40. Use Routh's stability criterion to determine how many roots with positive real parts the following equations have.

(a) $s^4 + 8s^3 + 32s^2 + 80s + 100 = 0.$

(b) $s^5 + 10s^4 + 30s^3 + 80s^2 + 344s + 480 = 0.$

(c) $s^4 + 2s^3 + 7s^2 - 2s + 8 = 0.$

Solution:

(a)

$$s^4 + 8s^3 + 32s^2 + 80s + 100 = 0$$

$$\begin{array}{lllll} s^4 & : & 1 & 32 & 100 \\ s^3 & : & 8 & 80 & \\ s^2 & : & 22 & 100 & \\ s^1 & : & 80 - \dfrac{800}{22} = 43.6 & & \\ s^0 & : & 100 & & \end{array}$$

$\Longrightarrow$ No roots not in the LHP

(b)

$$s^5 + 10s^4 + 30s^3 + 80s^2 + 344s + 480 = 0$$

$$\begin{array}{lllll} s^5 & : & 1 & 30 & 344 \\ s^4 & : & 10 & 80 & 480 \\ s^3 & : & 22 & 296 & \\ s^2 & : & -545 & 480 & \\ s^1 & : & 490 & & \\ s^0 & : & 480 & & \end{array}$$

$\Longrightarrow$ 2 roots not in the LHP.

(c)

$$s^4 + 2s^3 + 7s^2 - 2s + 8 = 0$$

There are roots in the RHP (not all coefficients are >0).

$$\begin{array}{lllll} s^4 & : & 1 & 7 & 8 \\ s^3 & : & 2 & -2 & \\ s^2 & : & 8 & 8 & \\ s^1 & : & -4 & & \\ s^0 & : & 8 & & \end{array}$$

$\Longrightarrow$ 2 roots not in the LHP.

42. The transfer function of a typical tape-drive system is given by

$$G(s) = \frac{K(s+4)}{s[(s+0.5)(s+1)(s^2+0.4s+4)]},$$

where time is measured in milliseconds. Using Routh's stability criterion, determine the range of K for which this system is stable when the characteristic equation is $1 + G(s) = 0$.

Solution:

$$1 + G(s) = s^5 + 1.9s^4 + 5.1s^3 + 6.2s^2 + (2+K)s + 4K = 0$$

s^5	:	1.0	5.1	$2+K$
s^4	:	1.9	6.2	$4K$
s^3	:	a_1	a_2	
s^2	:	b_1	$4K$	
s^1	:	c_1		
s^0	:	$4K$		

where

$$a_1 = \frac{(1.9)(5.1) - (1)(6.2)}{1.9} = 1.837 \qquad a_2 = \frac{(1.9)(2+K) - (1)(4K)}{1.9} = 2 - 1.1K$$

$$b_1 = \frac{(a_1)(6.2) - (a_2)(1.9)}{a_1} = 1.138(K + 3.63)$$

$$c_1 = \frac{(b_1)(a_2) - (4K)(a_1)}{b_1} = \frac{-(1.25K^2 + 9.61K - 8.26)}{1.138(K + 363)} = \frac{-(K + 8.47)(K - 0.78)}{0.91(K + 3.63)}$$

For stability:

$$(1)\ b_1 = K + 3.63 > 0 \Longrightarrow K > -3.63$$
$$(2)\ c_1 > 0 \Longrightarrow -8.43 < K < 0.78$$
$$(3)\ d_1 > 0 \Longrightarrow K > 0$$

Intersection of (1), (2), and (3) $\Longrightarrow 0 < K < 0.78$

44. Modify the Routh criterion so that it applies to the case where all the poles are to be to the left of $-\alpha$ when $\alpha > 0$. Apply the modified test to the polynomial

$$s^3 + (6+K)s^2 + (5+6K)s + 5K = 0,$$

finding those values of K for which all poles have a real part less than -1/2.

Solution:

Let $p = s + \alpha$ and substitute $s = p - \alpha$ to obtain a polynomial in terms of p. Apply the standard Routh test to the polynomial in p.

In our case, p=s+1/2. s=p-1/2. Substitute this in the polynomial to get

(p-1/2)^3+(6+K)(p-1/2)^2+(5+6K)(p-1/2)+5K=0

P^3:	1	-1/4+5K
P^2:	9/2+K	-9/8+9K/4
P^1:	5K(K+4)/(9/2+K)	
P^0:	18K/8-9/8	

Hence K>1/2

12. Consider the second-order plant

$$G(s) = \frac{1}{(s+1)(5s+1)}.$$

(a) Determine the system type and error constant with respect to tracking polynomial reference inputs of the system for P, PD, and PID controllers (as configured in Fig.4.2(b)) Let $k_p = 19$, $k_I = 9.5$, and $k_D = 4$.

(b) Determine the system type and error constant of the system with respect to disturbance inputs for each of the three regulators in part (a) with respect to rejecting polynomial disturbances $w(t)$ at the input to the plant.

(c) Is this system better at tracking references or rejecting disturbances? Explain your response briefly.

(d) Verify your results for parts (a) and (b) using MATLAB by plotting unit step and ramp responses for both tracking and disturbance rejection.

Solution:

(a)
- P:

$$\frac{Y(s)}{R(s)} = \frac{k_p G(s)}{1 + k_p G(s)} = \frac{19}{5s^2 + 6s + 20}$$

Using the FVT, $y_{ss} = \frac{19}{20}$ and the steady-state error is $e_{ss} = \frac{1}{20}$.
Type 0, $K_p = k_p = 19$

- PD:

$$\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)} = \frac{19 + 4s}{5s^2 + 10s + 20}$$

Using the FVT, $y_{ss} = \frac{19}{20}$ and the steady-state error is $e_{ss} = \frac{1}{20}$.
Type 0, $K_p = k_p = 19$

- PID:

$$\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)} = \frac{8s^2 + 38s + 19}{10s^3 + 20s^2 + 40s + 19}$$

Using the FVT, $y_{ss} = 1$ and the steady-state error is zero. Type 1, $K_v = k_I = 9.5$

- P:

$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1+k_pG(s)} = \frac{1}{5s^2+6s+20}$$

Using the FVT, $y_{ss} = \frac{1}{20}$. Type 0, $K_p = 19$.

- PD:

$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1+D(s)G(s)} = \frac{1}{5s^2+10s+20}$$

Using the FVT, $y_{ss} = \frac{1}{20}$. Type 0, $K_p = 19$

- PID:

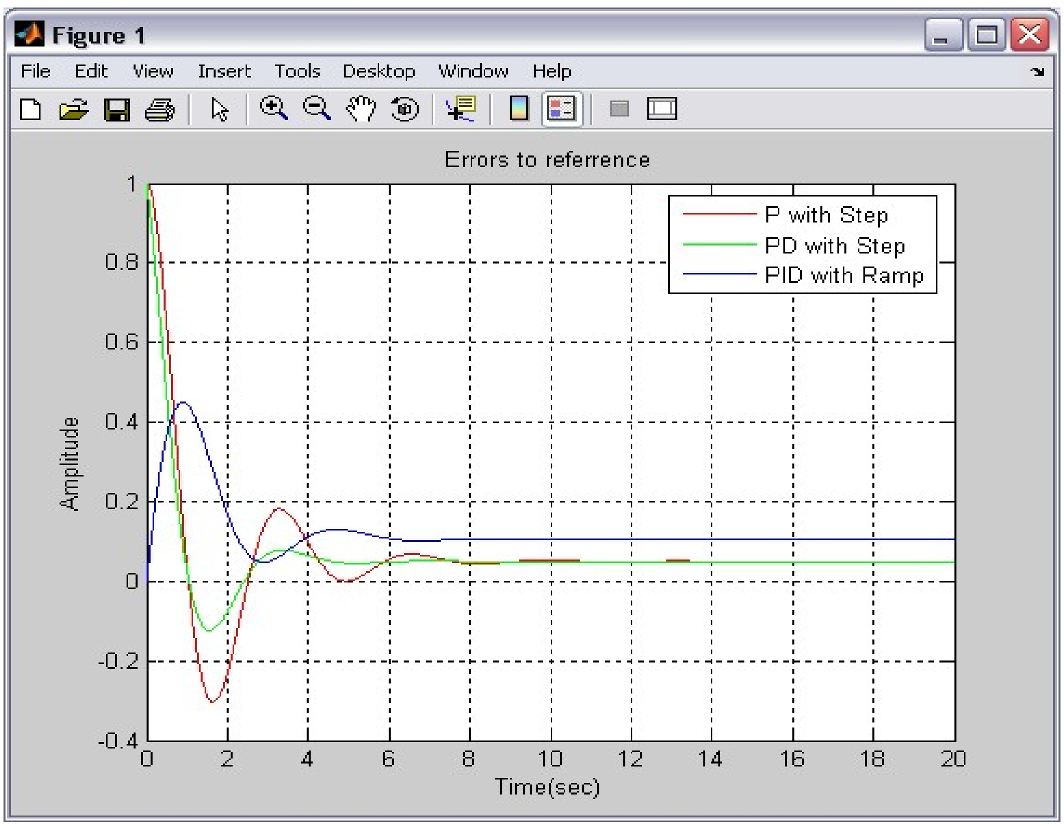
$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1+D(s)G(s)} = \frac{2s}{10s^3+20s^2+40s+19}$$

Using the FVT, $y_{ss} = 0$ and the steady-state error to a step is zero. Type 1, $K_v = 9.5$

d)clear all

```
t=0:0.01:20;

STEP=ones(1,2001); %Step input
RAMP=t;            %Ramp input

%Part (a)
%define Transfer Functions
num_a_P=[5 6 1];
den_a_P=[5 6 20];
num_a_PD=[5 6 1];
den_a_PD=[5 10 20];
num_a_PID=[10 12 2 0];
den_a_PID=[10 20 40 19];

a_P=tf(num_a_P,den_a_P);
a_PD=tf(num_a_PD,den_a_PD);
a_PID=tf(num_a_PID,den_a_PID);

%Part (b)
%define Transfer Functions
num_b_P=1;
den_b_P=[5 6 20];
num_b_PD=1;
den_b_PD=[5 10 20];
num_b_PID=[2 0];
den_b_PID=[10 20 40 19];

b_P=tf(num_b_P,den_b_P);
b_PD=tf(num_b_PD,den_b_PD);
b_PID=tf(num_b_PID,den_b_PID);

%Simulation
asim_P=lsim(a_P,STEP,t);
asim_PD=lsim(a_PD,STEP,t);
```

```
asim_PID=lsim(a_PID,RAMP,t);

bsim_P=lsim(b_P,STEP,t);
bsim_PD=lsim(b_PD,STEP,t);
bsim_PID=lsim(b_PID,RAMP,t);

%Plots
figure(1)
plot(t,asim_P,'r',t,asim_PD,'g',t,asim_PID,'b')
title('Errors to referrence')
xlabel('Time(sec)')
ylabel('Amplitude')
legend('P with Step','PD with Step','PID with Ramp')
grid

figure(2)
plot(t,bsim_P,'r',t,bsim_PD,'g',t,bsim_PID,'b')
title('Errors to disturbance')
xlabel('Time(sec)')
ylabel('Amplitude')
legend('P with Step','PD with Step','PID with Ramp')
grid
```



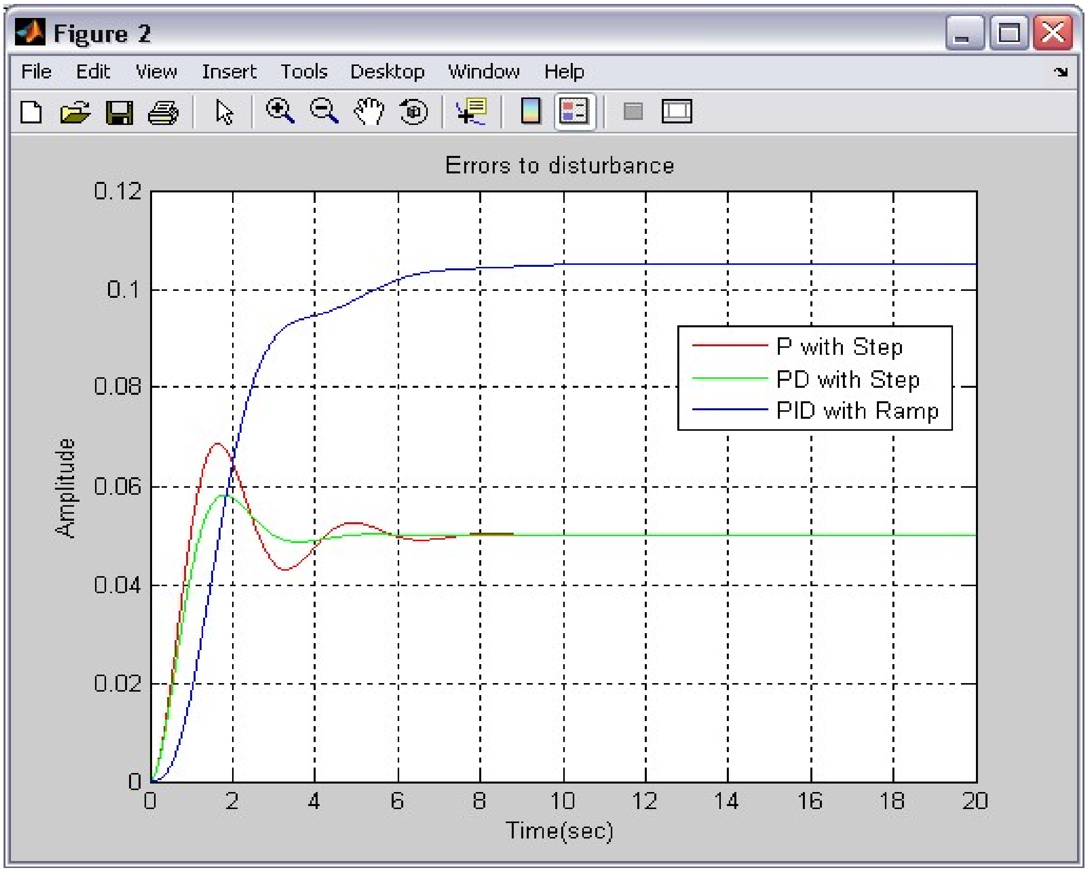


Figure 2
File Edit View Insert Tools Desktop Window Help
Errors to disturbance
0.12
0.1
0.08
0.06
0.04
0.02
0
Amplitude
0 2 4 6 8 10 12 14 16 18 20
Time(sec)
P with Step
PD with Step
PID with Ramp