MAE 141A

LINEAR CONTROL

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MIDTERM EXAM

October 30, 2001

NAME: SOLUTIONS

- Open books and notes.
- Present your reasoning and calculations clearly. Random or inconsistent etchings will not be graded.
- Write only on the paper provided. If you run out of space for a given problem, continue on the pages at the end of the set and indicate "Continued on page X."
- The problems are *not* ordered by difficulty.
- Total points: 30.
- Time: 2:20–3:40 (2.7 minutes/point)

Problem 1. (8 points)

For the time function

$$x(t) = 5e^{-2t} - 3t\,, \qquad t \geq 0$$

(a) Find the Laplace transform $X(s) = \mathcal{L}\{x(t)\}$.

(b) Based on point (a), and the initial value theorem, find $x(0)$.

(c) Based on points (a) and (b) and the theorem about the Laplace transform of a derivative of a time function, find $\mathcal{L}\{\dot{x}(t)\}$.

(a) $$X(s) = \frac{5}{s+2} - \frac{3}{s^2}$$

(b) $$x(0) = \lim_{s\to\infty} sX(s)$$

$$= \lim_{s\to\infty}\left[\frac{5s}{s+2} - \underbrace{\frac{3}{s}}_{0}\right]$$

$$= \lim_{s\to\infty} 5\underbrace{\frac{s}{s+2}}_{1} = 5$$

(c) $$\mathcal{L}\{\dot{x}(t)\} = s\,\mathcal{L}\{x(t)\} - x(0)$$

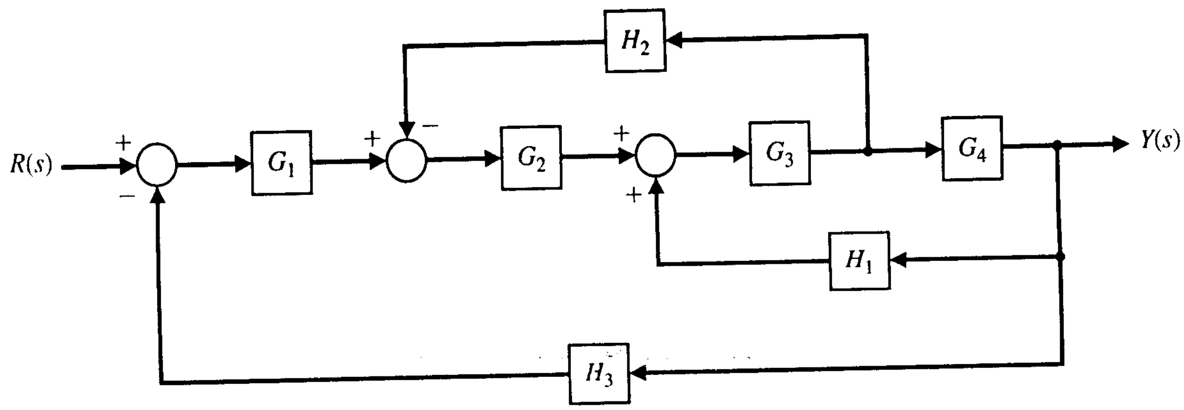
$$= s\left[\frac{5}{s+2} - \frac{3}{s^2}\right] - 5$$

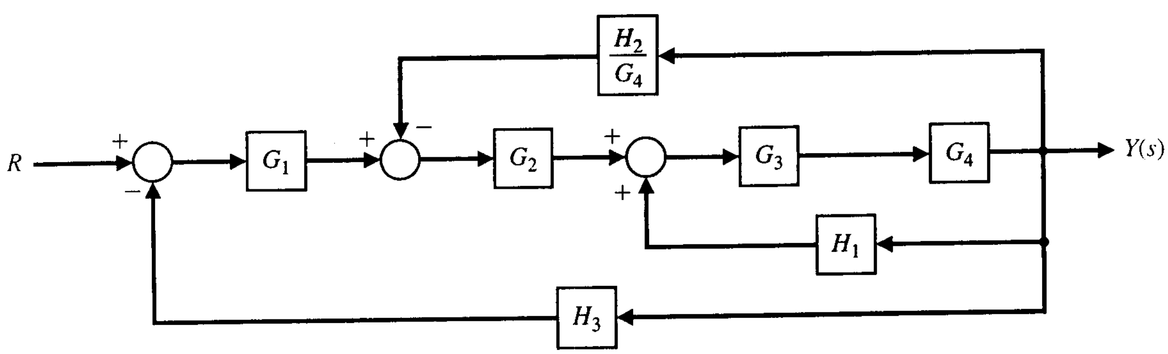
$$= \frac{5s}{s+2} - \frac{3}{s} - 5$$

$$= -\frac{13s+6}{s(s+2)}$$

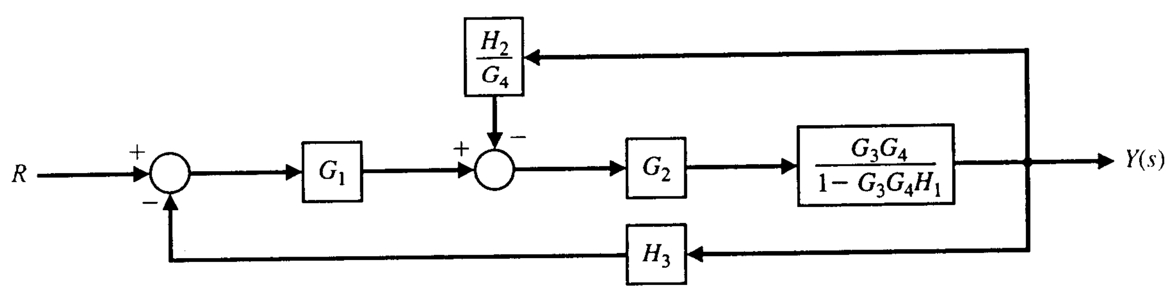
Problem 2. (10 points)

Find the closed-loop transfer function for the system

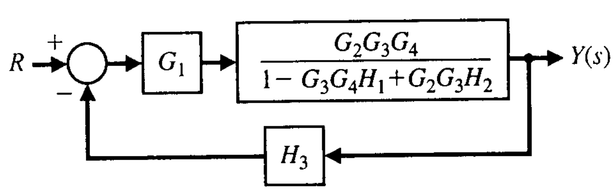




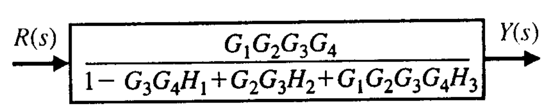
(a)



(b)



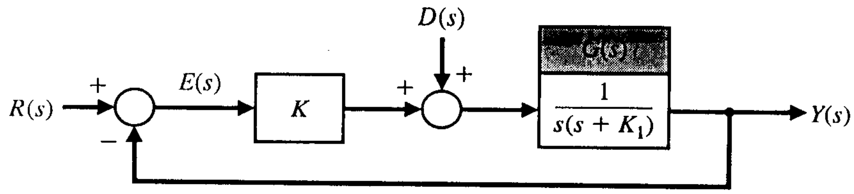
(c)



(d)

Problem 3. (12 points)

The Hubble space telescope control system has the following structure



where $Y(s)$ is the pointing angle and $R(s)$ is the commanded angle.

(a) Let the disturbance $D(s)$ be a unity step input and let $R(s) = 0$. Calculate the steady state value of the output, y_{ss}, as a function of the general gains K and/or K_1.

(b) Find the values of K and K_1 to achieve an overshoot of only 10% and a peak time of 2 seconds with respect to a unity step command in $R(s)$ (for zero disturbance).

(a) $$\frac{Y(s)}{D(s)} = \frac{G(s)}{1+KG(s)}$$

$$y_{ss} = \lim_{s\to 0} \cancel{s} \frac{G(s)}{1+KG(s)} \frac{1}{\cancel{s}}$$

$$= \lim_{s\to 0} \frac{1}{s^2+K_1 s+K} = \frac{1}{K}$$

(b) $$\frac{Y(s)}{R(s)} = \frac{KG(s)}{1+KG(s)} = \frac{K}{s^2 + \underbrace{K_1 s}_{2\zeta\omega_n} + \underbrace{K}_{\omega_n^2}},$$

$$\underbrace{\qquad\qquad}_{4\zeta^2 K = K_1^2}$$

We want

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 2 \text{ sec}$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.1$$

From $M_p(\xi)$ we have:

$$(\ln 10)^2(1-\xi^2) = \pi^2\xi^2$$

$$\rightarrow \xi^2 = \frac{(\ln 10)^2}{(\ln 10)^2+\pi^2} = \frac{K_1^2}{4K}$$

From $t_p(\xi,\omega_n)$ we have:

$$\frac{\pi^2}{4K} = 1-\xi^2 = 1-\frac{K_1^2}{4K}$$

$$= \frac{\pi^2}{(\ln 10)^2+\pi^2}$$

$$\rightarrow \boxed{K = \frac{(\ln 10)^2+\pi^2}{4}}$$

$$\rightarrow K_1^2 = 4K\frac{(\ln 10)^2}{(\ln 10)^2+\pi^2} = (\ln 10)^2$$

$$\Rightarrow \boxed{K_1 = \ln 10}$$

$K = 3.8 \qquad K_1 = 2.3$