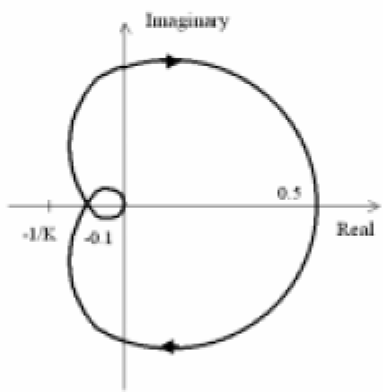
21. Draw the Nyquist plot for the system in Fig. 6.91. Using the Nyquist stability criterion, determine the range of K for which the system is stable. Consider both positive and negative values of K.

Solution :

The characteristic equation:

$$1 + K\frac{1}{(s^2+2s+2)}\frac{1}{(s+1)} = 0$$

$$G(s) = \frac{1}{(s+1)(s^2+2s+2)}$$



For positive K, note that the magnitude of the Nyquist plot as it crosses the negative real axis is 0.1, hence $K < 10$ for stability. For negative K, the entire Nyquist plot is essentially flipped about the imaginary axis, thus the magnitude where it crosses the negative real axis will be 0.5 and the stability limit is that $|K| < 2$ Therefore, the range of K for stability is $-2 < K < 10$.

23. The Nyquist plot for some actual control systems resembles the one shown in Fig.6.92. What are the gain and phase margin(s) for the system of

Fig. 6.92 given that $\alpha = 0.4$, $\beta = 1.3$, and $\phi = 40°$. Describe what happens to the stability of the system as the gain goes from zero to a very large value. Sketch what the corresponding root locus must look like for such a system. Also sketch what the corresponding Bode plots would look like for the system.

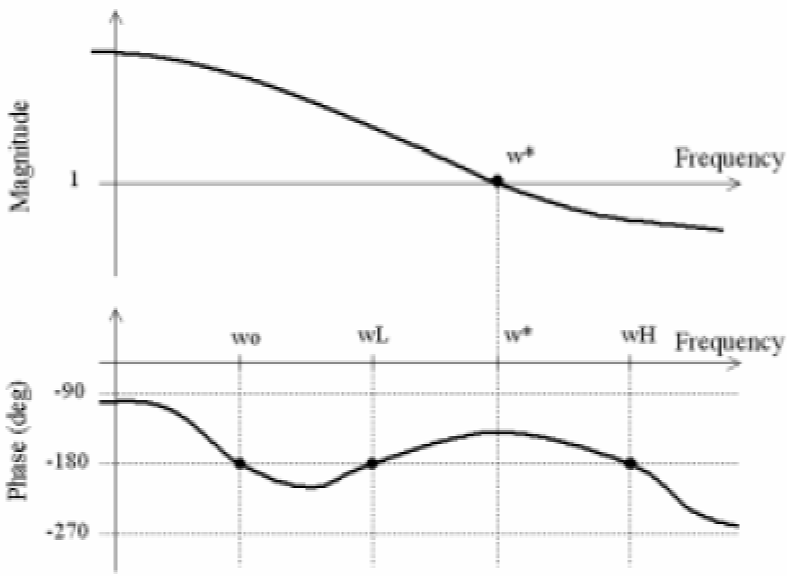
Solution :

The phase margin is defined as in Figure 6.33, $PM = \phi\ (\omega = \omega^*)$, but now there are several gain margins! If the system gain is increased (multiplied) by $\frac{1}{|\alpha|}$ or decreased (divided) by $|\beta|$, then the ststem will go unstable. This is a conditionally stable system. See Figure 6.39 for a typical root locus of a conditionally stable system.

$$\text{gain margin} = \text{-20 log}\,|\alpha|_{dB}\,(\omega = \omega_H)$$
$$\text{gain margin} = +20\text{ log}\,|\beta|_{dB}\,(\omega = \omega_L)$$

For a conditionally stable type of system as in Fig. *6.39*, the Bode phase plot crosses -180° twice; however, for this problem we see from the Nyquist plot that it crosses *3* times! For very low values of gain, the entire Nyquist plot would be shrunk, and the -1 point would occur to the left of the negative real axis crossing at ω_o, so there would be no encirclements and the system would be stable. As the gain increases, the -1 point occurs between ω_o and ω_L so there is an encirclement and the system is unstable.

Further increase of the gain causes the -1 point to occur between ω_L and ω_H (as shown in Fig. 6.92) so there is no encirclement and the system is stable. Even more increase in the gain would cause the -1 point to occur between ω_H and the origin where there is an encirclement and the system is unstable. The root locus would look like Fig. *6.39* except that the very low gain portion of the loci would start in the LHP before they loop out into the RHP as in Fig. *6.39*. The Bode plot would be vaguely like that drawn below:



24. The Bode plot for

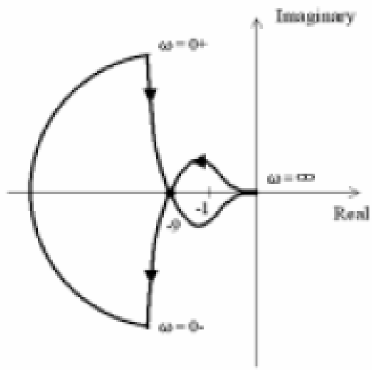
$$G(s) = \frac{100[(s/10)+1]}{s[(s/1)-1][(s/100)+1]}$$

is shown in Fig. *6.93*.

$$
\begin{aligned}
& \text{The phase at the point } \{s = j\omega(\omega = 0+)\} \\
= \quad & -180^\circ(\text{pole} : s = 1) - 90^\circ(\text{pole} : s = 0) + 0^\circ(\text{zero} : s = -10) + 0^\circ(\text{pole} : s = -100) \\
= \quad & -270^\circ
\end{aligned}
$$

Or, more simply, the RHP pole at $s = +1$ causes a -180° shift from the -90° that you would expect from a normal system with all the singularities in the LHP.

(b) The Nyquist plot for $G(s)$:



(c) As the Nyquist shows, there is one counter-clockwise encirclement of -1.

$$\Longrightarrow N = -1$$

We have one pole in RHP $\Longrightarrow P = 1$

$Z = N + P = -1 + 1 = 0 \Longrightarrow$ The closed-loop system is stable.

(d) The system goes unstable if the gain is lowered by a factor of 100.