Problem 1. (2+2+2+3+3 points) Draw the root locus for the five systems below. For each of the sub-problems, if proportional feedback (of any size of the gain $K$—small, medium, or large) is not sufficient to stabilize the system, namely, to place all the closed-loop poles in the left half plane, design a compensator $D(s)$ to stabilize the system and draw the root locus with respect to $K$ for the equation $1 + KD(s)G(s) = 0$.

(a) $G(s) = \frac{1}{s(s+5)}$  
(b) $G(s) = \frac{s}{(s-1)^2}$  
(c) $G(s) = \frac{1}{(s^2 - 2s + 2)}$  
(d) $G(s) = \frac{s-2}{s}$  
(e) $G(s) = \frac{s-1}{s(s-2)}$

Problem 2. (6 points) A second order system is controlled with unity feedback, as shown on the right. Draw the root locus with respect to $\zeta$.

Problem 3. (6 points) Sketch the magnitude and phase Bode plots of $G(s) = \frac{s(s^2 + 100)}{(s-1)^2}$.

Problem 4. (6 points) From the approximate Bode plots on the next page, determine the system transfer function $G(s)$. In case you are curious about the true Bode plots in Matlab, they are given below.
**Problem 1**

(a) \[ G(s) = \frac{1}{s(s+5)} \]

rel. deg. = 2

\[ x = -\frac{5}{2} \]

stable for all \( K > 0 \)

(b) \[ G(s) = \frac{s}{(s-1)^2} \]

rel. deg. = 1

stable for suffic. high \( K \)

(c) \[ G(s) = \frac{1}{(s-1)^2 + 1^2} \]

rel. deg. = 2

unstable for all \( K > 0 \)
\[ D(\delta) = \frac{\delta + z}{\delta + p}, \quad \lambda = -\frac{p + 2 + z}{2} \]

We need \( p > z + 2 \). Let \( z = 0, p = 4 \).

\( \lambda = -1 \)

Stable for suffic. high \( K \)

\((d)\) \[ G(\delta) = \frac{\delta - 2}{\delta} \]

Unstable for all \( K > 0 \).

Take \( D(\delta) = \frac{\delta - 1}{\delta + 1} \)

Stable for suffic. small \( K > 0 \)
(e) \[ G(s) = \frac{s-1}{s(s-2)} \quad \text{rel. deg.} = 1 \]

Unstable for all \( K > 0 \)

Non-robust stabilizing compensator: \( D(s) = \frac{s+1}{s-1} \)

If the pole-zero cancellation \( \frac{s-1}{s-1} \) is not exact, then we get a branch of RL on the positive real axis.
Robust but trickier compensator:

\[ D(s) = \frac{(s+0.3)(s-0.1)}{(s+0.22)(s-3.5)} \]

This is one of the many possible speculative solutions that happens to actually work. The stabilizing range for \( K \) is \([5.52, 5.96]\).

Obviously, this solution is FYI only. It required Matlab to verify.
Problem 2

Char. poly: \( \delta^2 + 2\xi \omega_n \delta + \omega_n^2 + \omega_n^2 = 0 \)

\[
1 + \delta \frac{2\omega_n \delta}{\delta^2 + 2\omega_n^2} = 0
\]

Diagram:

\( \delta_{\text{crit}} = \sqrt{2} \)
Draw the Bode Diagram

\[ G(s) = \frac{3(s^2 + 100)}{(s - 1)^2} \]

for the system using straight line approximations.

- Zero at the origin
- Double pole
- Unstable double pole
- Resonant zero

**Problem 3**
**Problem 4**

\[ G(s) = \frac{10^8 s (s-1)^2}{(s+10^2)(s-10^4)^2} \]

1. The initial phase of 90° and a slope of 20°/decade indicates a zero at the origin.
2. At 10°, the magnitude response increases by 40 dB/decade while the phase decreases, indicating an unstable double zero.
3. At 10^2, the magnitude decreases by 20 dB/decade and the phase decreases, indicating a stable pole.
4. The magnitude decreases by 40 dB at 10^5 while the phase increases, unstable double pole.

5. To find K, let S be smaller than 10:

\[
K \left(10^{-1}\right) \left(10^{-1}\right)^2 = 10^{-3} \frac{(10^2)}{(10^2)(10^4)^2} \]

K = 10^8