MAE 143B
LINEAR CONTROL

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MIDTERM

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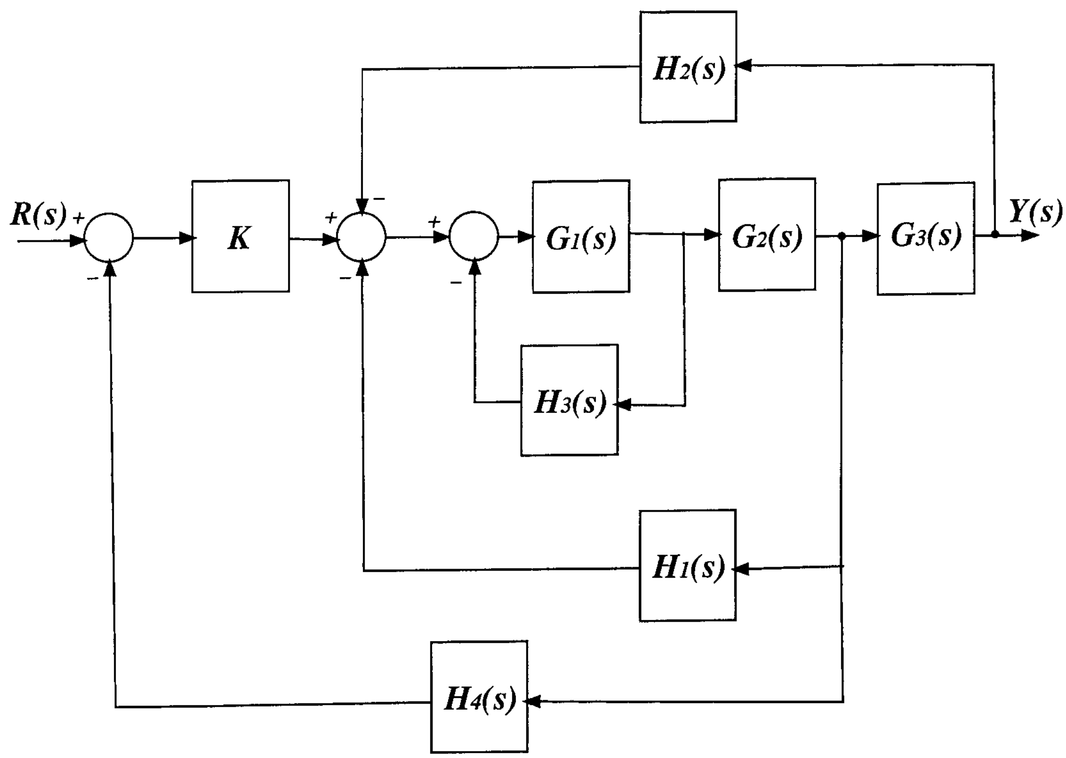
NAME: SOLUTIONS

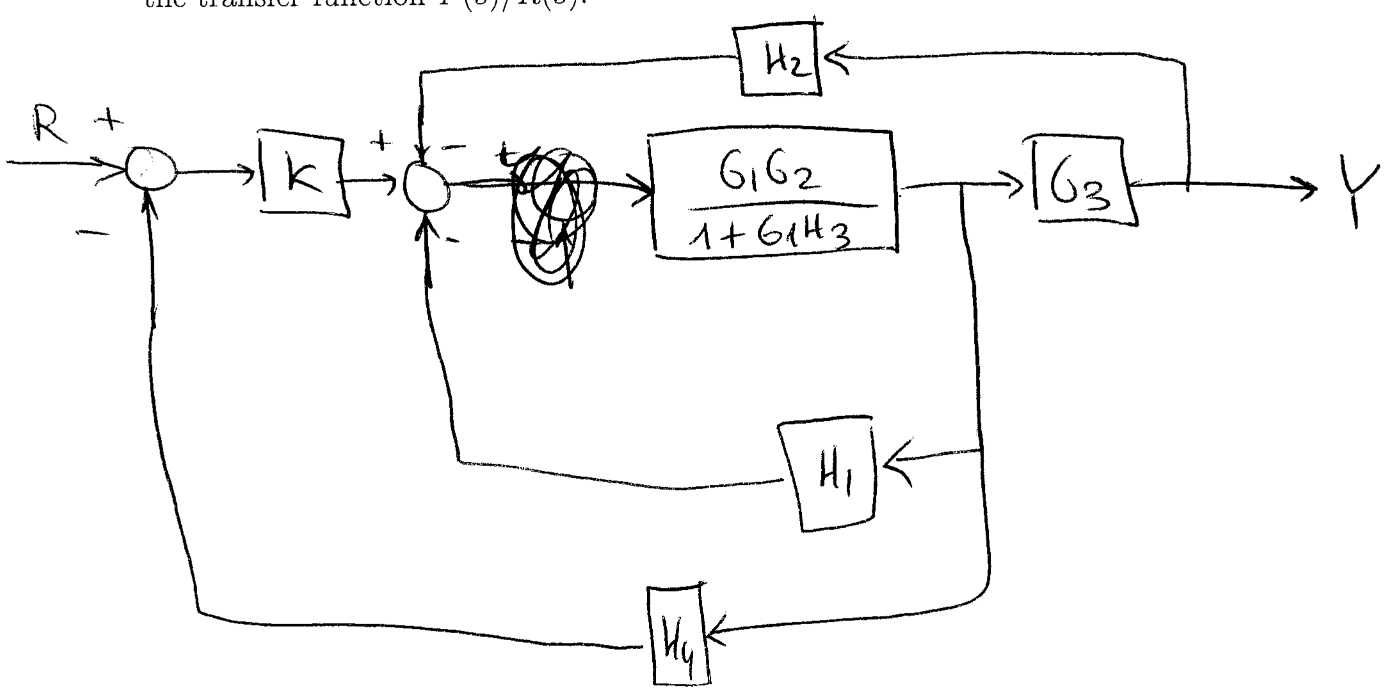
- Open books and notes.
- Present your reasoning and calculations clearly. Random or inconsistent etchings will not be graded.
- Write only on the paper provided. If you run out of space for a given problem, continue on the pages at the end of the set and indicate "Continued on page X."
- The problems are *not* ordered by difficulty.
- Total points: 25.
- Time: 1 hour.

Problem 1. (5 points)

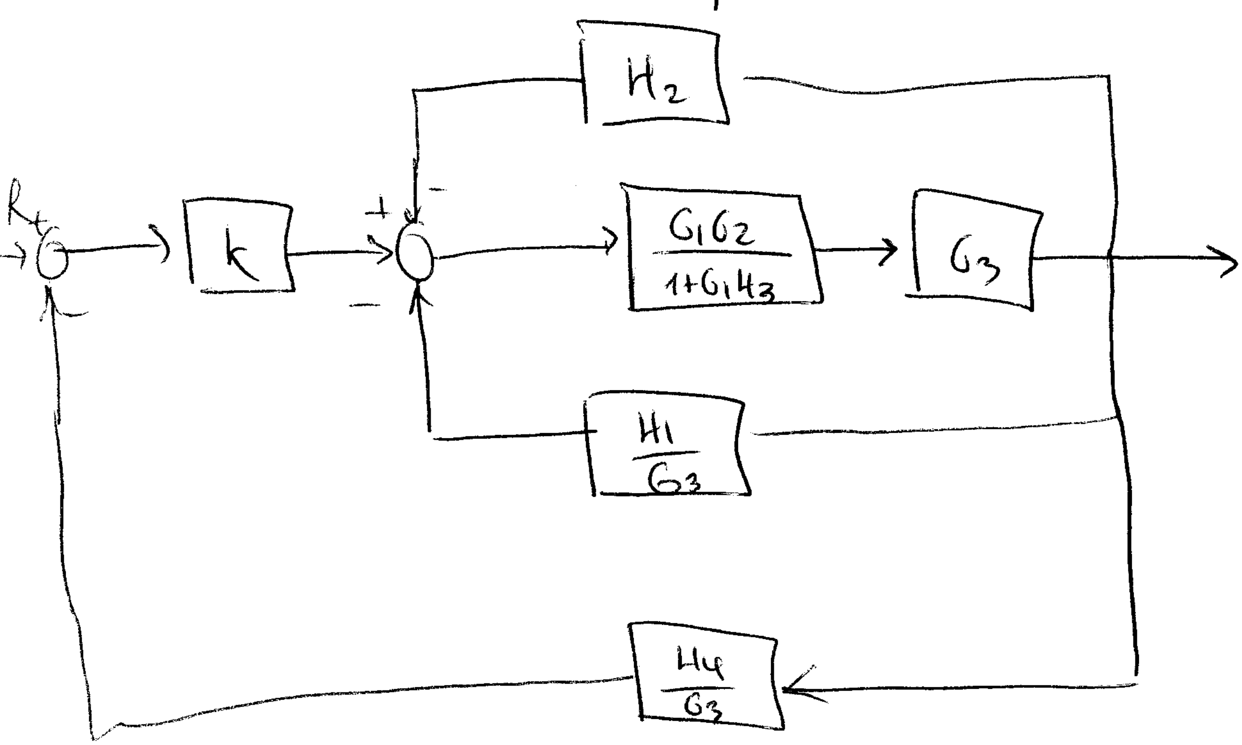
A ship steering system is represented in the following block diagram.

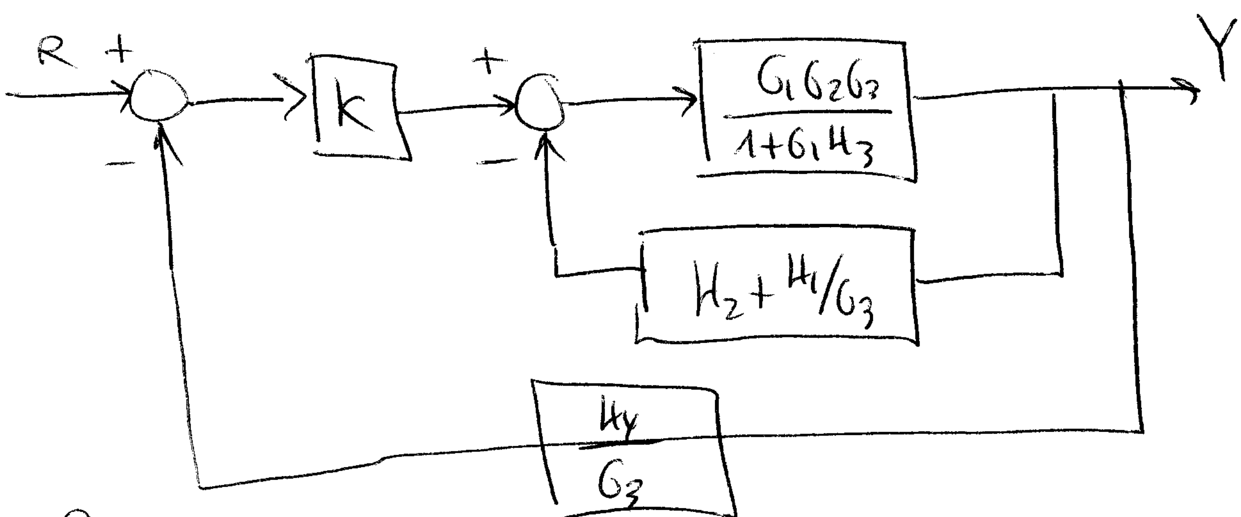


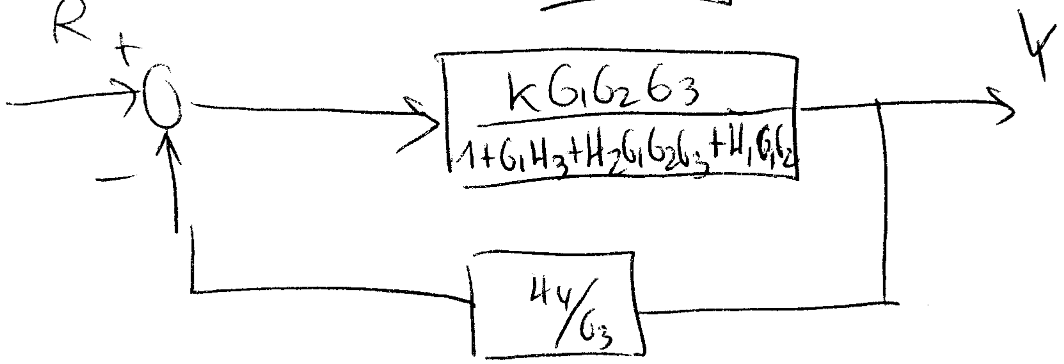
In the diagram, $Y(s)$ is the ship's course and $R(s)$ is the desired course (reference). Find the transfer function $Y(s)/R(s)$.



move G_3 to the left:



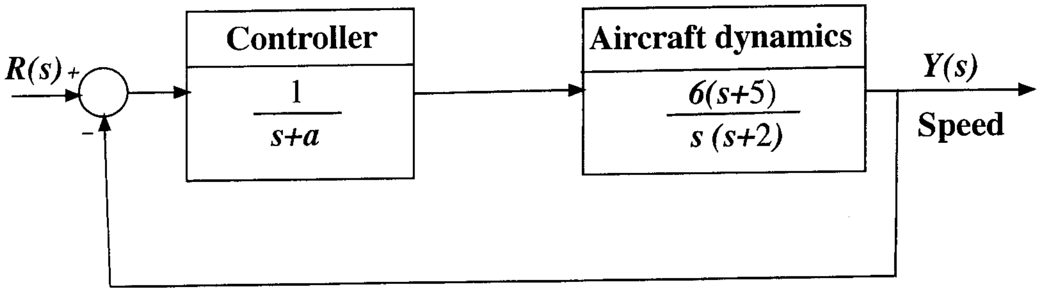




$$Y/R = \frac{KG_1G_2G_3}{1+G_1H_3+H_2G_1G_2G_3+H_1G_1G_2+H_4KG_1G_2}$$

Problem 2. (5 points)

The following feedback system is used to control the speed of an aircraft.



Find the sensitivity of the closed-loop transfer function $T(s)$ to a small change in the parameter a.

$$T(s) = \frac{\frac{1}{s+a}\,\frac{6(s+5)}{s(s+2)}}{1 + \frac{1}{s+a}\,\frac{6(s+5)}{s(s+2)}} = \frac{6(s+5)}{s(s+a)(s+2) + 6(s+5)}$$

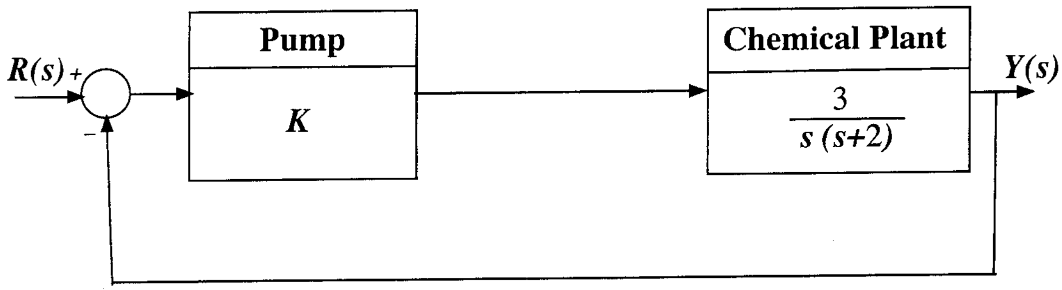
$$S_a^T = \frac{a}{T}\frac{dT}{da}\,;\quad \frac{dT}{da} = \frac{-6(s+5)s(s+2)}{\left(s(s+a)(s+2)+6(s+5)\right)^2}$$

$$S_a^T = \frac{a}{\frac{6(s+5)}{s(s+a)(s+2)+6(s+5)}}\;\frac{-6(s+5)s(s+2)}{\left(s(s+a)(s+2)+6(s+5)\right)^2}$$

$$= \frac{-a\cdot s(s+2)}{s(s+a)(s+2)+6(s+5)}$$

Problem 3. (5 points)

An automatic system for controlling the concentration of a certain chemical product is shown in the figure.



(a) (3 points) Determine K so that overshoot is 10%.

(b) (2 points) What are the settling time and the peak time for K determined in (a)?

$$T = \frac{\frac{3K}{s(s+2)}}{1 + \frac{3K}{s(s+2)}} = \frac{3K}{s^2+2s+3K}$$

$$\omega_n^2 = 3K \Rightarrow \omega_n = \sqrt{3K} \quad ; \quad 2\zeta\omega_n = 2 \Rightarrow \zeta = \frac{1}{\omega_n} = \frac{1}{\sqrt{3K}}$$

a) $$M_p = 0.1 \Rightarrow e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.1$$

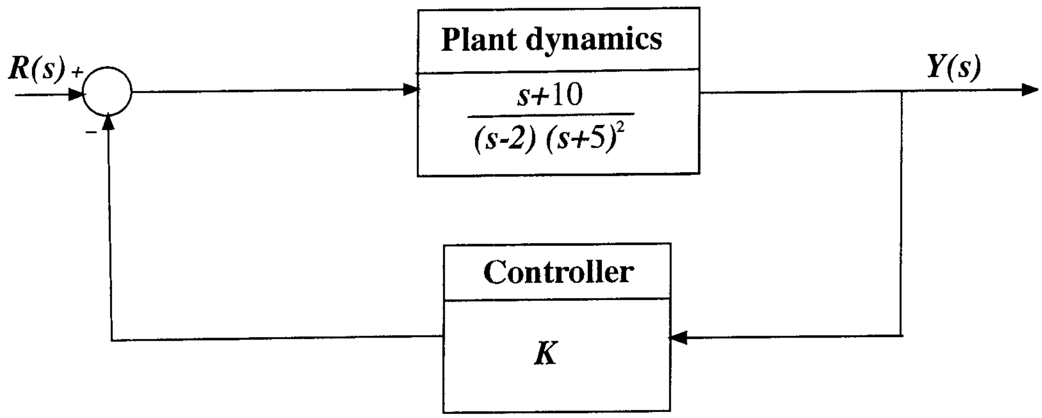
$$\Rightarrow \frac{-\pi\zeta}{\sqrt{1-\zeta^2}} = \ln 0.1 \Rightarrow \zeta = \sqrt{\frac{\left(-\frac{\ln 0.1}{\pi}\right)^2}{1+\left(\frac{\ln 0.1}{\pi}\right)^2}} = 0.591$$

$$K = \frac{1}{3(0.591)^2} = 0.954$$

b) $$t_s = \frac{4.6}{\zeta\omega_n} = 4.6\ s \quad , \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 2.3\ s$$

Problem 4. (5 points)

The plant in the figure is unstable and needs feedback to be stabilized.



Determine the range of possible gains K for stable operation.

$$T = \frac{\frac{s+10}{(s-2)(s+5)^2}}{1 + k\frac{s+10}{(s-2)(s+5)^2}} = \frac{s+10}{\underbrace{(s-2)(s+5)^2 + k(s+10)}_{s^3+8s^2+(5+k)s-50+10k}}$$

Routh's Criterion:

s^3: 1 $\quad 5+k$

s^2: 8 $\quad 10k-50$

s^1: $\frac{40+8K-10k+50}{8}$

s^0: $10k-50$

s^1: $\frac{90-2K}{8} > 0 \Rightarrow 90-2k>0 \Rightarrow 2k<90 \Rightarrow k<45$

s^0: $10k-50>0 \Rightarrow k>5$

$\Rightarrow \underline{\underline{k \in (5, 45)}}$

Problem 5. (5 points)

Are the following polynomials stable? If not, how many eigenvalues in the right-half plane do they have?

(a) (1 point)

$$p_1(s) = s^3 + 2s^2 + 2s - 1$$

(b) (2 points)

$$p_2(s) = s^4 + 2s^3 + 3s^2 + 2s + 3$$

(c) (2 points)

$$p_3(s) = s^4 + 4s^3 + 3s^2 + 2s + 1$$

a)
$$\begin{array}{lll} s^3 : & 1 & 2 \\ s^2 : & 2 & -1 \\ s^1 : & 5/2 & \\ s^0 : & -1 & \end{array}$$

1 RHP poles

b)
$$\begin{array}{llll} s^4 : & 1 & 3 & 3 \\ s^3 : & 2 & 2 & \\ s^2 : & 2 & 3 & \\ s^1 : & -1 & & \\ s^0 : & 3 & & \end{array}$$

2 RHP poles

c)
$$\begin{array}{llll} s^4 & 1 & 3 & 1 \\ s^3 & 4 & 2 & \\ s^2 & 5/2 & 1 & \\ s^1 & 2/5 & & \\ s^0 & 1 & & \end{array}$$

stable