1. Consider a system with the configuration of Fig.4.2(b) where D_c is the constant gain of the controller and G is that of the process. The nominal values of these gains are $D_c = 5$ and $G = 7$. Suppose a constant disturbance w is added to the control input u before the signal goes to the process.

   (a) Compute the gain from w to y in terms of D_c and G.

   (b) Suppose the system designer knows that an increase by a factor of 6 in the loop gain D_cG can be tolerated before the system goes out of specification. Where should the designer place the extra gain if the objective is to minimize the system error $r-y$ due to the disturbance? For example, either D_c or G could be increased by a factor of 6, or D_c could be doubled and G tripled, and so on. Which choice is the best?

Solution:

(a) Need y/w so set $r = 0$:

$$\frac{Y}{W} = \frac{G}{1+GD_c} = \frac{7}{1+35}$$

(b) Take $r = 0 :\Longrightarrow e = -y$

So to minimize e due to w, increase D_c to 30 (since it is given that beyond this dynamic response goes out of specifications)

6. A unity feedback control system has the open-loop transfer function

$$G(s) = \frac{A}{s(s+a)}.$$

   (a) Compute the sensitivity of the closed-loop transfer function to changes in the parameter A.

   (b) Compute the sensitivity of the closed-loop transfer function to changes in the parameter a.

   (c) If the unity gain in the feedback changes to a value of $\beta \neq 1$, compute the sensitivity of the closed-loop transfer function with respect to β.

   (d) Assuming $A = 1$ and $a = 1$, plot the magnitude of each of the above sensitivity functions for $s = j\omega$ using semilogy command in MATLAB. Comment on the relative effect of parameter variations in A, a, , and β at different frequencies ω, paying particular attention to DC (when $\omega = 0$).

Solution:

(a)

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{A}{s(s+a)}}{1+\frac{A}{s(s+a)}} = \frac{A}{s^2+as+A}$$

$$\frac{dT}{dA} = \frac{(s^2+as+A)-A}{(s^2+as+A)^2}$$

$$\mathcal{S}_A^T = \frac{A}{T}\frac{dT}{dA} = \frac{A(s^2+as+A)}{A}\frac{s^2+as}{(s^2+as+A)^2} = \frac{s(s+a)}{s(s+a)+A}$$

(b)

$$\frac{dT}{da} = \frac{-sA}{(s^2+as+A)^2}$$

$$\frac{a}{T}\frac{dT}{da} = \frac{a(s^2+as+A)}{A}\frac{-sA}{(s^2+as+A)^2}$$

$$\mathcal{S}_a^T = \frac{-as}{s(s+a)+A}$$

(c) In this case,

$$T(s) = \frac{G(s)}{1+\beta G(s)}$$

$$\frac{dT}{d\beta} = \frac{-G(s)^2}{(1+\beta G(s))^2}$$

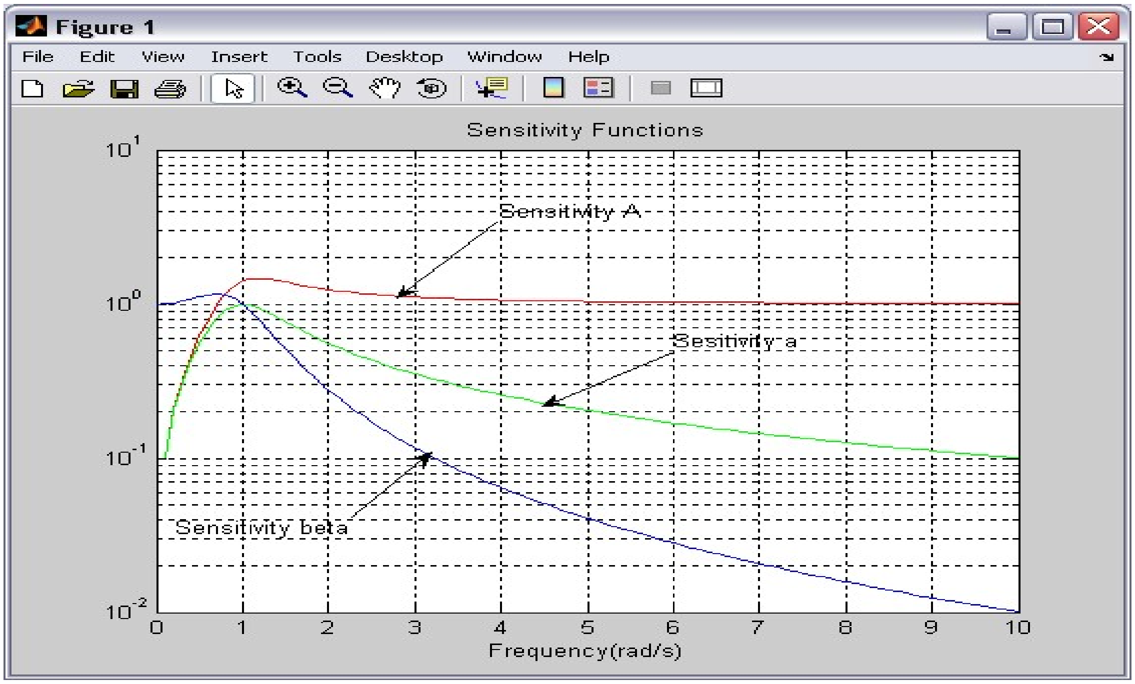
$$\frac{\beta}{T}\frac{dT}{d\beta} = \frac{\beta(1+\beta G)}{G}\frac{-G^2}{(1+\beta G)^2} = \frac{-\beta G}{1+\beta G}$$

$$\mathcal{S}_\beta^T = \frac{\frac{-\beta A}{s(s+A)}}{1+\frac{\beta A}{s(s+a)}} = \frac{-\beta A}{s(s+a)+\beta A}$$

- Transfer function is most sensitive to variations in a and A near $\omega = 1$ rad/sec (due to the fact that $a = 1$).
- Steady-state response is not affected by variations in A and a ($\mathcal{S}_A^T(0)$ and $\mathcal{S}_a^T(0)$ are both zeros).
- Steady-state response is heavily dependent on β since $|\mathcal{S}_\beta^T(0)| = 1$. See attached plots

d) clear all

```
A=1;
a=1;
beta=1;

%Sensitivity A
numA=[1 a 0];
denA=[1 a A];

%Sensitivity a
numa=[-a 0];
dena=[1 a A];

%Sensitivity beta
numbeta=[-beta*A];
denbeta=[1 a beta*A];

w=0:0.1:10;
[yA wA]=freqs(numA, denA, w);
[ya wa]=freqs(numa, dena, w);
[ybeta wbeta]=freqs(numbeta, denbeta, w);

AbsA=abs(yA);
Absa=abs(ya);
Absbeta=abs(ybeta);

figure(1)
semilogy(wA,AbsA,'r',wa,Absa,'g',wbeta,Absbeta,'b')
title('Sensitivity Functions')
xlabel('Frequency(rad/s)')
grid
```



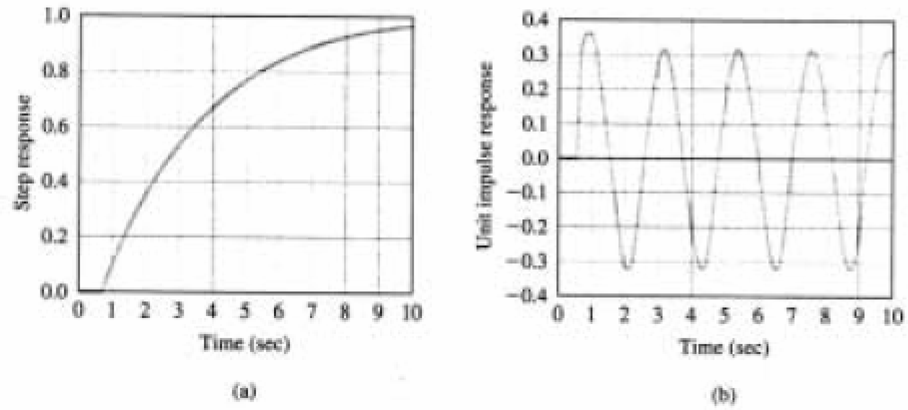


Figure 4.41: Paper machine response data for Problem 4.9

9. ¥The unit-step response of a paper machine is shown in Fig. 4.41(a) where the input into the system is stock flow onto the wire and the output is basis weight (thickness). The time delay and slope of the transient response may be determined from the figure.

   (a) Find the proportional, PI, and PID-controller parameters using the Zeigler–Nichols transient-response method.

   (b) Using proportional feedback control, control designers have obtained a closed-loop system with the unit impulse response shown in Fig. 4.41(b). When the gain $K_u = 8.556$, the system is on the verge of instability. Determine the proportional-, PI-, and PID-controller parameters according to the Zeigler–Nichols ultimate sensitivity method.

Solution:

(a) From step response: $L = \tau_d \simeq 0.65$ sec

$$R = \frac{1}{\tau} \simeq \frac{0.2}{1.25 - 0.65} = 0.33 \quad \text{sec}^{-1}$$

From Table 4.1:

Controller Gain P $\quad K = \frac{1}{RL} = 4.62$

PI $\quad K = \frac{0.9}{RL} = 4.15 \quad T_I = \dfrac{L}{0.3} = 2.17$

PID $\quad K = \frac{1.2}{RL} = 5.54 \quad T_I = 2L = 1.3 T_D = 0.5L = 0.33$

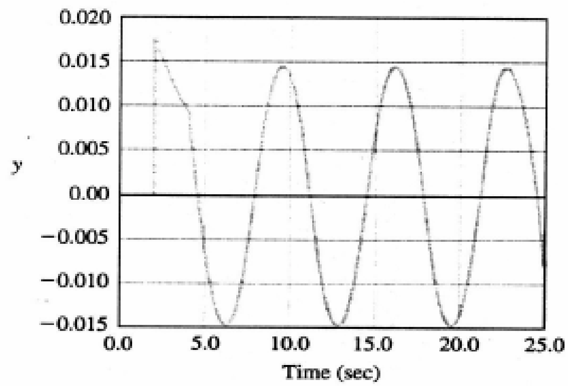


Figure 4.42: Unit impulse response for paper machine in Problem 4.10

(b) From the impulse response: $P_u \simeq 2.33$ sec. and from Table 4.2:

Controller Gain P $K = 0.5K_u = 4.28$

PI $K = 0.45K_u = 3.85$ $T_I = \frac{1}{1.2}P_u = 1.86$

PID $K = 0.6K_u = 5.13$ $T_I = \frac{1}{2}P_u = 1.12 T_D = \frac{1}{8}P_u = 0.28$