sharply} \end{cases}$

4) At $\omega = 10^1$, there is an unstable pole, so $\begin{cases} 0 \to -20\ dB/dec \\ -180° \to -90° \end{cases}$

Problem 5. (6 points) From the Bode plots below, determine the system's transfer function.



1) Initial phase 90° and magnitude -20dB/dec indicate pole at the origin. $\left(\frac{1}{s}\right)$

2) At $\omega = 10^0$, magnitude response increases by +40dB/dec while phase increases by +180°. This indicates a stable double zero $(s+1)^2$

3) At $\omega = 10^2$, magnitude response increases by +20dB/dec while phase decreases by 90°, which indicates unstable zero $(s-100)$

4) At $\omega = 10^4$, magnitude response decreases by 60dB/dec while phase drops by 270°, which indicates stable triple pole $\frac{1}{(s+10^4)^3}$.

$$G = \frac{K(s+1)^2(s-100)}{s(s+10^4)^3}$$

To find K, let $s = 10^{-1} \Rightarrow |G|_{dB} = 20\log\left|\frac{1.1^2(0.1-100)K}{0.1(0.1+10^4)^3}\right|$

$$\approx 20\log\left|\frac{100K}{10^{11}}\right| = 0 \text{ dB}$$

$$\Rightarrow 100K = 10^{11} \Rightarrow K = 10^9$$