MAE 143B
LINEAR CONTROL

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MIDTERM

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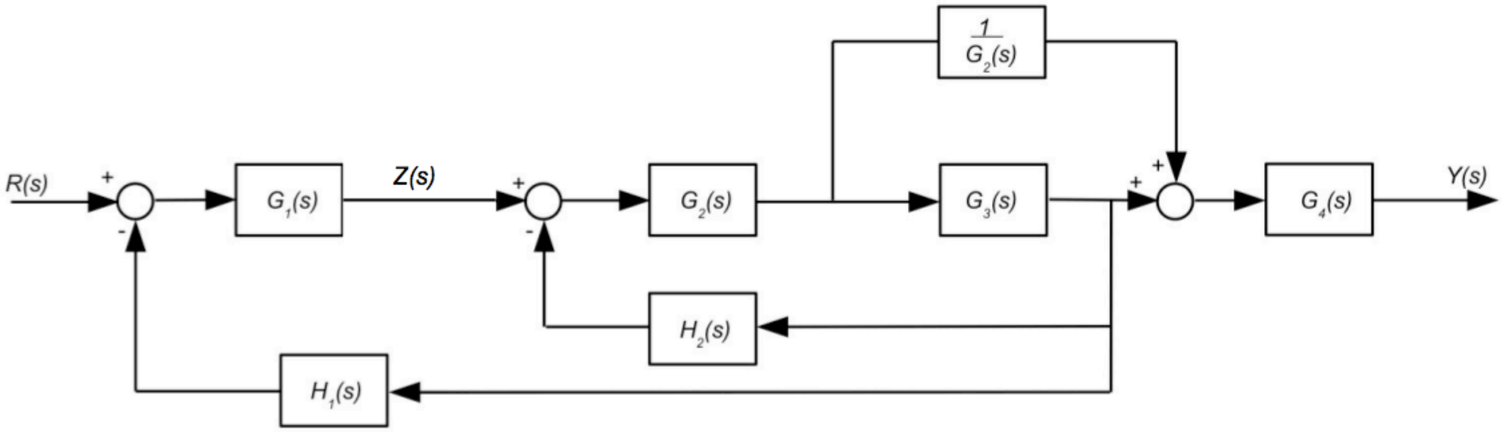
NAME: SOLUTIONS

- Open books and notes.
- Present your reasoning and calculations clearly. Random or inconsistent etchings will not be graded.
- Write only on the paper provided. If you run out of space for a given problem, continue on the pages at the end of the set and indicate "Continued on page X."
- The problems are *not* ordered by difficulty.
- Total points: 30.
- Time: 1 hour 30 minutes.

Problem 1. (6 points)

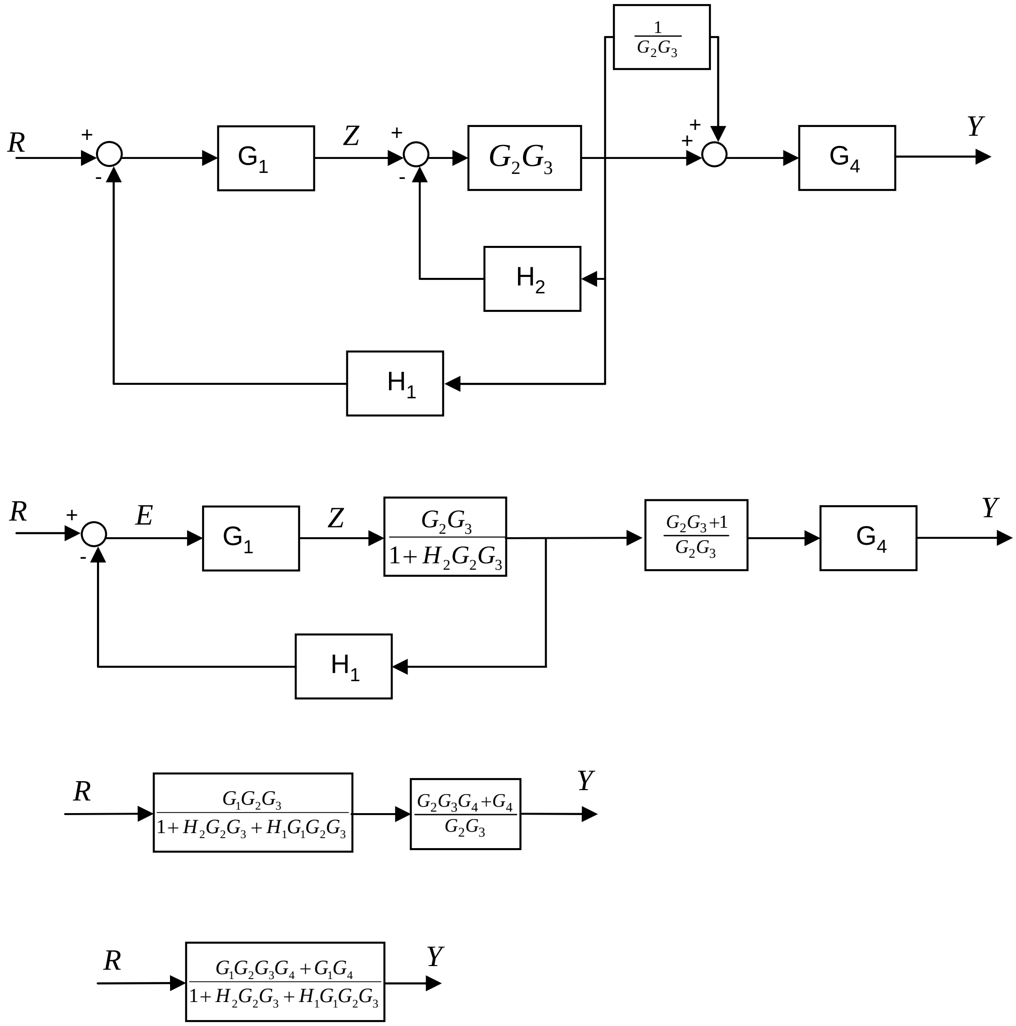


Using block diagram reduction techniques, find the following transfer functions:

(a) (4 point) $\dfrac{Y(s)}{R(s)}$

(b) (2 points) $\dfrac{Z(s)}{R(s)}$

Part (a)

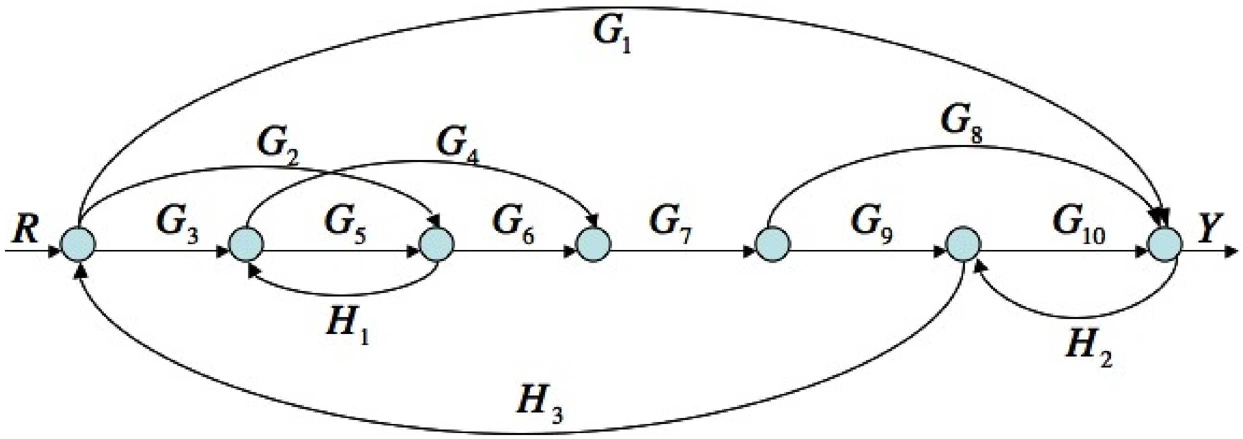


Part (b)

$$\frac{Z}{R} = G_1 \frac{E}{R} = \frac{G_1(1 + H_2 G_2 G_3)}{1 + H_2 G_2 G_3 + H_1 G_1 G_2 G_3}$$

Problem 2. (6 points)

Using Mason's rule, find the transfer function $\dfrac{Y}{R}$ in the following system:



Problem 2 See Signal Flow Graph

Forward Path 1	G1	Δ_1	1-G5H1
Forward Path 2	G3G5G6G7G9G10	Δ_2	1
Forward Path 3	G3G5G6G7G8	Δ_3	1
Forward Path 4	G3G4G7G8	Δ_4	1
Forward Path 5	G3G4G7G9G10	Δ_5	1
Forward Path 6	G2G6G7G9G10	Δ_6	1
Forward Path 7	G2G6G7G8	Δ_7	1
Forward Path 8	G2H1G4G7G8	Δ_8	1
Forward Path 9	G2H1G4G7G9G10	Δ_9	1

Loop 1	G1H2H3
Loop 2	G10H2
Loop 3	G8H2H3G3G5G6G7
Loop 4	G8H2H3G3G4G7
Loop 5	G8H2H3G2H1G4G7
Loop 6	G8H2H3G2G6G7
Loop 7	G5H1
Loop 8	G3G5G6G7G9H3
Loop 9	G2G6G7G9H3
Loop 10	G3G4G7G9H3
Loop 11	G2H1G4G7G9H3

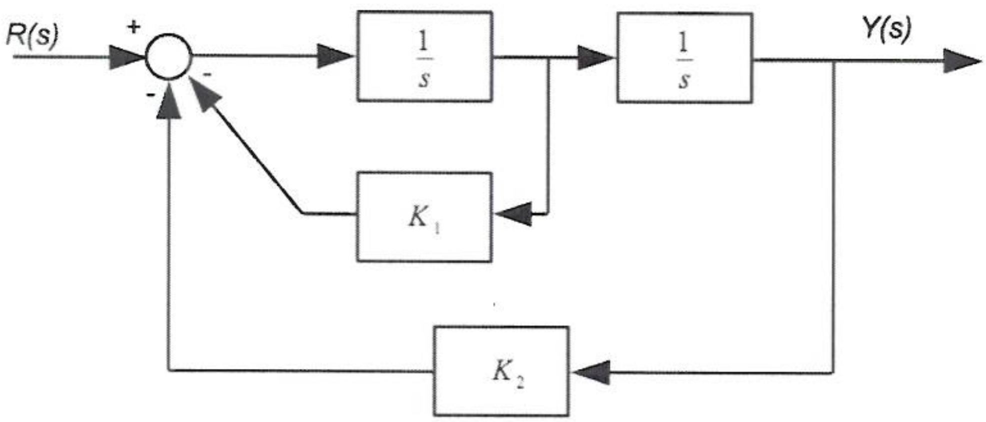
Loop 1 doesn't touch loop 7, Loop 2 doesn't touch loop 7

Transfer Function =

$$\frac{\mathrm{FP1}(\Delta 1)+\mathrm{FP2}(\Delta 2)+\ldots+\mathrm{FP9}(\Delta 9)}{1-(\sum_1^{11} Loop\ \_i)+Loop\,7(Loop\,1+Loop\,2)}$$

Problem 3. (6 points)

Consider the following feedback system



(a) (4 points) Find K_1 and K_2 such that the overshoot is approximately 17% and the settling time is 3 seconds.

(b) (2 points) What is the peak time and the rise time for K_1 and K_2 determined in (a)?

$$\frac{Y}{R} = \frac{1}{s^2 + K_1 s + K_2} \longrightarrow \begin{array}{l} K_1 = 2\zeta\omega_n \\ K_2 = \omega_n^2 \end{array} \qquad \begin{array}{l} M_p = 0.17 \\ t_s = 3 \end{array}$$

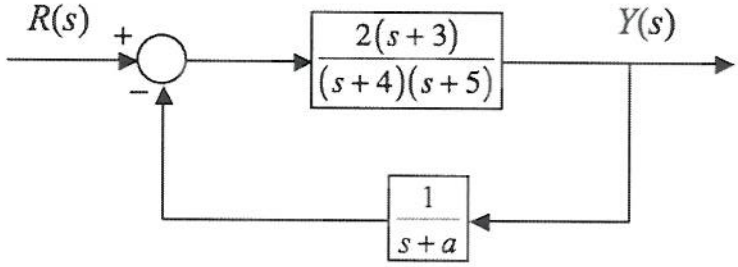
$$M_p = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}} \longrightarrow \zeta = \frac{\frac{|\ln M_p|}{\pi}}{\sqrt{1+\left(\frac{\ln M_p}{\pi}\right)^2}} = 0.4913$$

$$t_s = \frac{4.6}{\zeta\omega_n} \longrightarrow \boxed{K_1 = \frac{9.2}{t_s} = 3.067}$$

$$\omega_n = \frac{K_1}{2\zeta} = 3.12 \longrightarrow \boxed{K_2 = \omega_n^2 = 9.74}$$

$$\boxed{t_r = \frac{1.8}{\omega_n} = 0.577} \qquad \boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.1561}$$

Problem 4. (6 points)



Find the sensitivity of the closed-loop transfer function $T(s)$ to a small change in the parameter a.

$$\frac{Y}{R}(s) = T(s) = \frac{\dfrac{2(s+3)}{(s+4)(s+5)}}{1+\dfrac{1}{s+a}\,\dfrac{2(s+3)}{(s+4)(s+5)}} = \frac{2(s+3)(s+a)}{(s+a)(s+4)(s+5)+2(s+3)}$$

$$S_a^T = \frac{a}{T}\,T'(a)$$

$$T'(a) = \frac{2(s+3)}{(s+a)(s+4)(s+5)+2(s+3)} - \frac{2(s+3)(s+a)(s+4)(s+5)}{\left[(s+a)(s+4)(s+5)+2(s+3)\right]^2}$$

$$= \frac{\left(2(s+3)\right)^2}{\left[(s+a)(s+4)(s+5)+2(s+3)\right]^2} = \left(\frac{T(s)}{s+a}\right)^2$$

$$S_a^T = \frac{a}{(s+a)^2}\,T(s) = \frac{2(s+3)\,a}{(s+a)\left[(s+a)(s+4)(s+5)+2(s+3)\right]}$$

Problem 5. (5 points)

Consider the following step response:

$$y(t) = 6\left(1 - e^{-\frac{1}{10}(t-2)}\right) 1(t-2)\,.$$

Find the PID controller parameters (K, T_I, T_D) using the Ziegler-Nichols transient-response method with a 0.25 decay ratio (Method 1).

$$Y(s) = 6\left(\frac{1}{s} - \frac{1}{s+\frac{1}{10}}\right)e^{-2s}$$

$$= \frac{6e^{-2s}}{10s+1} \cdot \frac{1}{s}$$

So $A = 6$, $\tau = 10$, $t_d = 2$

From ZN table for Method 1,

$$K = 1.2\frac{\tau}{A t_d} = 1$$

$$T_I = 2t_d = 4$$

$$T_D = 0.5t_d = 1$$

Problem 6. (7 points)

Consider the transfer function

$$G(s) = \frac{2s^2 + 2s + 1}{s^4 + 2s^3 + s^2 + 2s + 2}$$

(a) (3 point) Show that $G(s)$ is unstable and determine the number of its unstable poles.

(b) (4 points) Close the loop around $G(s)$ with a gain K and determine the range of K for which the closed-loop system is asymptotically stable.

(a)

$$\begin{array}{c|ccc}
s^4 & 1 & 1 & 2 \\
s^3 & 2 & 2 & \\
\hline
s^2 & 0 & 2 & \leftarrow \text{zero element in first column} \\
s^2 & \epsilon & 2 & \\
s^1 & 2-\frac{4}{\epsilon} \to -\infty & \text{as } \epsilon \to 0^+ & \\
s^0 & 2 & &
\end{array}$$

One sign change $\to$ two unstable poles

(b)

$$\frac{G(s)}{1+KG(s)} = \frac{2s^2+2s+1}{s^4+2s^3+(2K+1)s^2+(2K+2)s+K+2}$$

$$\begin{array}{c|ccc}
s^4 & 1 & 2K+1 & 2+K \\
s^3 & 2 & 2K+2 & \\
\hline
s^2 & K & K+2 & \\
s^1 & \frac{2K^2-4}{K} & & \\
s^0 & 2+K & &
\end{array}$$

$\Longrightarrow$ Asymp. stability condition:

$$\boxed{K > \sqrt{2}}$$

(Dominates the other conditions, $K>-2$, $K>0$.)