MAE 143B
LINEAR CONTROL

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MIDTERM

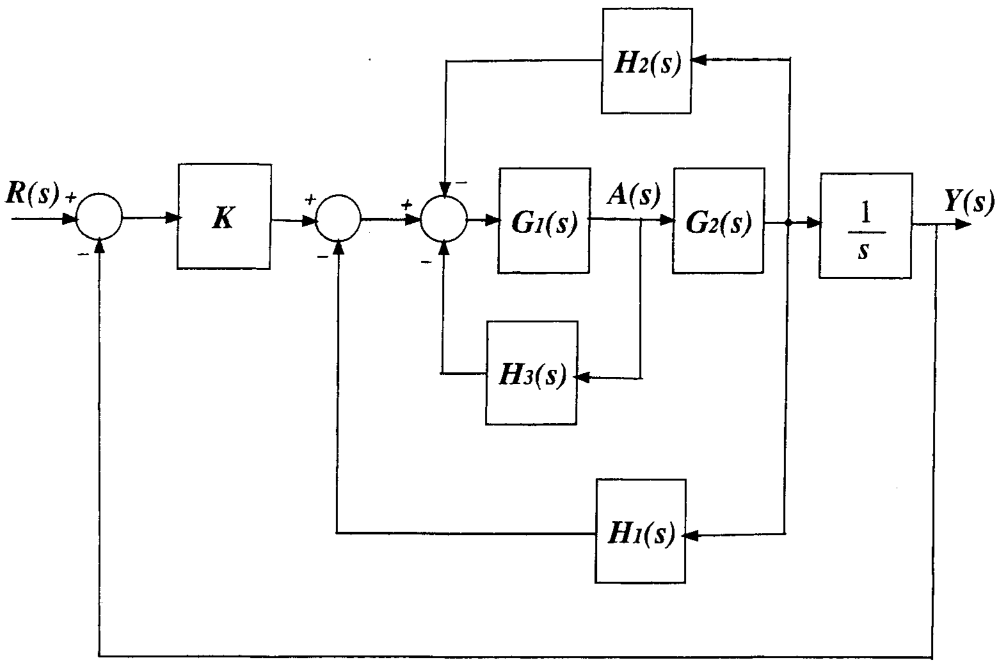
May 11, 2006

NAME: Solutions

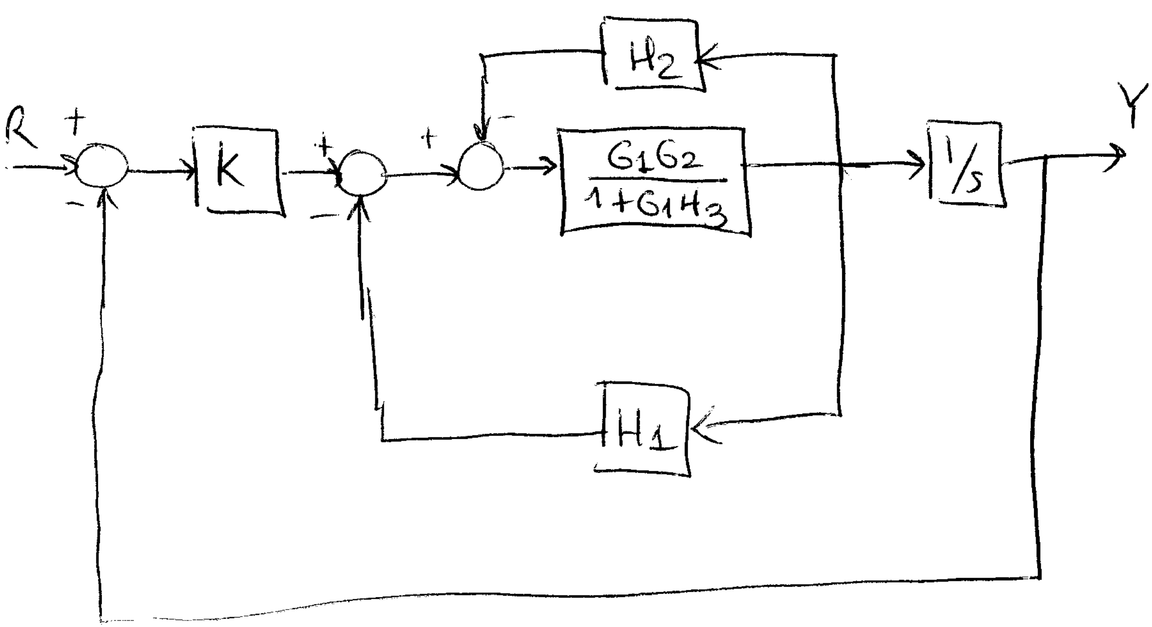
- Open books and notes.
- Present your reasoning and calculations clearly. Random or inconsistent etchings will not be graded.
- Write only on the paper provided. If you run out of space for a given problem, continue on the pages at the end of the set and indicate "Continued on page X."
- The problems are *not* ordered by difficulty.
- Total points: 30.
- Problem 7 is optional, it can be done for extra credit of 5 points.
- Time: 2:00–3:20.

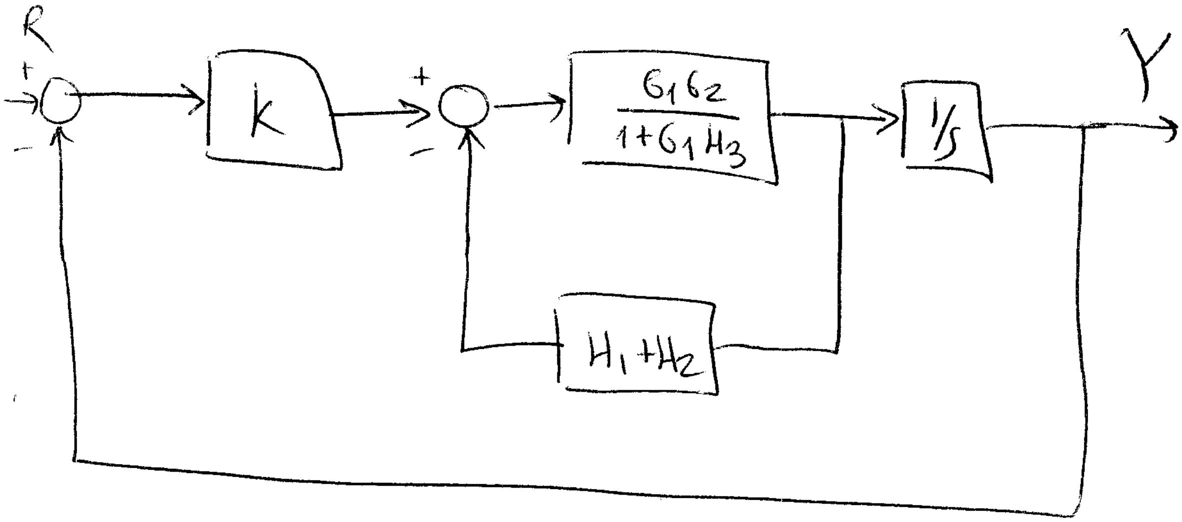
Problem 1. (5 points)

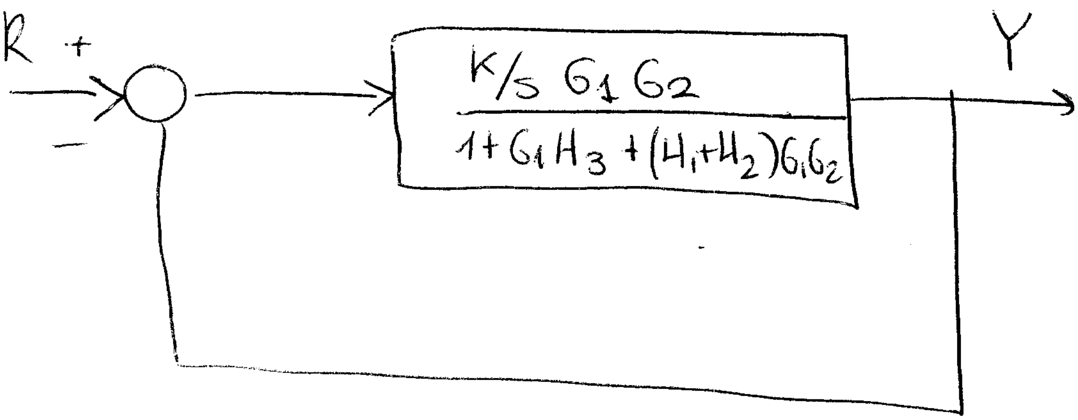
A control engineer, N. Minorsky, designed an innovative ship steering system in the 1930s for the U.S. Navy. The system is represented in the following block diagram.

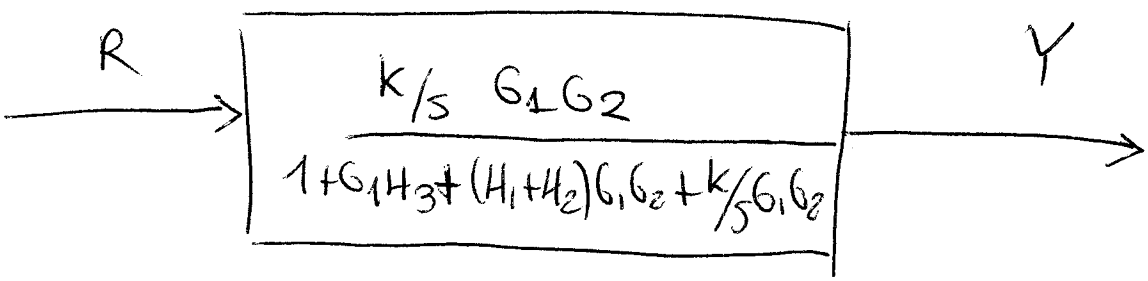


In the diagram, $Y(s)$ is the ship's course, $R(s)$ is the desired course (reference), and $A(s)$ is the rudder angle. Find the transfer function $Y(s)/R(s)$.




R
+
−
k
+
−
$\frac{G_1 G_2}{1+G_1 H_3}$
$1/s$
Y
H_1+H_2

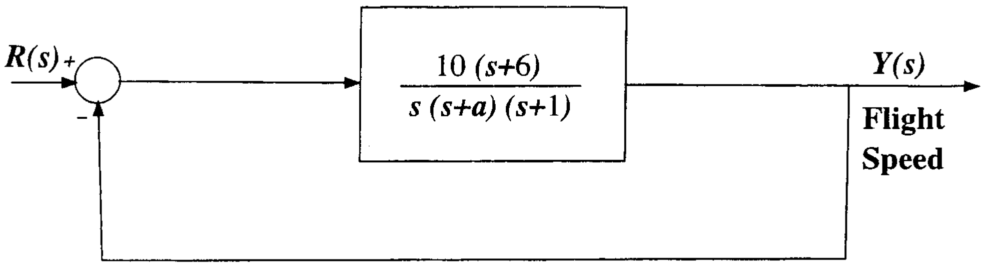

R
+
−
$\frac{K/s \, G_1 G_2}{1+G_1 H_3+(H_1+H_2)G_1 G_2}$
Y


R
$\frac{K/s \, G_1 G_2}{1+G_1 H_3+(H_1+H_2)G_1 G_2+K/s \, G_1 G_2}$
Y

Problem 2. (5 points)

A proposed hypersonic plane would climb to 100,000 feet, fly 3800 miles per hour, and cross the Pacific in 2 hours. (These data are not relevant for solving the problem, they are just given as information of general interest.)

Control of speed of the aircraft could be represented by the following model:



Find the sensitivity of the closed-loop transfer function $T(s)$ to a small change in the parameter a.

$$T = \frac{\dfrac{10(s+6)}{s(s+a)(s+1)}}{1+\dfrac{10(s+6)}{s(s+a)(s+1)}} = \frac{10(s+6)}{s(s+a)(s+1)+10(s+6)}$$

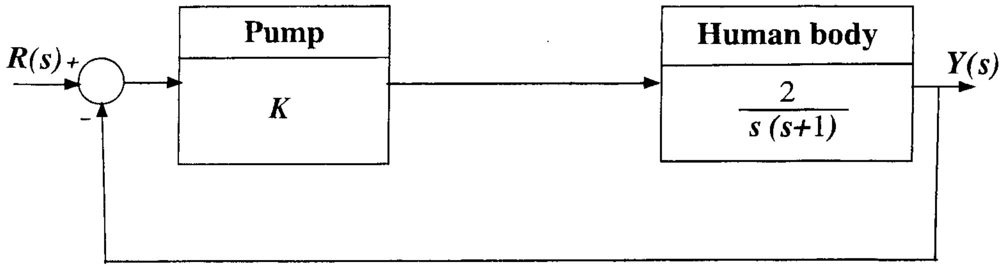
$$S_a^T = \frac{a}{T}\frac{dT}{da} \;;\; \frac{dT}{da} = \frac{-10(s+6)\,s(s+1)}{\left(s(s+a)(s+1)+10(s+6)\right)^2}$$

$$\Rightarrow S_a^T = \frac{a}{\dfrac{10(s+6)}{s(s+a)(s+1)+10(s+6)}} \cdot \frac{-10(s+6)s(s+1)}{\left(s(s+a)(s+1)+10(s+6)\right)^2}$$

$$= \frac{-a\,s(s+1)}{s(s+a)(s+1)+10(s+6)}$$

Problem 3. (6 points)

An automatic insulin injection sytem for blood-sugar level control in diabetic persons is shown in the figure:



(a) (4 points) Determine K so that overshoot is 7%.

(b) (2 points) What is the settling time for K determined in (a)?

$$T(s) = \frac{K\frac{2}{s(s+1)}}{1+K\frac{2}{s(s+1)}} = \frac{2K}{s^2+s+2K}$$

$$\Rightarrow \omega_n^2 = 2K \Rightarrow \omega_n = \sqrt{2K}, \quad 2\xi\omega_n = 1 \Rightarrow \xi = \frac{1}{2\omega_n} = \frac{1}{2\sqrt{2K}}$$

a) $$M_p = 0.07 \Rightarrow e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.07$$

$$\Rightarrow -\frac{\pi\xi}{\sqrt{1-\xi^2}} = \ln 0.07 \; ; \; \frac{\xi^2}{1-\xi^2} = \left(-\frac{\ln 0.07}{\pi}\right)^2$$

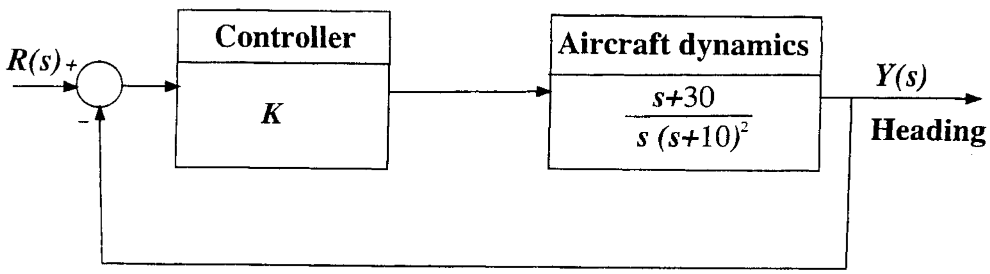
$$\Rightarrow \xi = \sqrt{\frac{\left(-\frac{\ln 0.07}{\pi}\right)^2}{1+\left(-\frac{\ln 0.07}{\pi}\right)^2}} = 0.646$$

$$0.646 = \frac{1}{2\sqrt{2K}} \Rightarrow K = \frac{1}{8\cdot(0.646)^2} = 0.299 \approx 0.3$$

b) $$t_s = \frac{4.6}{\xi\omega_n} = \frac{4.6}{1/2} = 9.2\,s$$

Problem 4. (5 points)

A vertical-takeoff aircraft with turning jet nozzles to steer the airplane is shown in a feedback loop with a controller:



Determine the maximum gain K for stable operation.

$$T = \frac{K\dfrac{S+30}{S(S+10)^2}}{1+K\dfrac{S+30}{S(S+10)^2}} = \frac{K(S+30)}{\underbrace{S(S+10)^2 + K(S+30)}_{S^3+20S^2+(100+K)S+30K}}$$

Routh's Criterion:

S^3 : 1 $\quad 100+K$

S^2 : 20 $\quad 30K$

S^1 : $\dfrac{2000+20K-30K}{20}$

S^0 : $30K$

from S^1 : $\dfrac{2000-10K}{20} > 0 \Rightarrow 2000-10K > 0 \Rightarrow 10K < 2000 \Rightarrow K < 200$

from S^0 : $30K > 0 \Rightarrow K > 0$

maximum gain: $\underline{200}$

Problem 5. (5 points)

(a) (2.5 points) Is the following polynomial stable?

$$s^4 + s^3 + 3s^2 + 2s + 1$$

(b) (2.5 points) How many eigenvalues in the right-half plane does the following polynomial have?

$$s^4 + s^3 + 2s^2 + s + 5$$

a)

$$\begin{array}{llll} s^4: & 1 & 3 & 1 \\ s^3: & 1 & 2 & \\ s^2: & 1 & 1 & \\ s^1: & 1 & & \\ s^0: & 1 & & \end{array}$$

Stable

b)

$$\begin{array}{llll} s^4: & 1 & 2 & 5 \\ s^3: & 1 & 1 & \\ s^2: & 1 & 5 & \\ s^1: & -4 & & \\ s^0: & 5 & & \end{array}$$

2 RHP

Problem 6. (4 points)

Find the step response of the transfer function

$$H(s) = \frac{s+2}{(s+1)(s+3)}$$

$$\mathcal{L}^{-1}\left\{H(s)\cdot\frac{1}{s}\right\} \equiv \text{step response.}$$

$$H(s)\cdot\frac{1}{s} = \frac{s+2}{s(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$A = \left(\cancel{s}\cdot\frac{H(s)}{\cancel{s}}\right)_{s=0} = \frac{2}{3}$$

$$B = \left((s+1)\frac{H(s)}{s}\right)_{s=-1} = \frac{1}{2\cdot(-1)} = \frac{-1}{2}$$

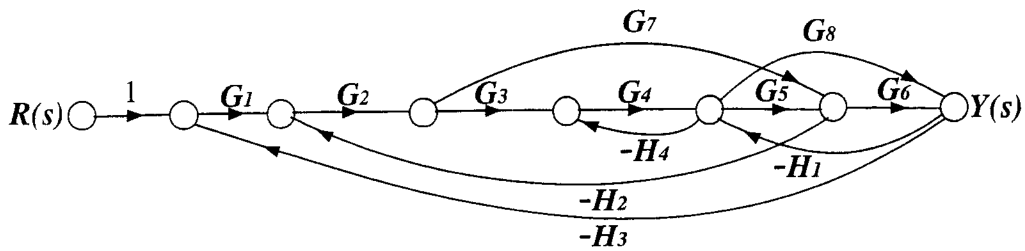
$$C = \left((s+3)\frac{H(s)}{s}\right)_{s=-3} = \frac{-1}{-2\cdot(-3)} = \frac{-1}{6}$$

$$\frac{H(s)}{s} = \frac{2}{3}\frac{1}{s} - \frac{1}{2}\frac{1}{s+1} - \frac{1}{6}\frac{1}{s+3}$$

$$\mathcal{L}^{-1}\left\{\frac{H(s)}{s}\right\} = \left(\frac{2}{3} - \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t}\right)1(t)$$

Problem 7. (Extra credit)

Find the transfer function of the following system, represented by its signal-flow graph.



FORWARD PATHS:

$F_1 = G_1 G_2 G_3 G_4 G_5 G_6$

$F_2 = G_1 G_2 G_7 G_6$

$F_3 = G_1 G_2 G_3 G_4 G_8$

LOOP PATHS:

$L_1 = G_4 H_4$, $L_2 = G_8 H_1$, $L_3 = G_2 G_7 H_2$, $L_4 = G_2 G_3 G_4 G_5 H_2$

$L_5 = G_1 G_2 G_7 G_6 H_3$, $L_6 = G_1 G_2 G_3 G_4 G_8 H_3$, $L_7 = G_1 G_2 G_3 G_4 G_5 G_6 H_3$

$L_8 = G_5 G_6 H_1$

$\Delta = 1 + L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8 + L_1 L_3 + L_1 L_5 + L_3 L_2$

$\Delta_1 = 1$, $\Delta_2 = L_1$, $\Delta_3 = 1$

$$G(s) = \frac{F_1 \Delta_1 + F_2 \Delta_2 + F_3 \Delta_3}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 G_5 G_6 + G_5 H_4 G_1 G_2 G_7 G_6 + G_1 G_2 G_3 G_4 G_8}{1 + G_4 H_4 + G_8 H_1 + G_2 G_7 H_2 + G_2 G_3 G_4 G_5 H_2 + G_1 G_2 G_7 G_6 H_3 + G_1 G_2 G_3 G_4 G_8 H_3 + G_1 G_2 G_3 G_4 G_5 G_6 H_3 + G_5 G_6 H_1 + G_4 H_4 G_2 G_7 H_2 + G_4 H_4}$$

$\times G_1 G_2 G_7 G_6 H_3 + G_8 H_1 G_2 G_7 H_2$