MAE 143B Linear Control — Prof. M. Krstic

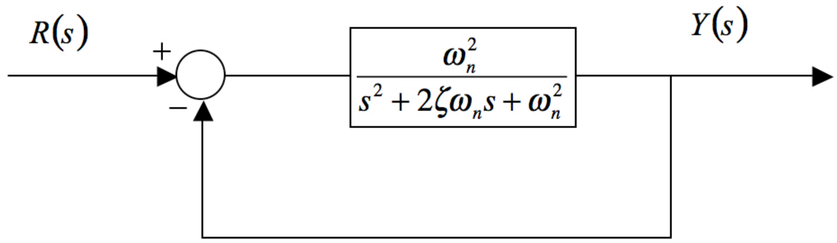
MIDTERM 2 — **SOLUTIONS** — May 27, 2008

Problem 1. (2+2+2+3+3 points) Draw the root locus for the five systems below. For each of the sub-problems, if proportional feedback (of any size of the gain *K*—small, medium, or large) is not sufficient to stabilize the system, namely, to place all the closed-loop poles in the left half plane, design a compensator $D(s)$ to stabilize the system and draw the root locus with respect to *K* for the equation $1+KD(s)G(s)=0$.

(a) $G(s)=\dfrac{1}{s(s+5)}$ (b) $G(s)=\dfrac{s}{(s-1)^2}$ (c) $G(s)=\dfrac{1}{(s^2-2s+2)}$

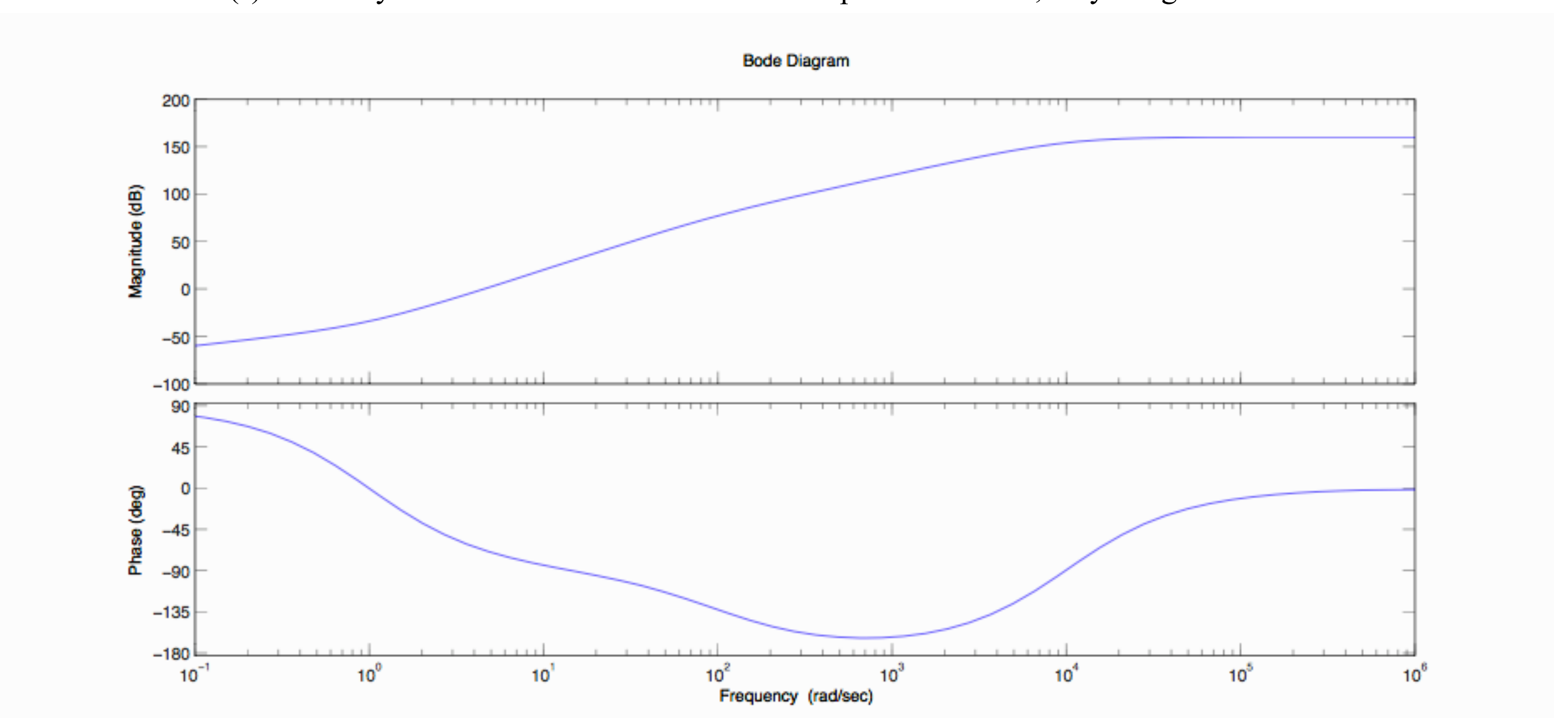
(d) $G(s)=\dfrac{s-2}{s}$ (e) $G(s)=\dfrac{s-1}{s(s-2)}$

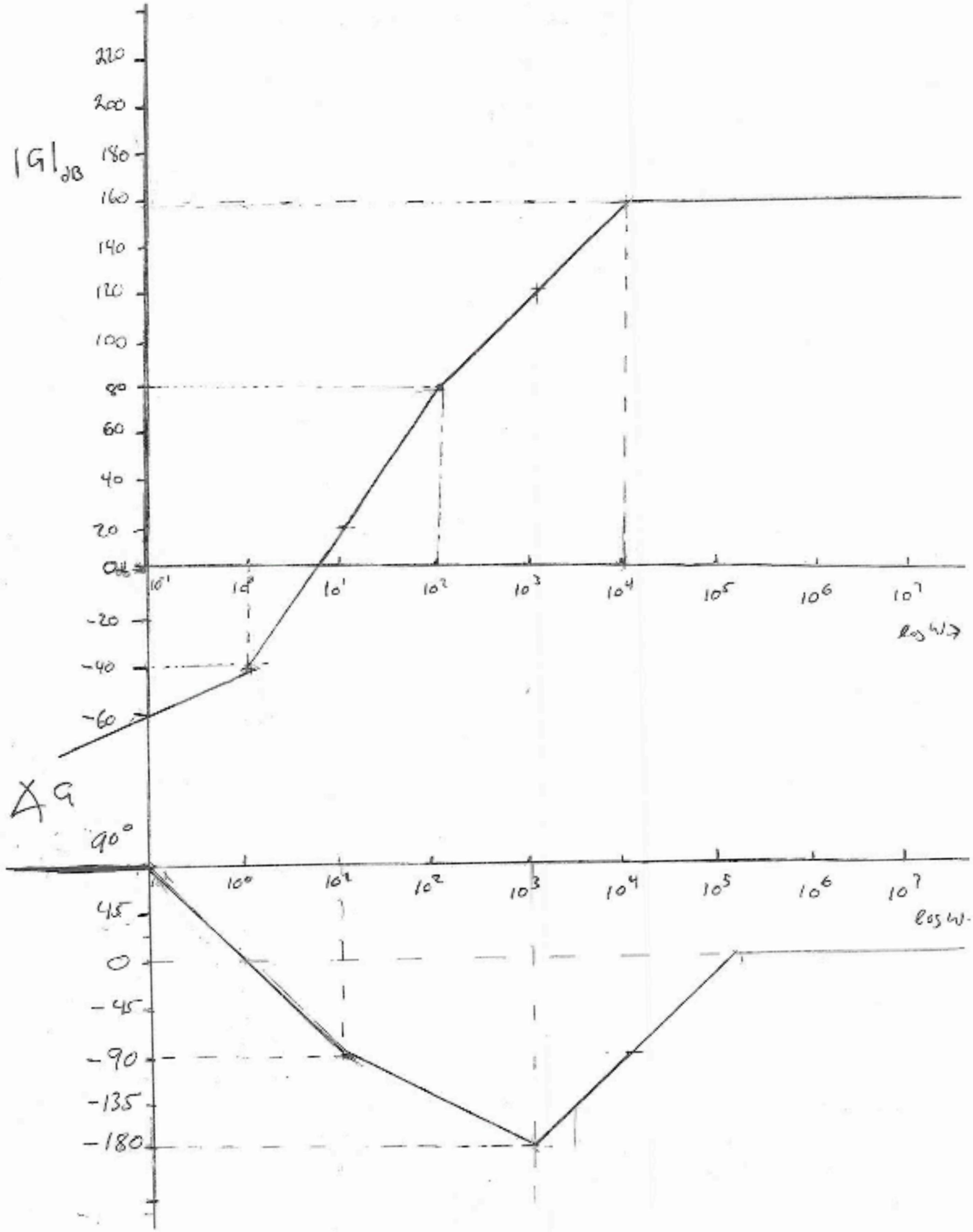
Problem 2. (6 points) A second order system is controlled with unity feedback, as shown on the right. Draw the root locus with respect to ζ.



Problem 3. (6 points) Sketch the magnitude and phase Bode plots of $G(s)=\dfrac{s(s^2+100)}{(s-1)^2}$.

Problem 4. (6 points) From the *approximate* Bode plots on the *next page*, determine the system transfer function *G*(*s*). In case you are curious about the true Bode plots in Matlab, they are given below.





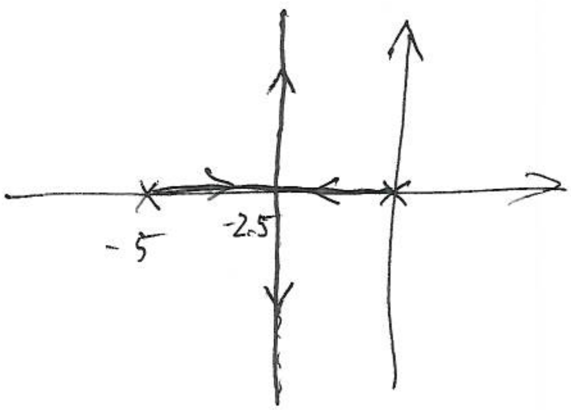
|G|dB
220
200
180
160
140
120
100
80
60
40
20
0dB
-20
-40
-60
10^-1
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
log ω →
∡G
90°
45
0
-45
-90
-135
-180
log ω

PROBLEM 1

(a) $G(s) = \dfrac{1}{s(s+5)}$

rel. deg. = 2

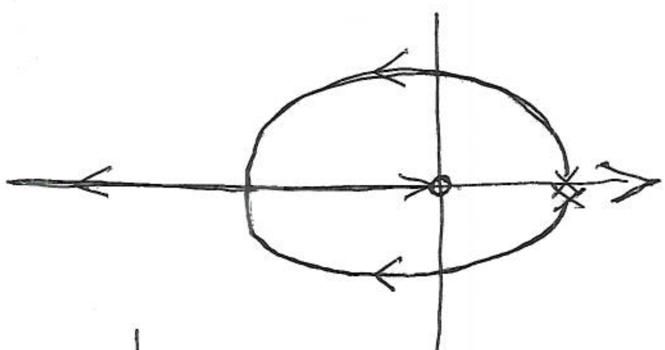
$\alpha = -\dfrac{5}{2}$



stable for all K > 0

(b) $G(s) = \dfrac{s}{(s-1)^2}$

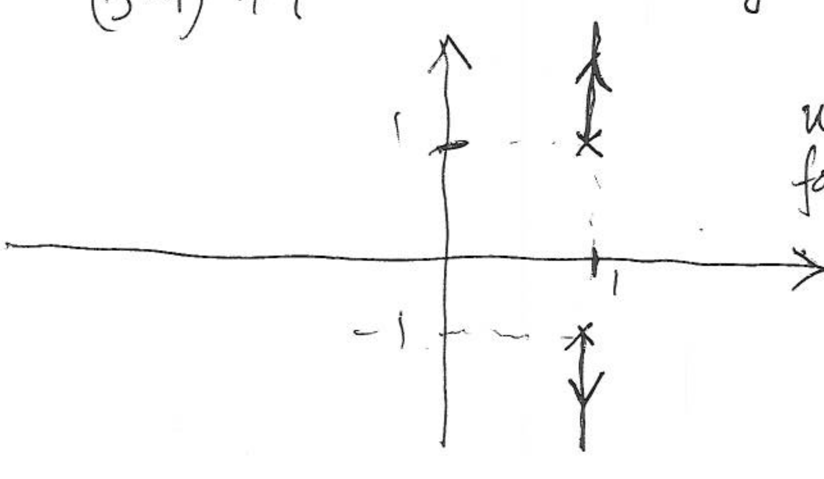
rel. deg. = 1



stable for suffic. high K

(c) $G(s) = \dfrac{1}{(s-1)^2+1^2}$

rel. deg. = 2

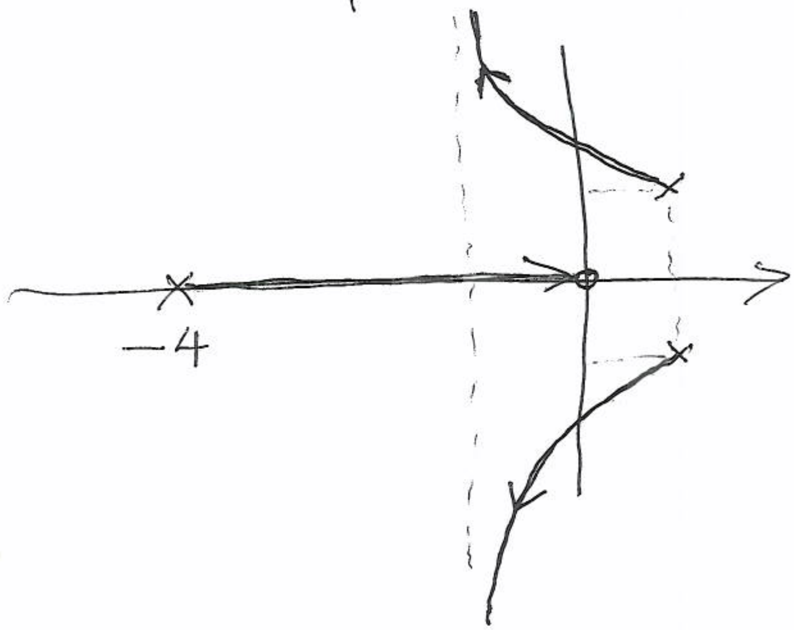


unstable for all K > 0

$D(s) = \frac{s+z}{s+p}$, $\quad \alpha = \frac{-p+2+z}{2}$

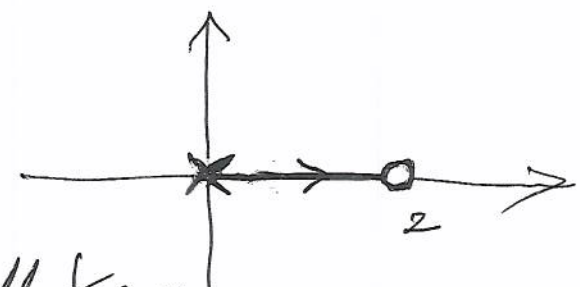
We need $p > z+2$. Let $z=0$, $p=4$.

$\alpha = -1$



Stable for suffic. high K

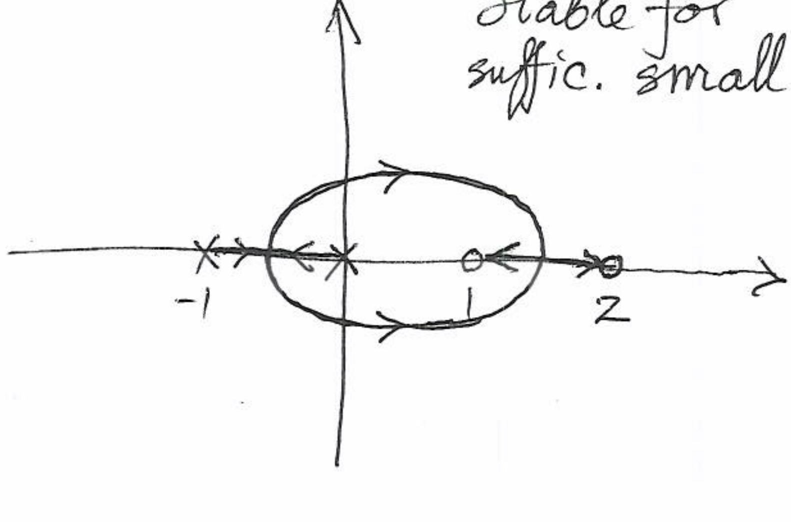
(d) $G(s) = \frac{s-2}{s}$



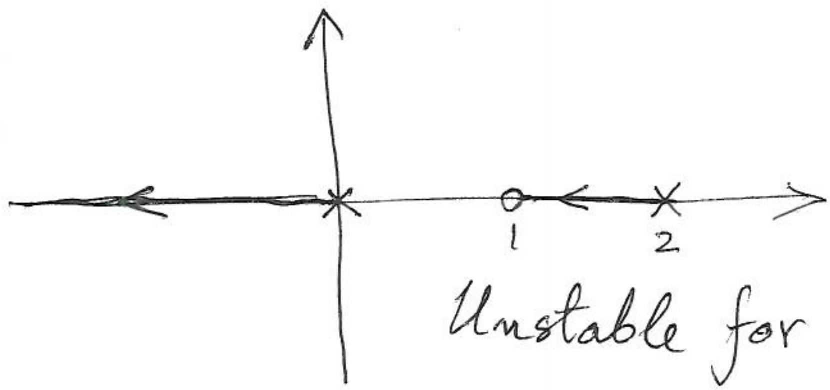
Unstable for all $K>0$.

Take $D(s) = \frac{s-1}{s+1}$

Stable for suffic. small $K>0$

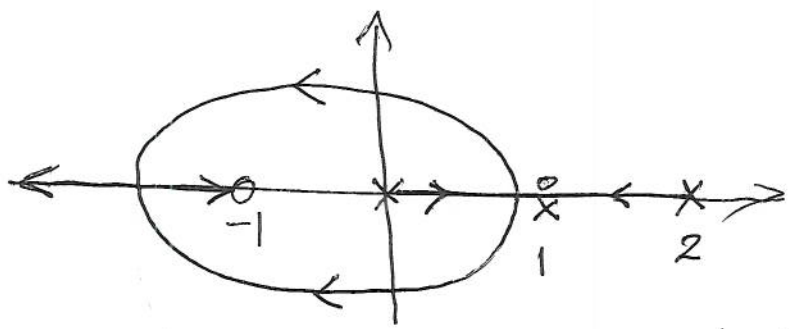


(e) $G(s) = \dfrac{s-1}{s(s-2)}$ rel. deg. = 1



Unstable for all $K > 0$

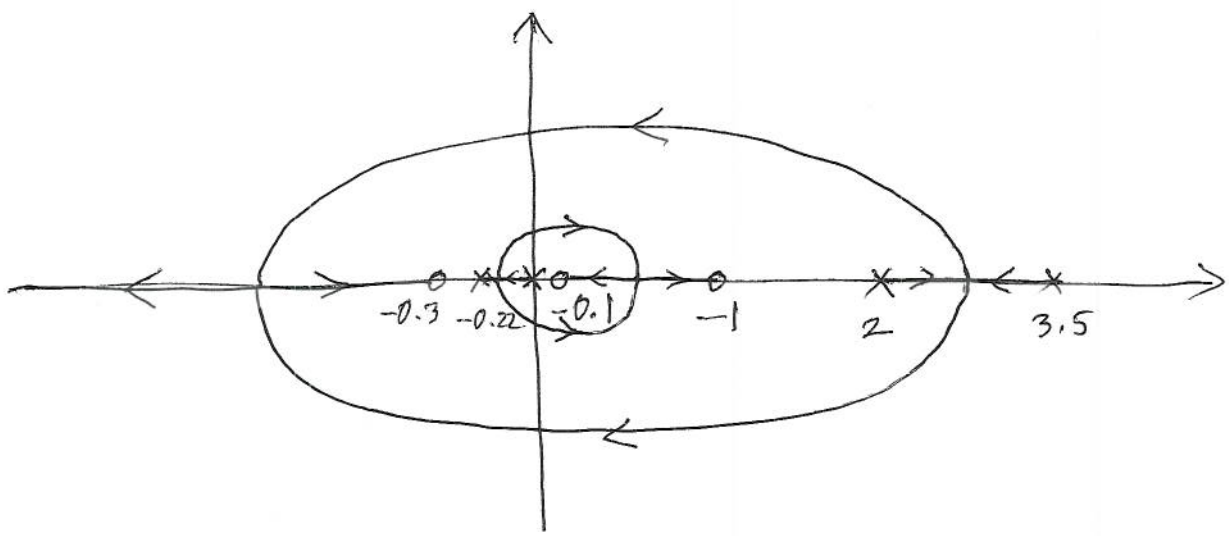
<u>Non-robust</u> stabilizing compensator: $D(s) = \dfrac{s+1}{s-1}$



If the pole-zero cancellation $\frac{s-1}{s-1}$ is not exact, then we get a branch of RL on the positive real axis.

Robust but trickier compensator:

$$D(s) = \frac{(s+0.3)(s-0.1)}{(s+0.22)(s-3.5)}$$



This is one of the many possible speculative solutions that happens to actually work.

The stabilizing range for K is $[5.52, 5.96)$.

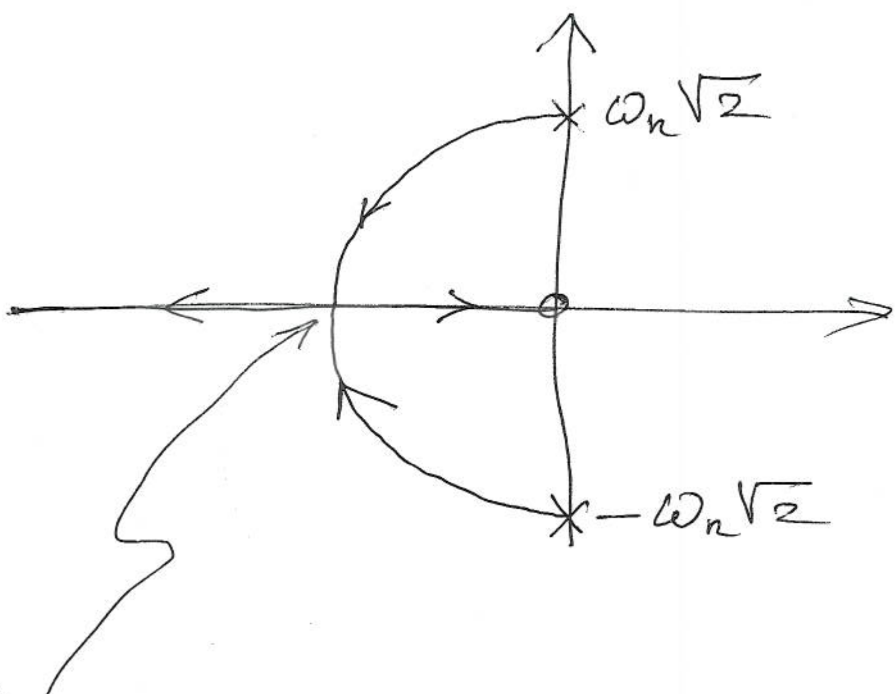
Obviosly, this solution is FYI only.

It required Matlab to verify.

PROBLEM 2

Char. poly: $s^2 + 2\zeta\omega_n s + \omega_n^2 + \omega_n^2 = 0$

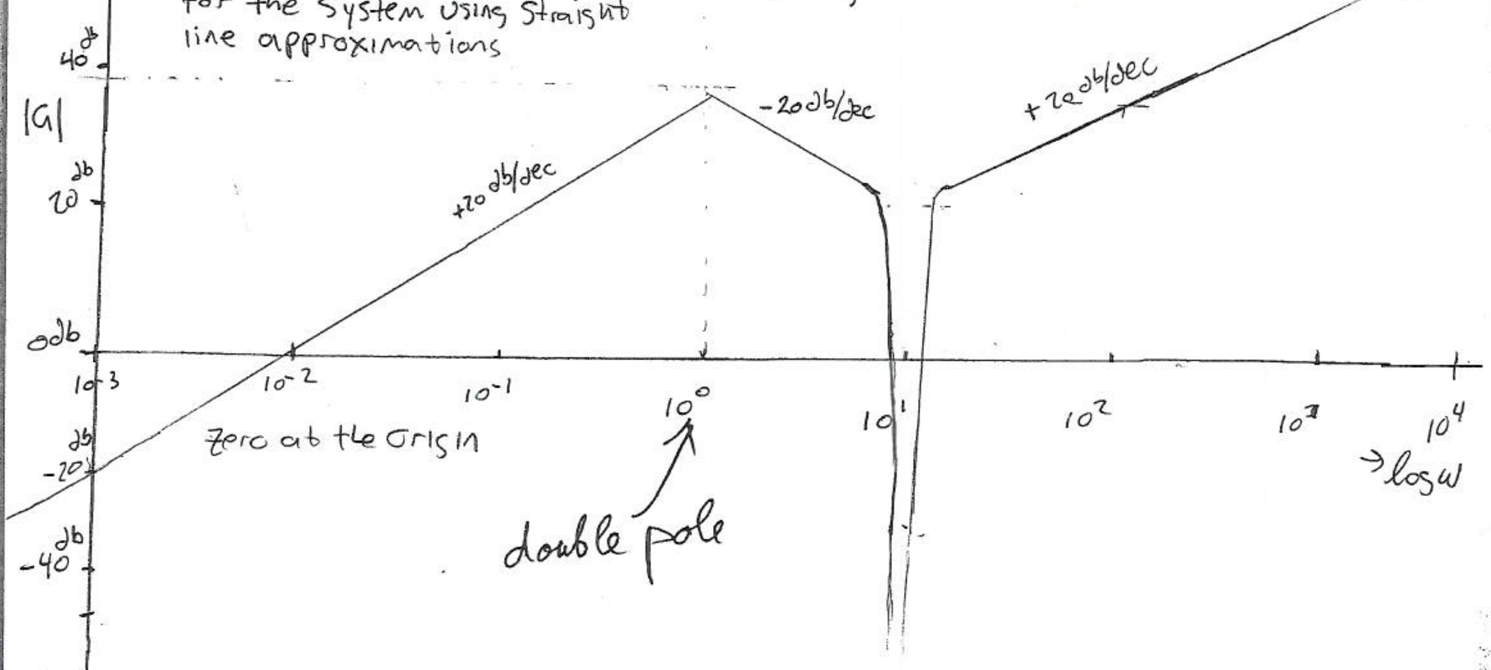
$$1 + \zeta \frac{2\omega_n s}{s^2 + 2\omega_n^2} = 0$$

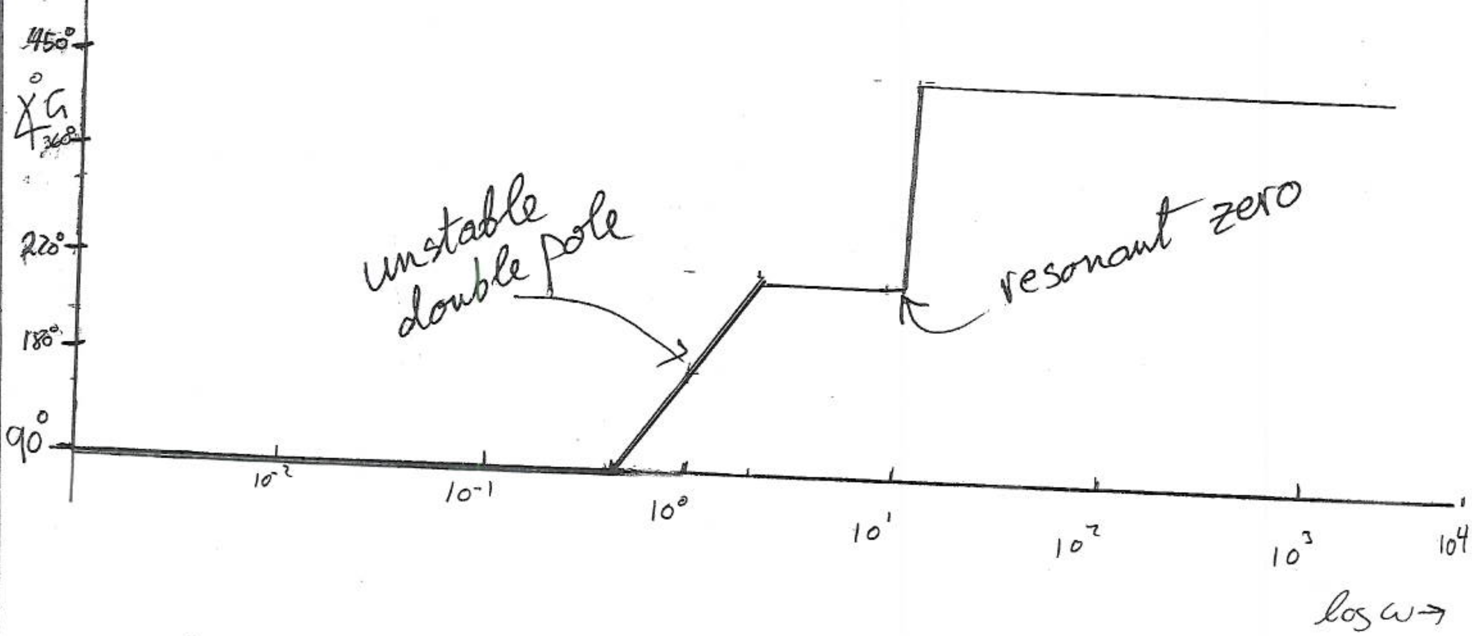


$\zeta_{crit} = \sqrt{2}$

Draw the Bode Diagram $G(s) = \frac{s(s^2+100)}{(s-1)^2}$ for the system using straight line approximations

(8)

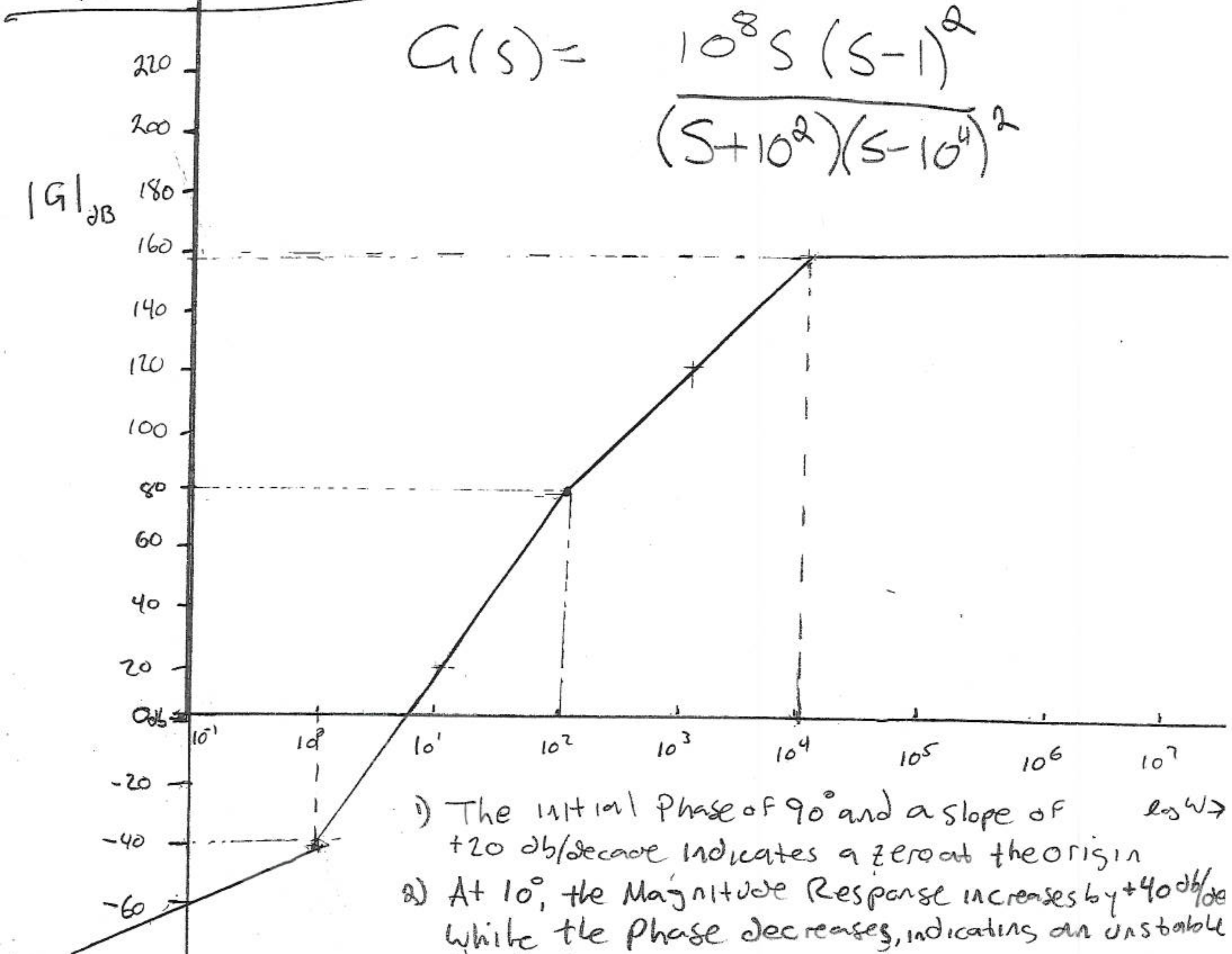




PROBLEM 3

PROBLEM 4

$$G(s) = \frac{10^8 s (s-1)^2}{(s+10^2)(s-10^4)^2}$$

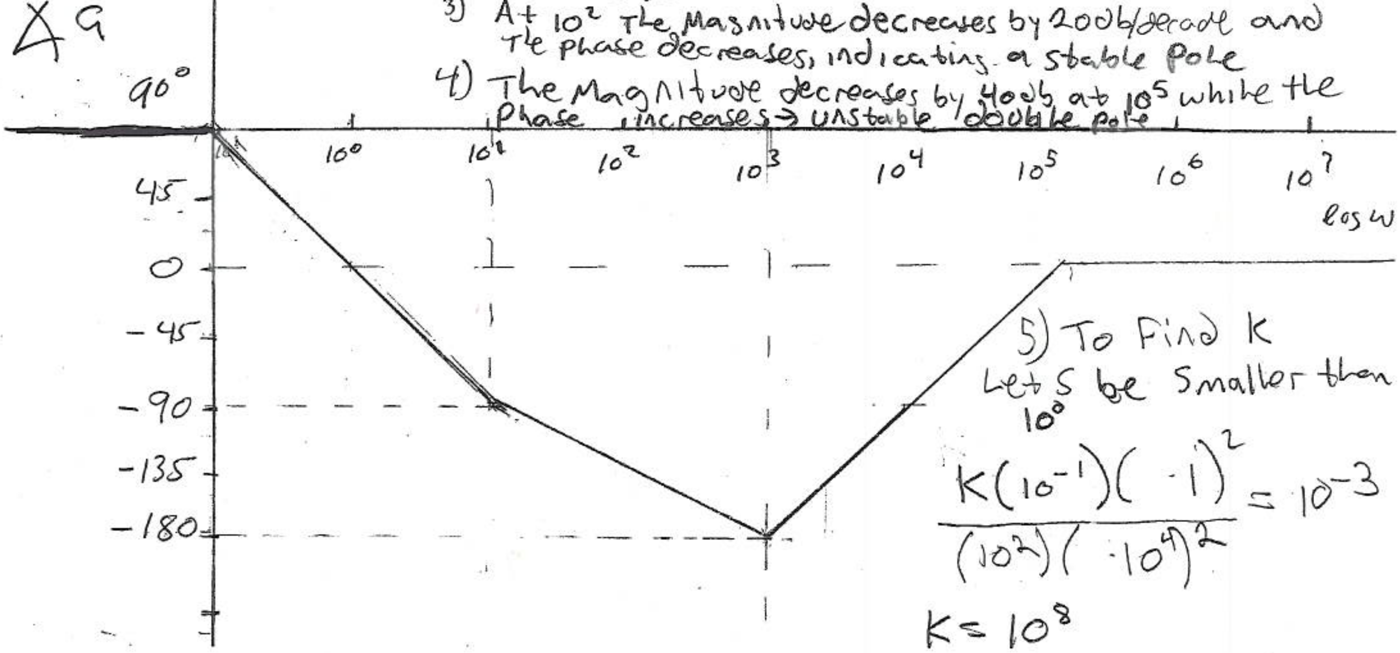


1) The initial Phase of 90° and a slope of +20 db/decade indicates a zero at the origin

2) At 10^0, the Magnitude Response increases by +40 db/dec while the Phase decreases, indicating an unstable double zero

3) At 10^2 The Magnitude decreases by 20db/decade and the phase decreases, indicating a stable Pole

4) The Magnitude decreases by 40db at 10^5 while the Phase increases → unstable double pole



5) To Find K
Let s be Smaller than 10^0

$$\frac{K(10^{-1})(-1)^2}{(10^2)(-10^4)^2} = 10^{-3}$$

$K = 10^8$