3. For the characteristic equation

$$1 + \frac{K}{s(s+1)(s+5)} = 0 :$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \to \infty$.

(c) For what value of K are the roots on the imaginary axis?

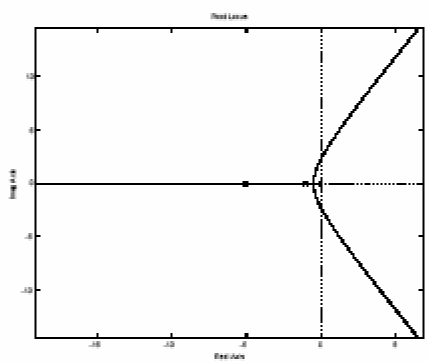
(d) Verify your sketch with a MATLAB plot.

Solution:

(a) The real axis segments are $0 > \sigma > -1$; $-5 > \sigma$

(b) $\alpha = -6/3 = -2$; $\phi_i = \pm 60, 180$

(c) $K_o = 30$



(d) Solution for Problem 5.3

<u>I put more solutions for 5.4, 5.5, and 5.6 for your practice.</u>

4. Real poles and zeros. Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{1}{s(s+1)(s+5)(s+10)}$

(b) $L(s) = \dfrac{(s+2)}{s(s+1)(s+5)(s+10)}$

(c) $L(s) = \dfrac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$

(d) $L(s) = \dfrac{(s+2)(s+4)}{s(s+1)(s+5)(s+10)}$

Solution:

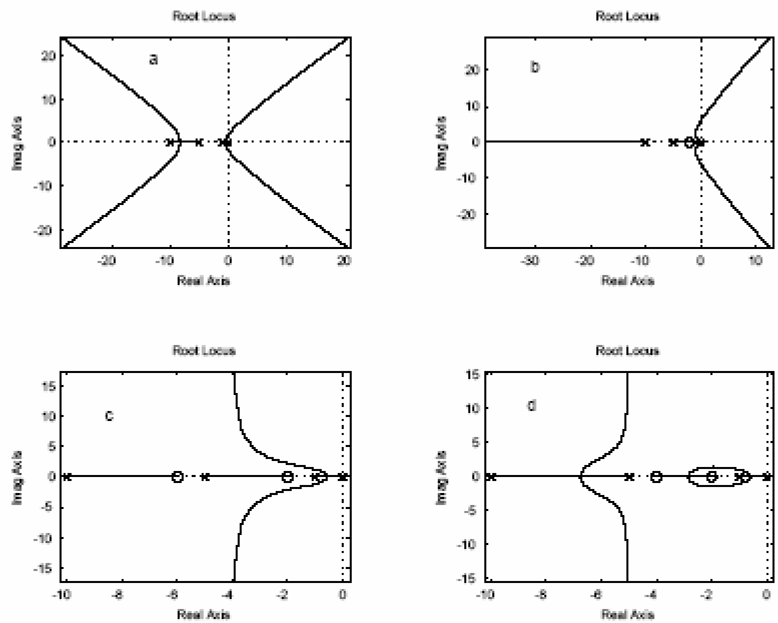
All the root locus plots are displayed at the end of the solution set for this problem.

(a) $\alpha = -4$; $\phi_i = \pm 45$; ± 135; $\omega_o = 1.77$

(b) $\alpha = -4.67$; $\phi_i = \pm 60$; ± 180; $\omega_o = 5.98$

(c) $\alpha = -4$; $\phi_i = \pm 90$; $\omega_o -> none$

(d) $\alpha = -5$; $\phi_i = \pm 90$; $\omega_o -> none$



Solution for Problem 5.4

5. **Complex poles and zeros** Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{1}{s^2 + 3s + 10}$

(b) $L(s) = \dfrac{1}{s(s^2 + 3s + 10)}$

(c) $L(s) = \dfrac{(s^2 + 2s + 8)}{s(s^2 + 2s + 10)}$

(d) $L(s) = \dfrac{(s^2 + 2s + 12)}{s(s^2 + 2s + 10)}$

(e) $L(s) = \dfrac{(s^2 + 1)}{s(s^2 + 4)}$

(f) $L(s) = \dfrac{(s^2 + 4)}{s(s^2 + 1)}$

Solution:

All the root locus plots are displayed at the end of the solution set for this problem.

(a) $\alpha = -3$; $\phi_i = \pm 90$; $\theta_d = \pm 90$ $\omega_o -> none$

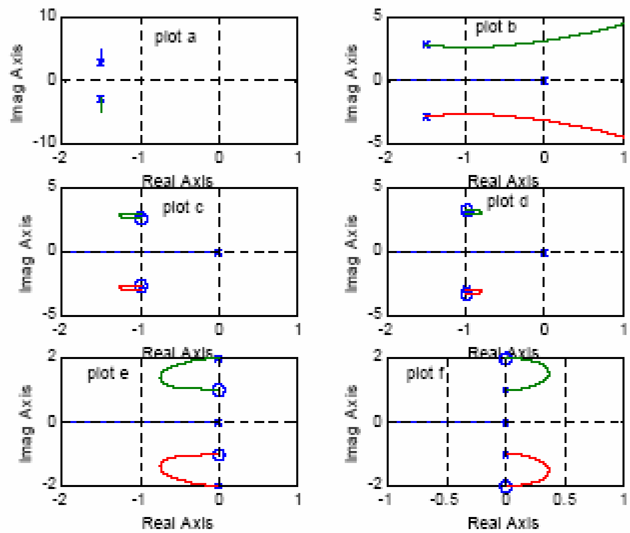
(b) $\alpha = -3$; $\phi_i = \pm 60, \pm 180$; $\theta_d = \pm 28.3$ $\omega_o = 3.16$

(c) $\alpha = -2$; $\phi_i = \pm 180$; $\theta_d = \pm 161.6$; $\theta_a = \pm 200.7$; $\omega_o -> none$

(d) $\alpha = -2$; $\phi_i = \pm 180$; $\theta_d = \pm 18.4$; $\theta_a = \pm 16.8$; $\omega_o -> none$

(e) $\alpha = 0$; $\phi_i = \pm 180$; $\theta_d = \pm 180$; $\theta_a = \pm 180$; $\omega_o -> none$

(f) $\alpha = 0$; $\phi_i = \pm 180$; $\theta_d = 0$; $\theta_a = 0$; $\omega_o -> none$



Solution for Problem 5.5

6. **Multiple poles at the origin** Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{1}{s^2(s+8)}$

(b) $L(s) = \dfrac{1}{s^3(s+8)}$

(c) $L(s) = \dfrac{1}{s^4(s+8)}$

(d) $L(s) = \dfrac{(s+3)}{s^2(s+8)}$

(e) $L(s) = \dfrac{(s+3)}{s^3(s+4)}$

(f) $L(s) = \dfrac{(s+1)^2}{s^3(s+4)}$

(g) $L(s) = \dfrac{(s+1)^2}{s^3(s+10)^2}$

Solution:

(a) $\alpha = -2.67$; $\phi_i = \pm 60$; ± 180; $w_0 - > none$

(b) $\alpha = -2$; $\phi_i = \pm 45$; ± 135; $w_0 - > none$

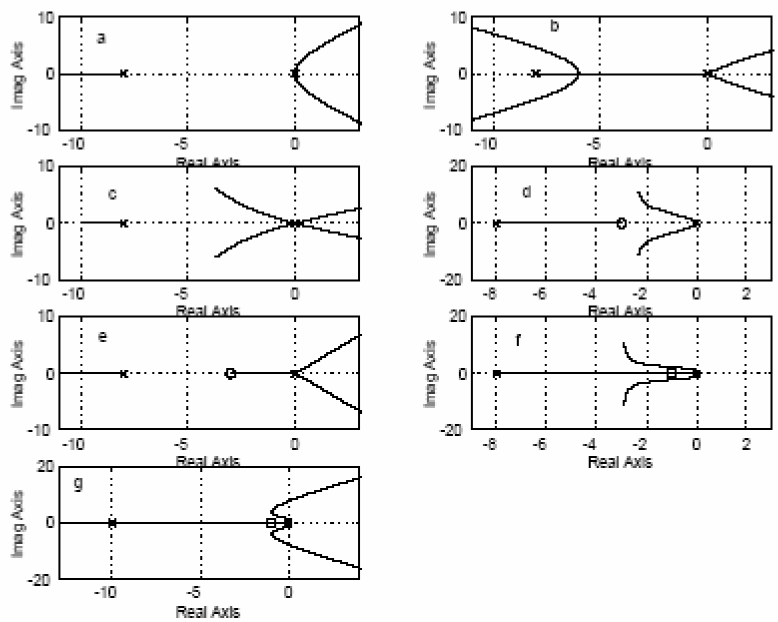
(c) $\alpha = -1.6$; $\phi_i = \pm 36$; ± 108; $w_0 - > none$

(d) $\alpha = -2.5$; $\phi_i = \pm 90$; $w_0 - > none$

(e) $\alpha = -0.33$; $\phi_i = \pm 60$; ± 180; $w_0 - > none$

(f) $\alpha = -3$; $\phi_i = \pm 90$; $w_0 = \pm 1.414$

(g) $\alpha = -6$; $\phi_i = \pm 60; 180$; $w_0 = \pm 1.31; \pm 7.63$



Solution for Problem 5.6

30. Assume the closed-loop system of Fig. 5.71 has a feed forward transfer function $G(s)$ given by

$$G(s) = \frac{1}{s(s+2)}.$$

Design a lag compensation so that the dominant poles of the closed-loop system are located at $s = -1 \pm j$ and the steady-state error to a unit ramp input is less than 0.2.

Solution:

The poles can be put in the desired location with proportional control alone, with a gain of $k_p = 2$ resulting in a $K_v = 1$. To get a $K_v = 5$, we add a compensation with zero at 0.1 and a pole at 0.02. $D(s) = 2\frac{s+0.1}{s+0.02}$.

31. An elementary magnetic suspension scheme is depicted in Fig. 5.72. For small motions near the referencev position, the voltage e on the photo detector is related to the ball displacement x (in meters) by $e = 100x$. The upward force (in newtons) on the ball caused by the current i (in amperes) may be approximated by $f = 0.5i + 20x$. The mass of the ball is 20 g, and the gravitational force is 9.8 N/kg. The power amplifier is a voltage-to-current device with an output (in amperes) of $i = u + V_0$.

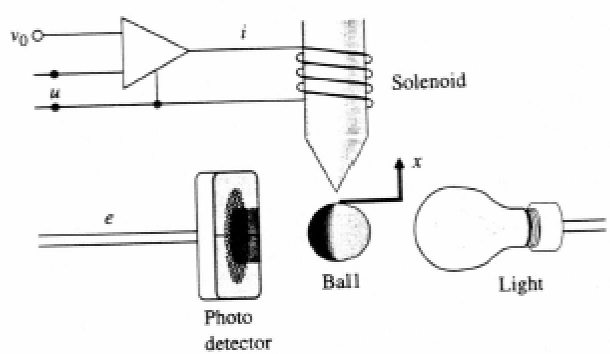


Figure 5.72: Elementary magnetic suspension

(a) Write the equations of motion for this setup.

(b) Give the value of the bias V_0 that results in the ball being in equilibrium at $x = 0$.

(c) What is the transfer function from u to e?

(d) Suppose the control input u is given by $u = -Ke$. Sketch the root locus of the closed-loop system as a function of K.

(e) Assume that a lead compensation is available in the form $\frac{U}{E} = D(s) = K\frac{s+z}{s+p}$. Give values of K, z, and p that yields improved performance over the one proposed in part (d).

Solution:

(a) $m\ddot{x} = 20x + 0.5i - mg$. Substituting numbers, $0.02\ddot{x} = 20x + 0.5(u + V_o) - 0.196$.

(b) To have the bias cancel gravity, the last two terms must add to zero. Thus $V_o = 0.392$.

(c) Taking transfors of the equation and substituting $e = 100x$,

$$\frac{E}{U} = \frac{2500}{s^2 - 1000}$$

(d) The locus starts at the two poles symmetric to the imaginary axis, meet at the origin and cover the imaginary axis. The locus is plotted below.

(e) The lead can be used to cancel the left-hand-plane zero and the pole at m−150 which will bring the locus into the left-hand plane where K can be selected to give a damping of, for example 0.7. See the plot below.