

3. Sketch the asymptotes of the Bode plot magnitude and phase for each of the following open-loop transfer functions. After completing the hand sketches verify your result using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

$$(a) L(s) = \frac{4000}{s(s+400)}$$

$$(b) L(s) = \frac{100}{s(0.1s+1)(0.5s+1)}$$

$$(c) L(s) = \frac{1}{s(s+1)(0.02s+1)}$$

$$(d) L(s) = \frac{1}{(s+1)^2(s^2+2s+4)}$$

$$(e) L(s) = \frac{10(s+4)}{s(s+1)(s^2+2s+5)}$$

$$(f) L(s) = \frac{1000(s+0.1)}{s(s+1)(s^2+8s+64)}$$

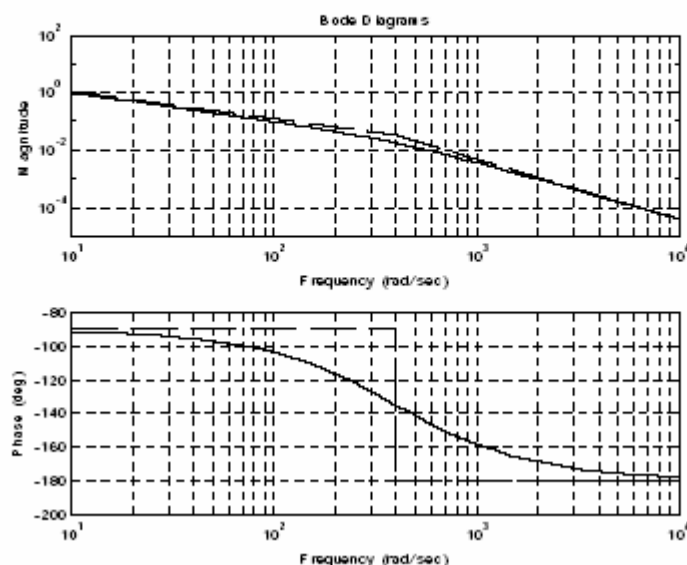
$$(g) L(s) = \frac{(s+5)(s+3)}{s(s+1)(s^2+s+4)}$$

$$(h) L(s) = \frac{4s(s+10)}{(s+100)(4s^2+5s+4)}$$

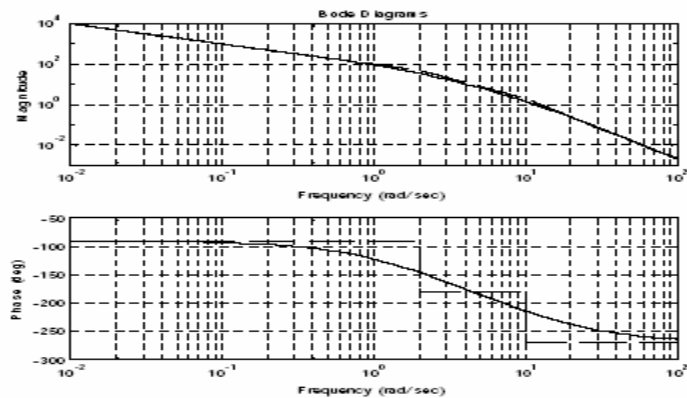
$$(i) L(s) = \frac{s}{(s+1)(s+10)(s^2+2s+2500)}$$

Solution:

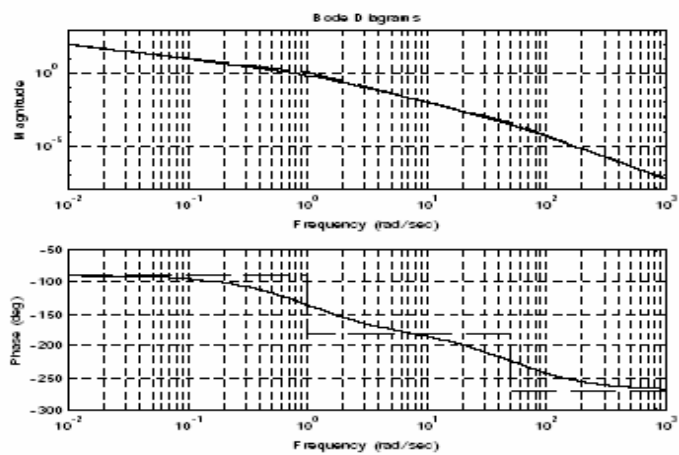
$$(a) L(s) = \frac{10}{s \left[ \left( \frac{s}{20} \right)^2 + 1 \right]}$$



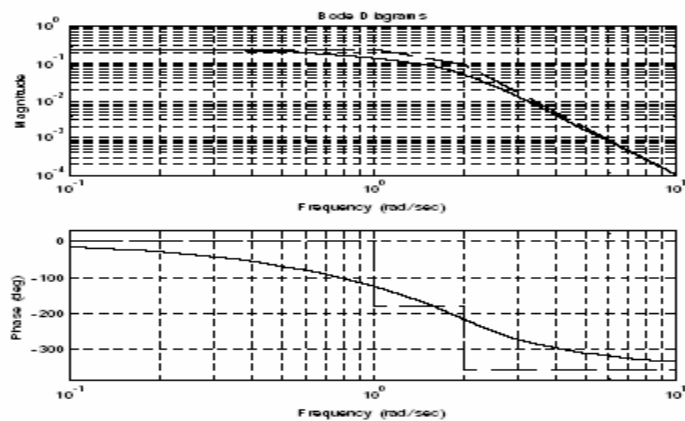
$$(b) L(s) = \frac{100}{s \left( \frac{s}{10} + 1 \right) \left( \frac{s}{2} + 1 \right)}$$



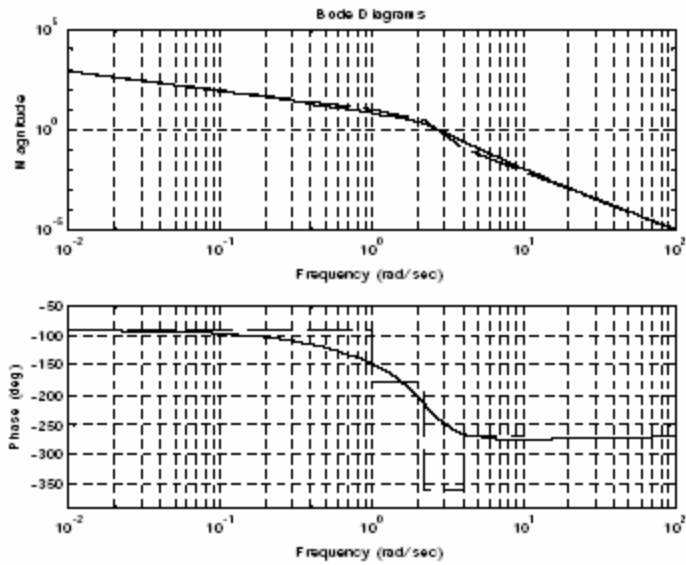
$$(c) L(s) = \frac{1}{s(s+1) \left( \frac{s}{50} + 1 \right)}$$



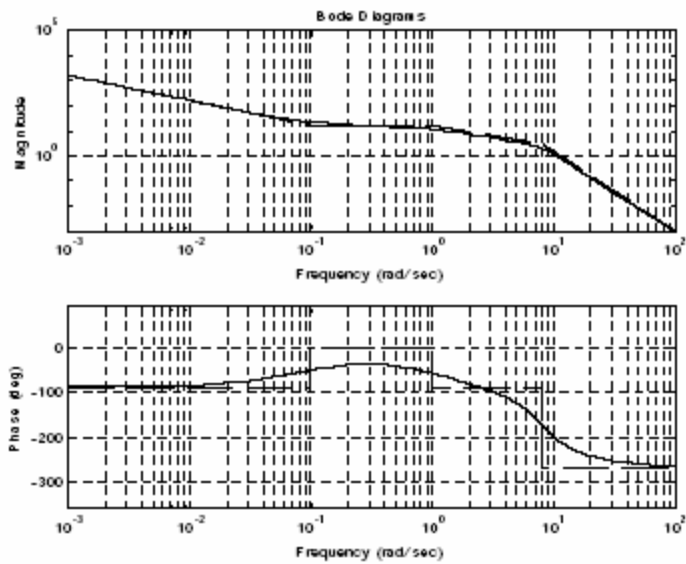
$$(d) L(s) = \frac{1}{(s+1)^2 \left[ \left( \frac{s}{2} \right)^2 + \frac{s}{2} + 1 \right]}$$



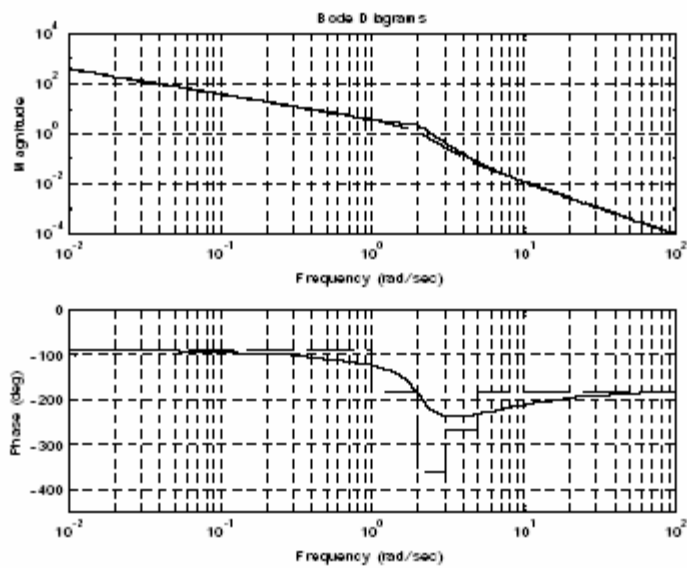
$$(e) \quad L(s) = \frac{s\left(\frac{s}{4} + 1\right)}{s(s+1)\left[\left(\frac{s}{\sqrt{5}}\right)^2 + \frac{2}{5}s + 1\right]}$$



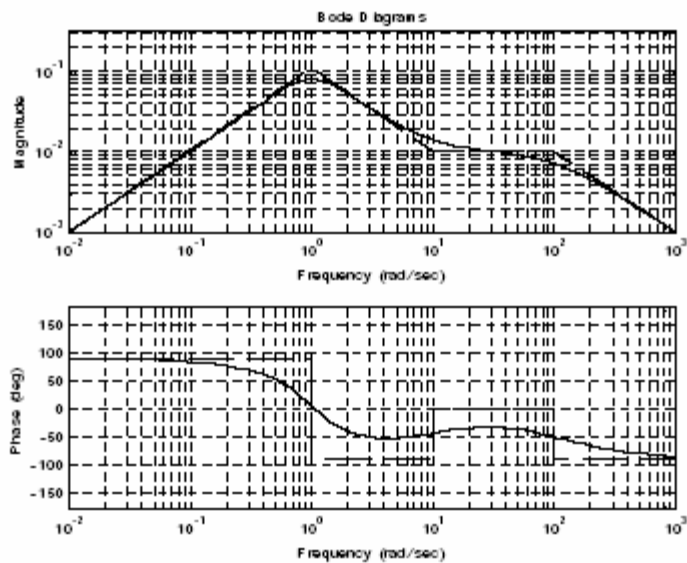
$$(f) \quad L(s) = \frac{\left(\frac{25}{16}\right)(10s+1)}{s(s+1)\left[\left(\frac{s}{8}\right)^2 + \frac{s}{8} + 1\right]}$$



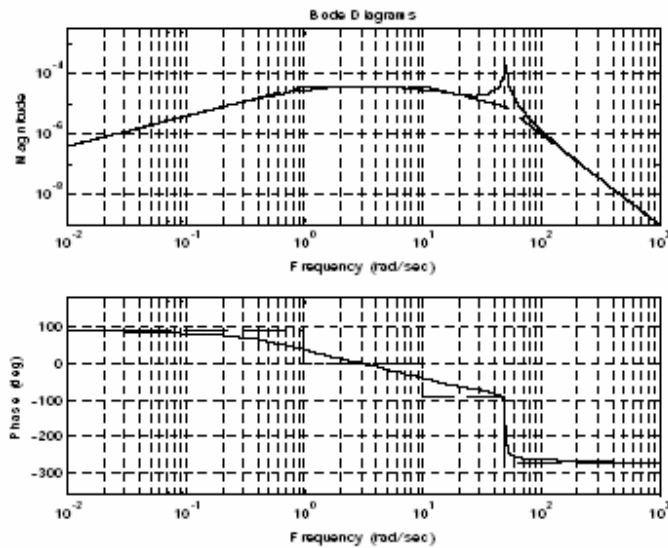
$$(g) \quad L(s) = \frac{\left(\frac{15}{4}\right) \left(\frac{s}{5} + 1\right) \left(\frac{s}{3} + 1\right)}{s(s+1) \left[\left(\frac{s}{2}\right)^2 + \frac{s}{4} + 1\right]}$$



$$(h) \quad L(s) = \frac{\left(\frac{1}{10}\right) s \left(\frac{s}{10} + 1\right)}{\left(\frac{s}{100} + 1\right) \left(s^2 + \frac{5}{4}s + 1\right)}$$



$$(i) L(s) = \frac{\left(\frac{1}{25000}\right)s}{(s+1)\left(\frac{s}{10}+1\right)\left[\left(\frac{s}{50}\right)^2 + \frac{1}{1250}s + 1\right]}$$



4. *Real poles and zeros.* Sketch the asymptotes of the Bode plot magnitude and phase for each of the following open-loop transfer functions. After completing the hand sketches verify your result using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

$$(a) L(s) = \frac{1}{s(s+1)(s+5)(s+10)}$$

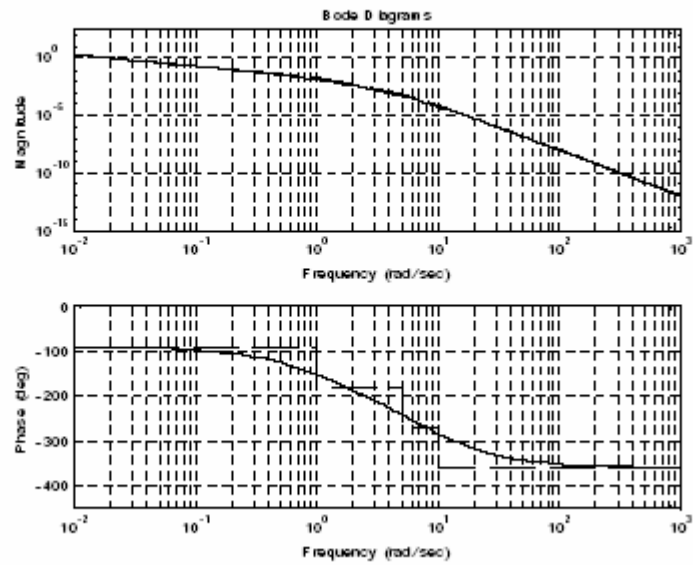
$$(b) L(s) = \frac{(s+2)}{s(s+1)(s+5)(s+10)}$$

$$(c) L(s) = \frac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$$

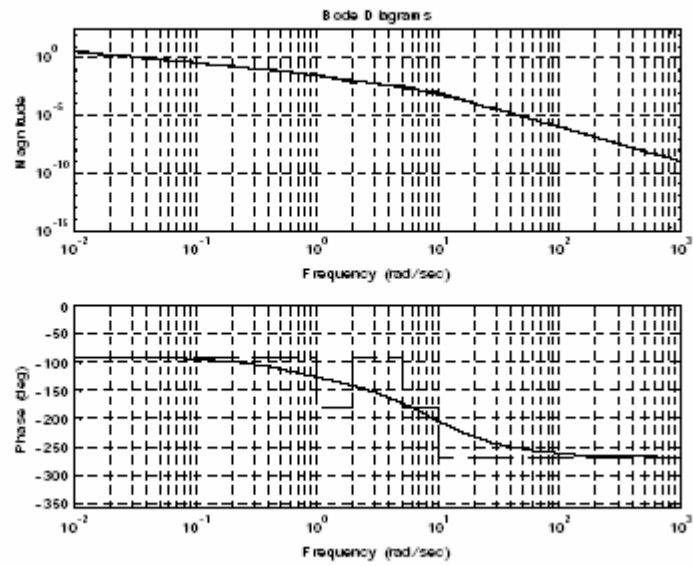
$$(d) L(s) = \frac{(s+2)(s+4)}{s(s+1)(s+5)(s+10)}$$

Solution:

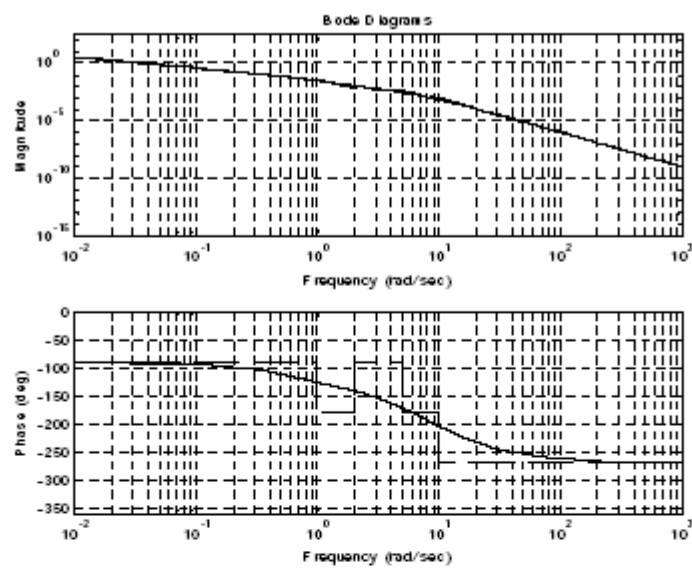
$$(a) L(s) = \frac{1}{s(s+1)\left(\frac{s}{5}+1\right)\left(\frac{s}{10}+1\right)}$$



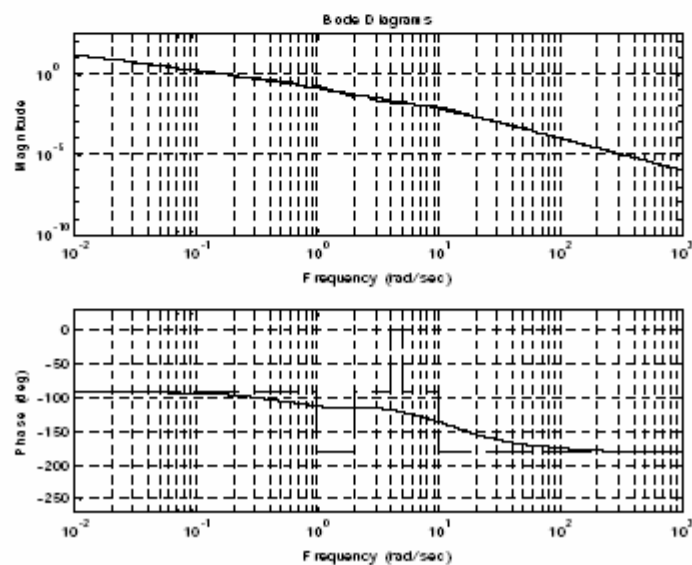
$$(b) L(s) = \frac{1}{25} \left( \frac{s}{2} + 1 \right) \frac{1}{s(s+1)\left(\frac{s}{5}+1\right)\left(\frac{s}{10}+1\right)}$$



$$(c) L(s) = \frac{\frac{6}{25} \left(\frac{s}{2} + 1\right) \left(\frac{s}{6} + 1\right)}{s(s+1) \left(\frac{s}{5} + 1\right) \left(\frac{s}{10} + 1\right)}$$



$$(d) L(s) = \frac{\frac{4}{25} \left(\frac{s}{2} + 1\right) \left(\frac{s}{4} + 1\right)}{s(s+1) \left(\frac{s}{5} + 1\right) \left(\frac{s}{10} + 1\right)}$$



5. *Complex poles and zeros* Sketch the asymptotes of the Bode plot magnitude and phase for each of the following open-loop transfer functions and approximate the transition at the second order break point based on the value of the damping ratio. After completing the hand sketches verify your result using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a)  $L(s) = \frac{1}{s^2 + 3s + 10}$

(b)  $L(s) = \frac{1}{s(s^2 + 3s + 10)}$

(c)  $L(s) = \frac{(s^2 + 2s + 8)}{s(s^2 + 2s + 10)}$

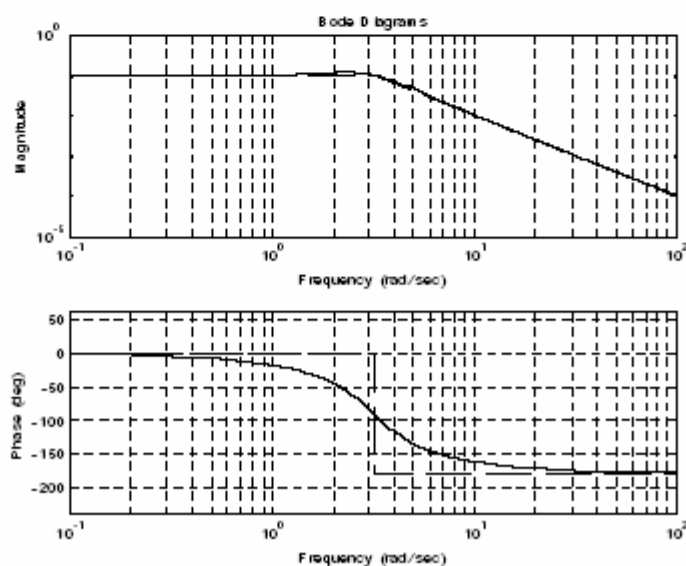
(d)  $L(s) = \frac{(s^2 + 2s + 12)}{s(s^2 + 2s + 10)}$

(e)  $L(s) = \frac{(s^2 + 1)}{s(s^2 + 4)}$

(f)  $L(s) = \frac{(s^2 + 4)}{s(s^2 + 1)}$

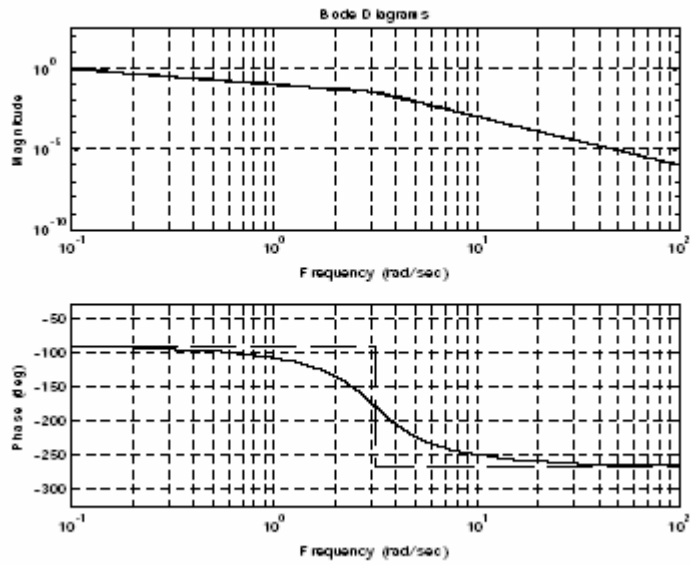
Solution:

(a)  $L(s) = \frac{\frac{1}{10}}{\left(\frac{s}{\sqrt{10}}\right)^2 + \frac{3}{10}s + 1}$

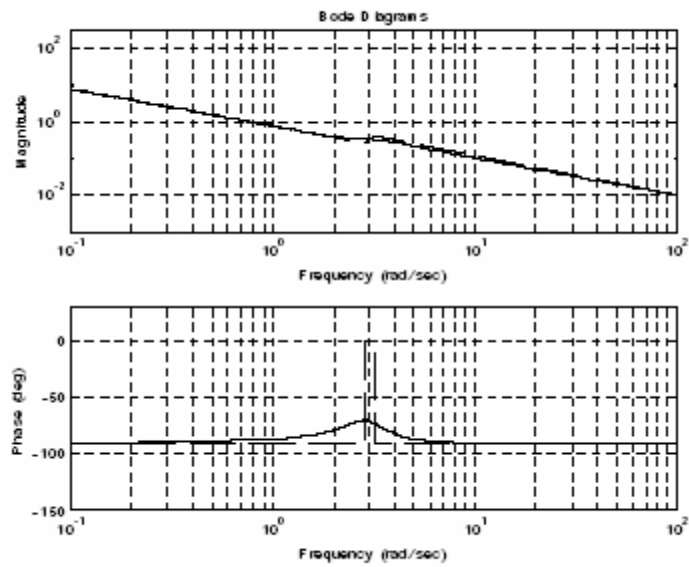




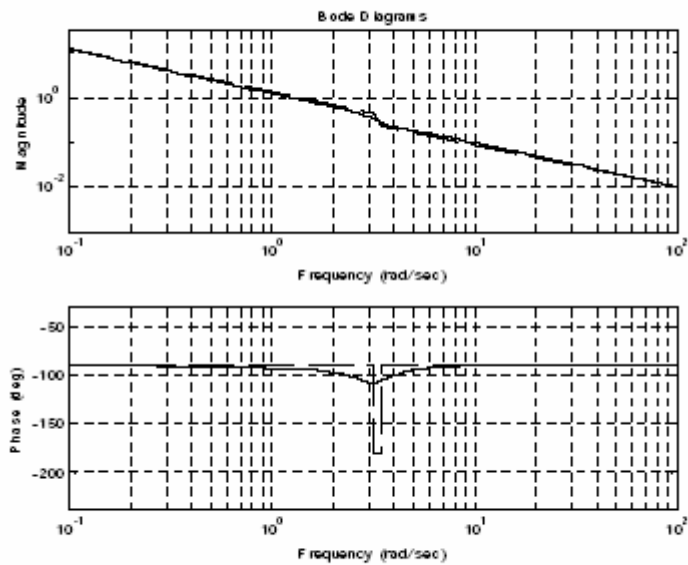
$$(b) L(s) = \frac{\frac{1}{10}}{s \left[ \left( \frac{s}{\sqrt{10}} \right)^2 + \frac{3}{10}s + 1 \right]}$$



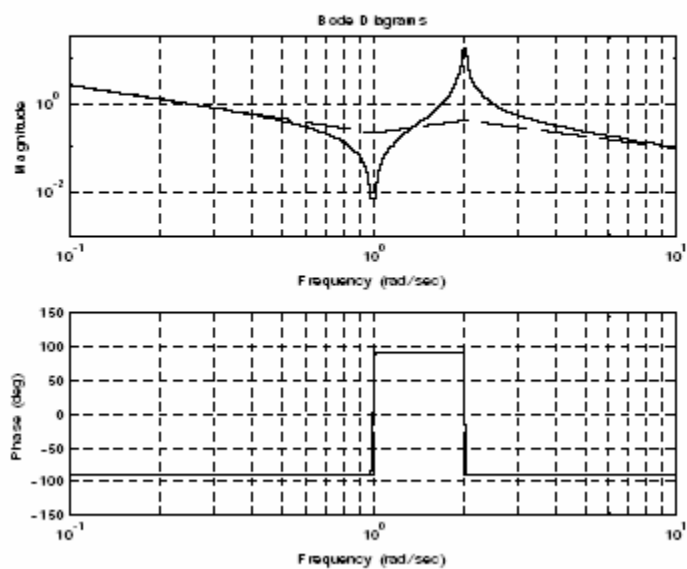
$$(c) L(s) = \frac{\frac{4}{5} \left[ \left( \frac{s}{2\sqrt{2}} \right)^2 + \frac{1}{4}s + 1 \right]}{s \left[ \left( \frac{s}{\sqrt{10}} \right)^2 + \frac{1}{5}s + 1 \right]}$$



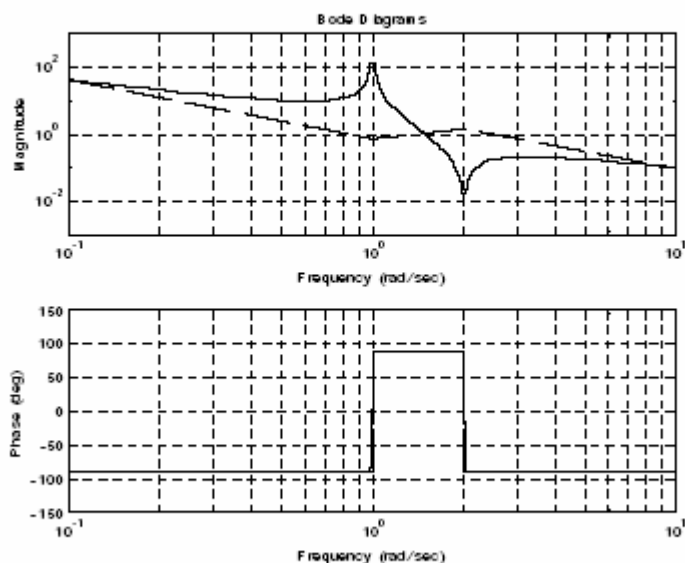
$$(d) L(s) = \frac{s/6 \left[ \left( \frac{s}{2\sqrt{3}} \right)^2 + \frac{1}{6}s + 1 \right]}{s \left[ \left( \frac{s}{\sqrt{10}} \right)^2 + \frac{1}{5}s + 1 \right]}$$



$$(e) L(s) = \frac{\frac{1}{4}(s^2 + 1)}{s \left[ \left( \frac{s}{2} \right)^2 + 1 \right]}$$



$$(f) L(s) = \frac{4 \left[ \left( \frac{s}{2} \right)^2 + 1 \right]}{s(s^2 + 1)}$$



6. *Multiple poles at the origin* Sketch the asymptotes of the Bode plot magnitude and phase for each of the following open-loop transfer functions. After completing the hand sketches verify your result using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

$$(a) L(s) = \frac{1}{s^2(s+8)}$$

$$(b) L(s) = \frac{1}{s^3(s+8)}$$

$$(c) L(s) = \frac{1}{s^4(s+8)}$$

$$(d) L(s) = \frac{(s+3)}{s^2(s+8)}$$

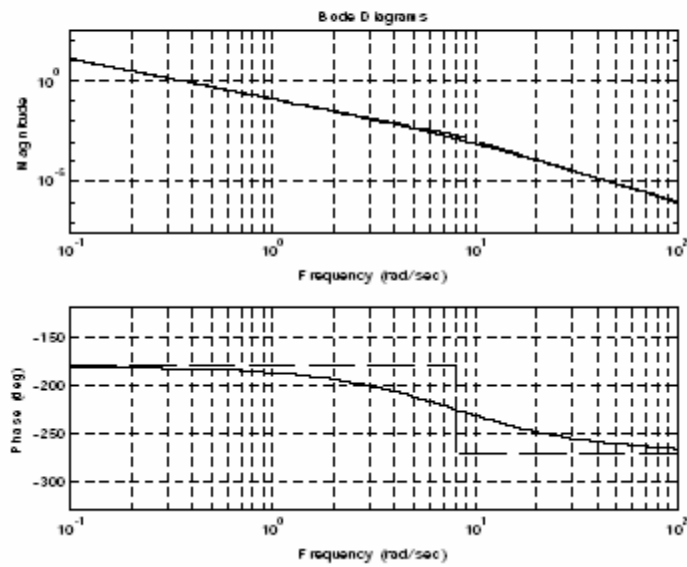
$$(e) L(s) = \frac{(s+3)}{s^3(s+4)}$$

$$(f) L(s) = \frac{(s+1)^2}{s^3(s+4)}$$

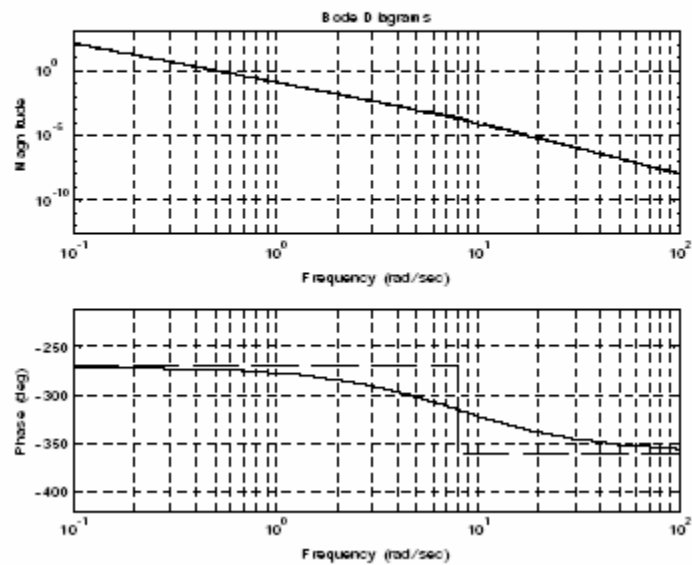
$$(g) L(s) = \frac{(s+1)^2}{s^3(s+10)^2}$$

Solution:

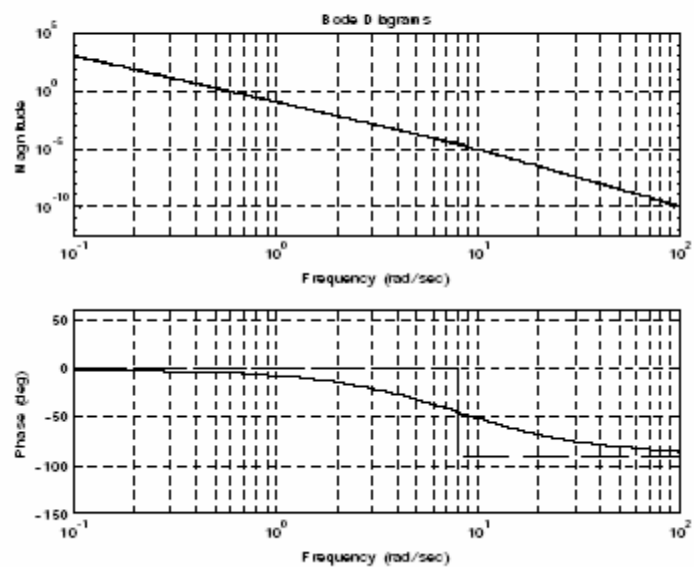
$$(a) L(s) = \frac{1}{s^2 \left( \frac{s}{8} + 1 \right)}$$



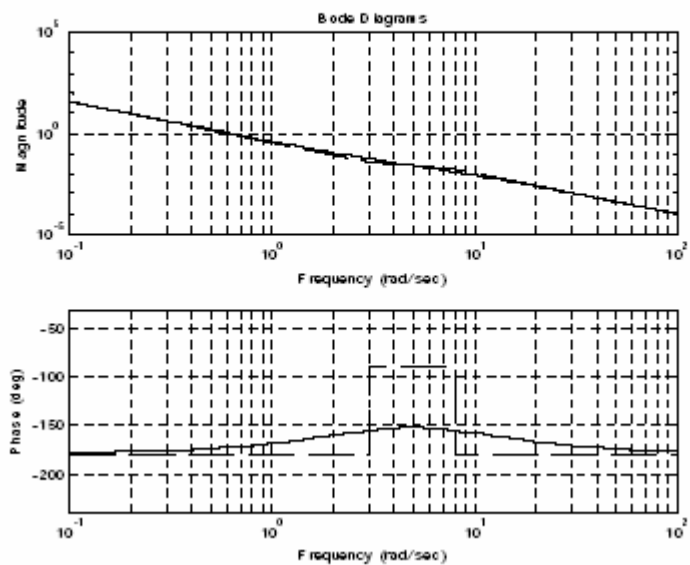
$$(b) L(s) = \frac{1}{s^3 \left( \frac{s}{8} + 1 \right)}$$



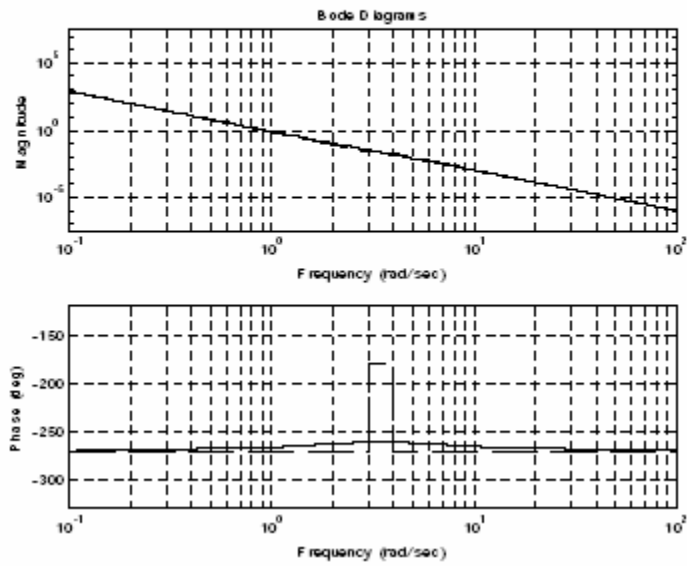
$$(c) \quad L(s) = \frac{1}{s^4 \left( \frac{s}{8} + 1 \right)}$$



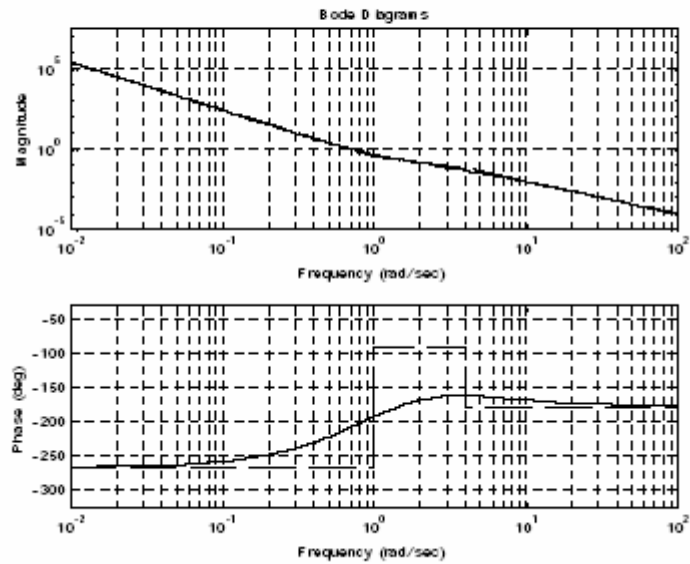
$$(d) \quad L(s) = \frac{3}{s^2 \left( \frac{s}{3} + 1 \right)}$$



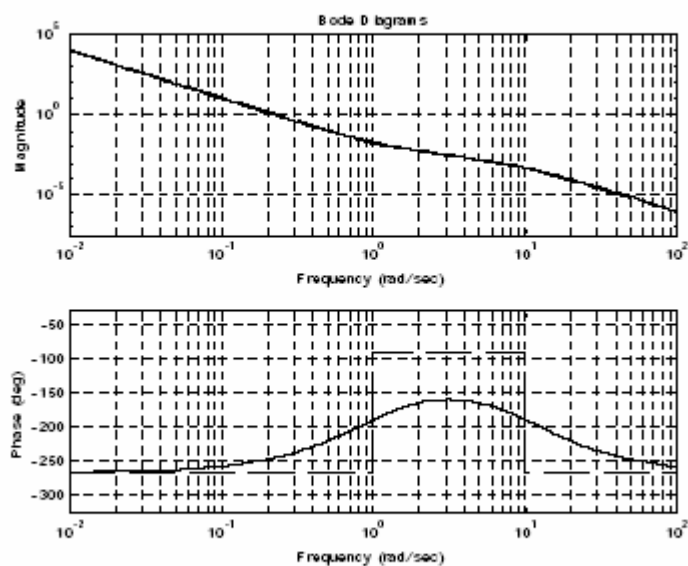
$$(e) L(s) = \frac{\frac{3}{4} \left( \frac{s}{3} + 1 \right)}{s^3 \left( \frac{s}{4} + 1 \right)}$$



$$(f) L(s) = \frac{\frac{1}{4} (s + 1)^2}{s^3 \left( \frac{s}{4} + 1 \right)}$$



$$(g) L(s) = \frac{1}{100} \frac{(s+1)^2}{s^3 \left(\frac{s}{10} + 1\right)^2}$$



7. *Mixed real and complex poles* Sketch the asymptotes of the Bode plot magnitude and phase for each of the following open-loop transfer functions. After completing the hand sketches verify your result using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

$$(a) L(s) = \frac{(s+2)}{s(s+10)(s^2+2s+2)}$$

$$(b) L(s) = \frac{(s+2)}{s^2(s+10)(s^2+6s+25)}$$

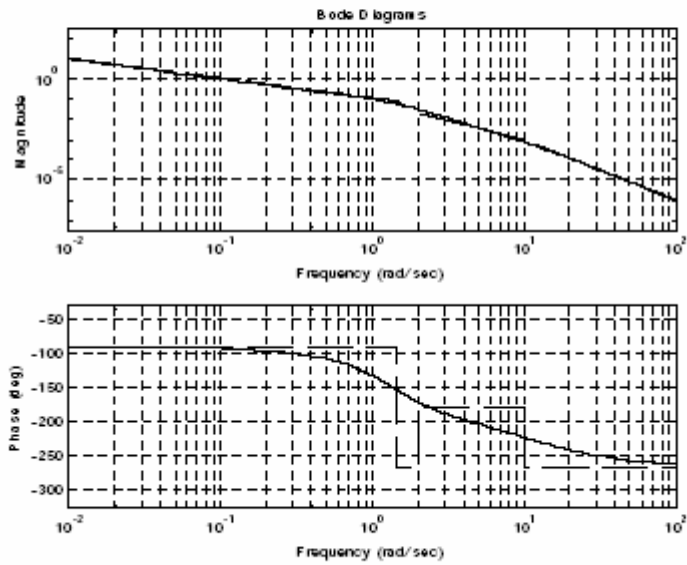
$$(c) L(s) = \frac{(s+2)^2}{s^2(s+10)(s^2+6s+25)}$$

$$(d) L(s) = \frac{(s+2)(s^2+4s+68)}{s^2(s+10)(s^2+4s+85)}$$

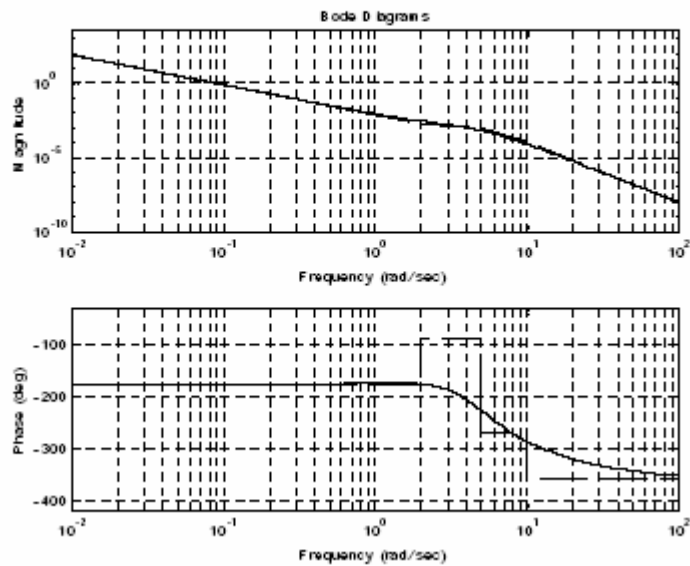
$$(e) L(s) = \frac{[(s+1)^2+1]}{s^2(s+2)(s+3)}$$

Solution:

$$(a) L(s) = \frac{\frac{1}{10} \left( \frac{s}{2} + 1 \right)}{s \left( \frac{s}{10} + 1 \right) \left[ \left( \frac{s}{\sqrt{2}} \right)^2 + s + 1 \right]}$$

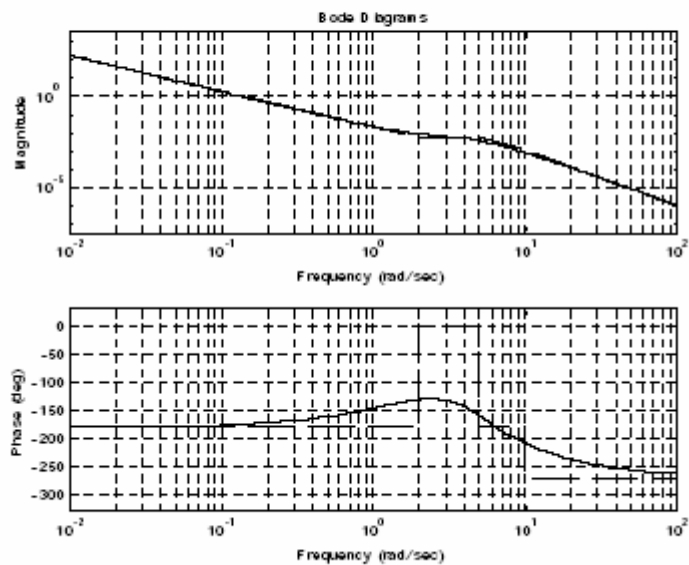


$$(b) L(s) = \frac{\frac{1}{125} \left( \frac{s}{2} + 1 \right)}{s^2 \left( \frac{s}{10} + 1 \right) \left[ \left( \frac{s}{5} \right)^2 + \frac{6}{25}s + 1 \right]}$$

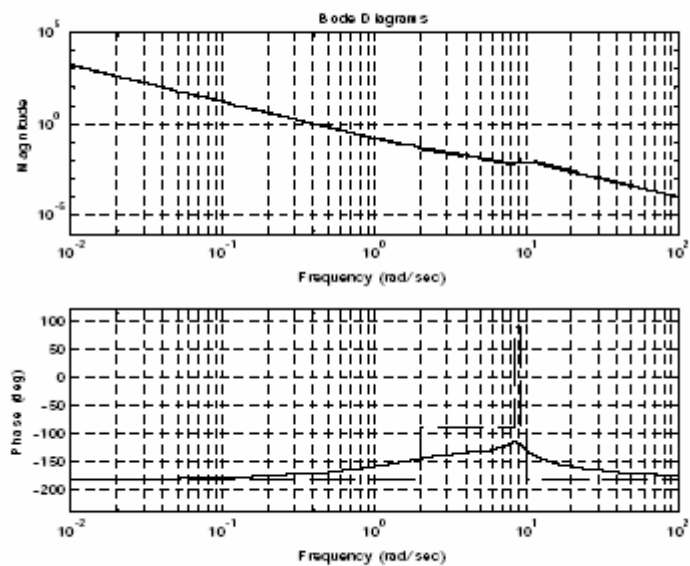




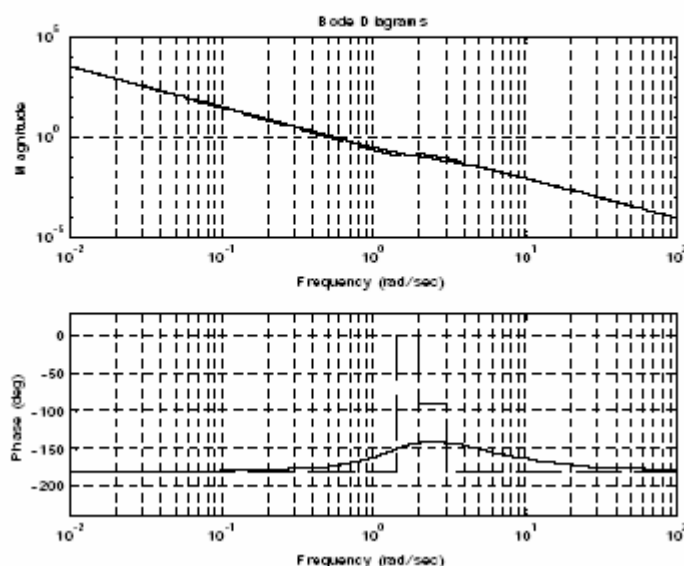
$$(c) L(s) = \frac{\frac{2}{125} \left(\frac{s}{2} + 1\right)^2}{s^2 \left(\frac{s}{10} + 1\right) \left[\left(\frac{s}{5}\right)^2 + \frac{6}{25}s + 1\right]}$$



$$(d) L(s) = \frac{\frac{4}{25} \left(\frac{s}{2} + 1\right) \left[\left(\frac{s}{2\sqrt{17}}\right)^2 + \frac{1}{17}s + 1\right]}{s^2 \left(\frac{s}{10} + 1\right) \left[\left(\frac{s}{\sqrt{85}}\right)^2 + \frac{4}{85}s + 1\right]}$$



$$(e) L(s) = \frac{\frac{1}{3} \left[ \left( \frac{s}{\sqrt{2}} \right)^2 + s + 1 \right]}{s^2 \left( \frac{s}{2} + 1 \right) \left( \frac{s}{3} + 1 \right)}$$



8. *Right half plane poles and zeros* Sketch the asymptotes of the Bode plot magnitude and phase for each of the following open-loop transfer functions. Make sure the phase asymptotes properly take the RHP singularity into account by sketching the complex plane to see how the  $\angle L(s)$  changes as  $s$  goes from 0 to  $+j\infty$ . After completing the hand sketches verify your result using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a)  $L(s) = \frac{s+2}{s+10} \frac{1}{s^2-1}$ ; The model for a case of magnetic levitation with lead compensation.

(b)  $L(s) = \frac{s+2}{s(s+10)} \frac{1}{(s^2-1)}$ ; The magnetic levitation system with integral control and lead compensation.

(c)  $L(s) = \frac{s-1}{s^2}$

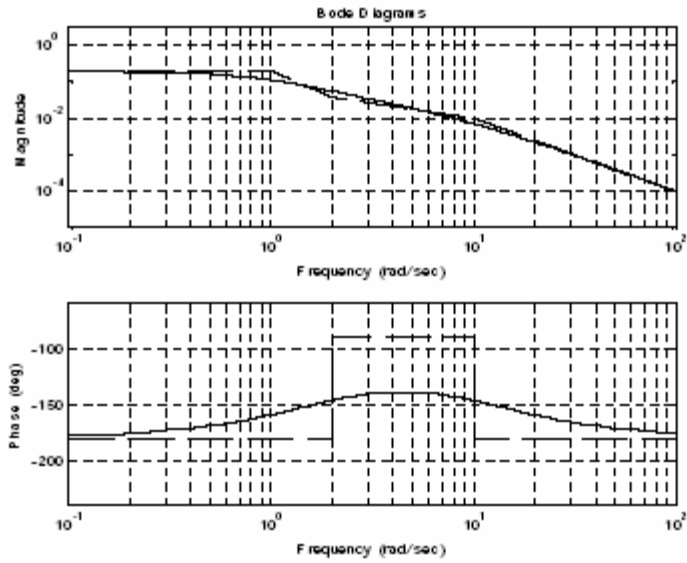
(d)  $L(s) = \frac{s^2+2s+1}{s(s+20)^2(s^2-2s+2)}$

(e)  $L(s) = \frac{(s+2)}{s(s-1)(s+6)^2}$

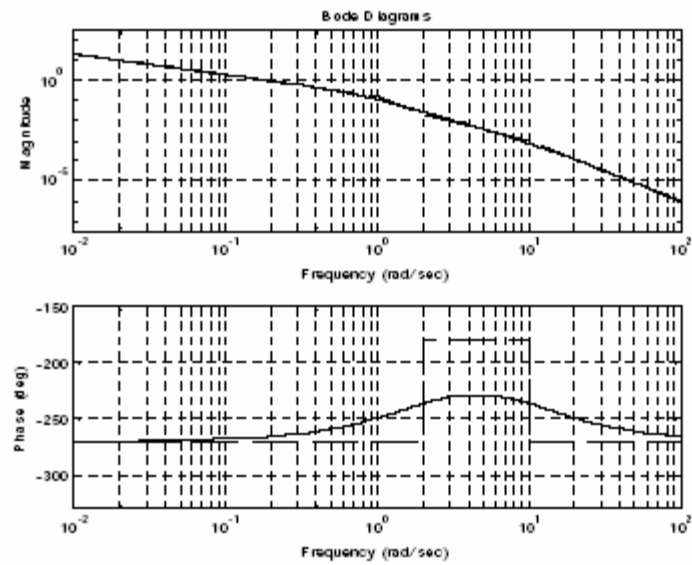
(f)  $L(s) = \frac{1}{(s-1)[(s+2)^2+3]}$

Solution:

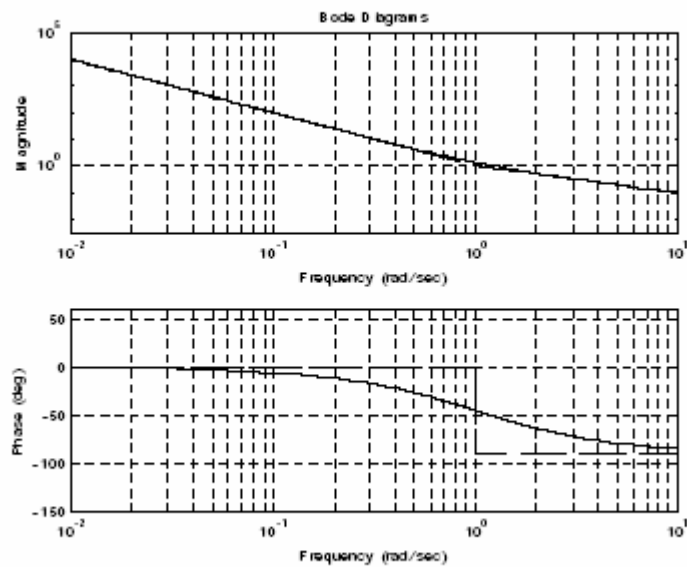
$$(a) L(s) = \frac{\frac{1}{5} \left( \frac{s}{2} + 1 \right)}{s + 10} \frac{1}{s^2 - 1}$$



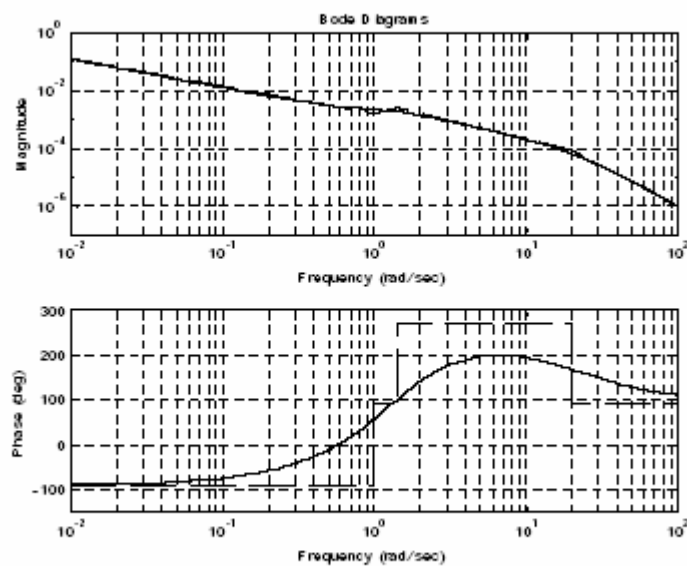
$$(b) L(s) = \frac{\frac{1}{5} \left( \frac{s}{2} + 1 \right)}{s(s + 10)} \frac{1}{s^2 - 1}$$



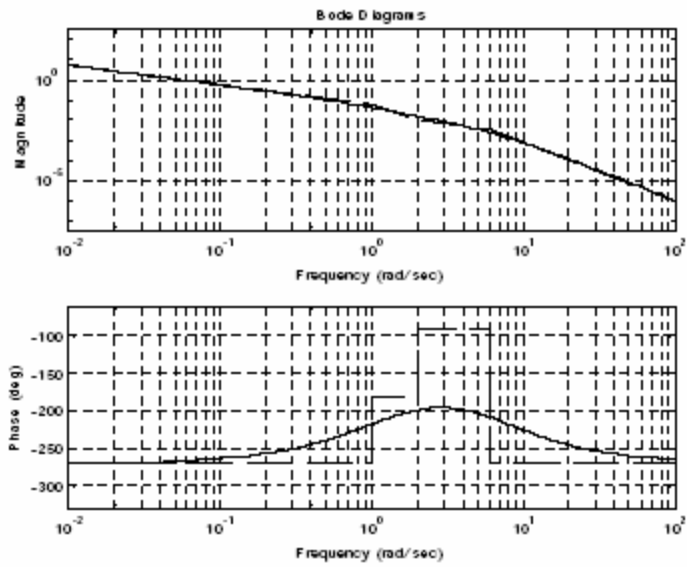
(c)  $L(s) = \frac{s-1}{s^2}$



(d)  $L(s) = \frac{\frac{1}{40}(s^2 + 2s + 1)}{s \left(\frac{s}{20} + 1\right)^2 \left[\left(\frac{s}{\sqrt{2}}\right)^2 - s + 1\right]}$



$$(e) \quad L(s) = \frac{\frac{1}{18} \left( \frac{s}{2} + 1 \right)}{s(s-1) \left( \frac{s}{6} + 1 \right)^2}$$



$$(f) \quad L(s) = \frac{1}{(s-1) \left[ \left( \frac{s}{\sqrt{7}} \right)^2 + \frac{4}{7}s + 1 \right]}$$

