25. For the unity feedback system shown in Fig. 3.60, specify the gain and pole location of the compensator so that the overall closed-loop response to a unit-step input has an overshoot of no more than 25%, and a 1% settling time of no more than 0.1 sec. Verify your design using MATLAB.

Solution: The transfer function from R to Y is given by

$$\frac{Y(s)}{R(s)} = \frac{100K}{s^2 + (25+a)s + 25a + 100K} = \frac{100K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The given information is

$$R(s) = \frac{1}{s} \quad \text{(unit step)}$$
$$M_p \le 25\%$$
$$t_s \le 0.1$$

Since $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$, we have

$$\zeta = \sqrt{\frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}} \ge 0.4037$$

Furthermore, it follows that

$$\omega_n \ge \frac{46.05}{\zeta}$$

from

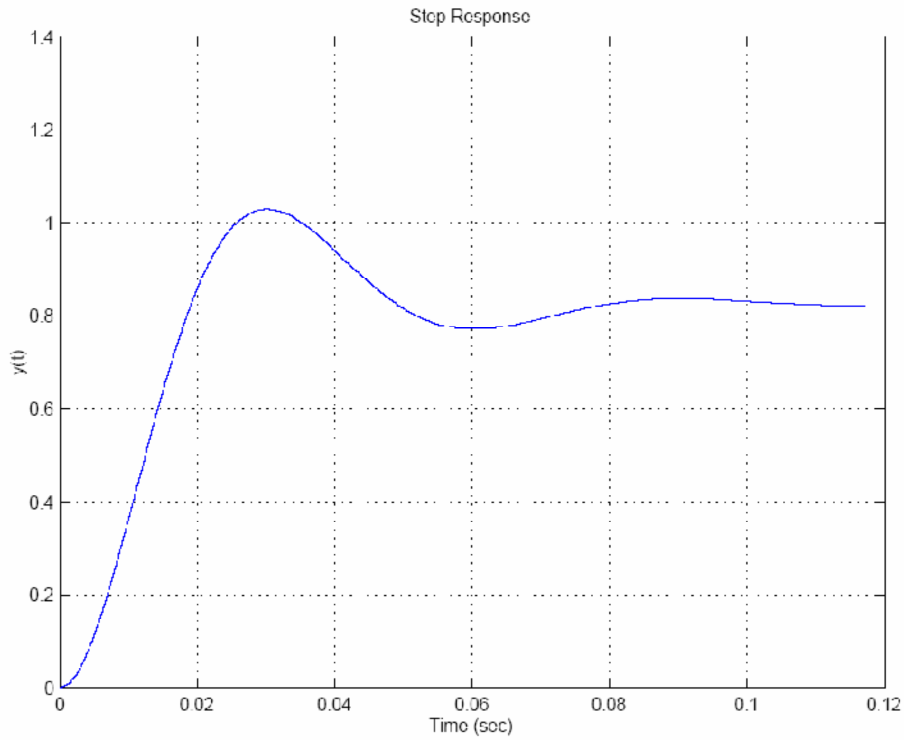
$$t_s = \frac{4.605}{\zeta\omega_n} \le 0.1$$

Let $\zeta = 0.4037$ and $\omega_n = 46.05/\zeta = 114.07$. Then

$$2\zeta\omega_n = 25 + a \quad \Rightarrow \quad a = 67.10$$

$$\omega_n^2 = 25a + 100K \quad \Rightarrow \quad K = 113.34$$

The step response of the system using MATLAB is shown below.



Step Response for Problem 3.25

29. The equations of motion for the DC motor shown in Fig. 2.26 were given in Eqs. (2.63-64) as

$$J_m \ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a} \right) \dot{\theta}_m = \frac{K_t}{R_a} v_a.$$

Assume that

$$\begin{aligned} J_m &= 0.01 \text{ kg} \cdot \text{m}^2, \\ b &= 0.001 \text{ N} \cdot \text{m} \cdot \text{sec}, \\ K_e &= 0.02 \text{ V} \cdot \text{sec}, \\ K_t &= 0.02 \text{ N} \cdot \text{m/A}, \\ R_a &= 10\ \Omega. \end{aligned}$$

(a) Find the transfer function between the applied voltage v_a and the motor speed $\dot{\theta}_m$.

(b) What is the steady-state speed of the motor after a voltage $v_a = 10$ V has been applied?

(c) Find the transfer function between the applied voltage v_a and the shaft angle θ_m.

(d) Suppose feedback is added to the system in part (c) so that it becomes a position servo device such that the applied voltage is given by

$$v_a = K(\theta_r - \theta_m),$$

where K is the feedback gain. Find the transfer function between θ_r and θ_m.

(e) What is the maximum value of K that can be used if an overshoot $M_p < 20\%$ is desired?

(f) What values of K will provide a rise time of less than 4 sec? (Ignore the M_p constraint.)

(g) Use MATLAB to plot the step response of the position servo system for values of the gain $K = 0.5$, 1, and find the overshoot and rise time of the three step responses by examining your plots. Are the plots consistent with your calculations in parts (e) and (f)?

Solution:

$$J_m \ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a} v_a$$

(a)

$$J_m \Theta_m s^2 + \left(b + \frac{K_t K_e}{R_a}\right)\Theta_m s = \frac{K_t}{R_a} V_a(s)$$

$$\frac{s\Theta_m(s)}{V_a(s)} = \frac{\frac{K_t}{R_a J_m}}{s + \frac{b}{J_m} + \frac{K_t K_e}{R_a J_m}}$$

$$\begin{aligned}
J_m &= 0.01 kg \cdot m^2, \\
b &= 0.001 N \cdot m \cdot \text{sec}, \\
K_e &= 0.02 V \cdot \text{sec}, \\
K_t &= 0.02 N \cdot m/A, \\
R_a &= 10\Omega.
\end{aligned}$$

$$\frac{s\Theta_m(s)}{V_a(s)} = \frac{0.2}{s + 0.104}$$

(b) Final Value Theorem

$$\dot{\theta}(\infty) = \frac{s(10)(0.2)}{s(s+0.104)}|_{s=0} = \frac{2}{0.104} = 19.23$$

(c)

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{0.2}{s(s+0.104)}$$

(d)

$$\begin{aligned}
\Theta_m(s) &= \frac{0.2K(\Theta_r - \Theta_m)}{s(s+0.104)} \\
\frac{\Theta_m(s)}{\Theta_r(s)} &= \frac{0.2K}{s^2 + 0.104s + 0.2K}
\end{aligned}$$

(e)

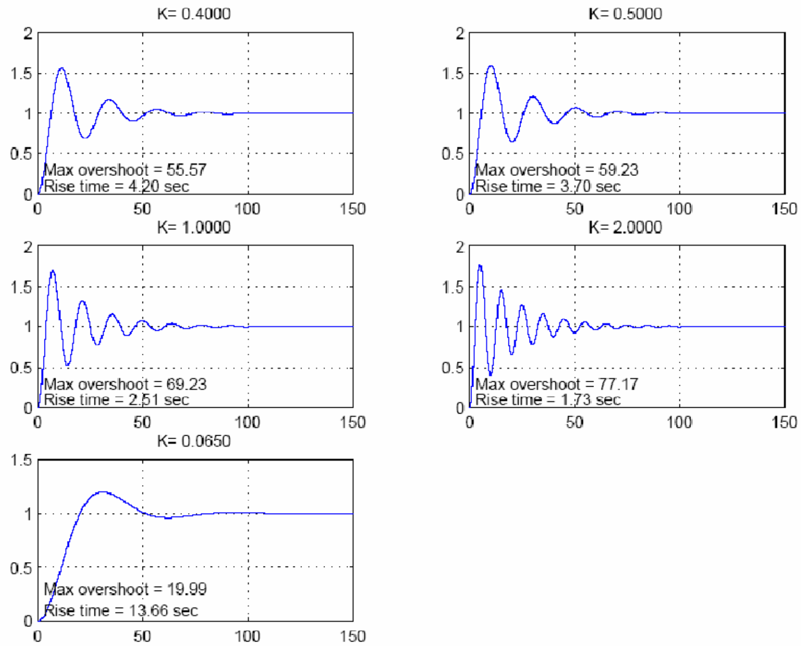
$$\begin{aligned}
M_p &= e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.2 \qquad (20\%) \\
\zeta &= 0.4559 \\
Y(s) &= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \\
2\zeta\omega_n &= 0.104 \\
\omega_n &= \frac{0.104}{2(0.4559)} = 0.114 rad/\sec \\
\omega_n^2 &= 0.2K \\
K &< 6.50 \times 10^{-2}
\end{aligned}$$

(f)

$$\begin{aligned}
\omega_n &\geq \frac{1.8}{t_r} \\
\omega_n^2 &= 0.2K \\
K &\geq 1.01
\end{aligned}$$

(g) MATLAB

```
clear all
close all
K1=[0.5 1.0 2.0 6.5e-2];
t=0:0.01:150;
for i=1:1:length(K1)
K = K1(i);
titleText = sprintf(' K= %1.4f ', K);
wn = sqrt(0.2*K);
num=wn^2;
den=[1 0.104 wn^2];
zeta=0.104/2/wn;
sys = tf(num, den);
y= step(sys, t);
% Finding maximum overshoot
if zeta < 1
Mp = (max(y) - 1)*100;
overshootText = sprintf(' Max overshoot = %3.2f %', Mp);
else
overshootText = sprintf(' No overshoot');
end
% Finding rise time
idx_01 = max(find(y<0.1));
idx_09 = min(find(y>0.9));
t_r = t(idx_09) - t(idx_01);
risetimeText = sprintf(' Rise time = %3.2f sec', t_r);
% Plotting
subplot(3,2,i);
plot(t,y);
grid on;
title(titleText);
text( 0.5, 0.3, overshootText);
text( 0.5, 0.1, risetimeText);
end
```



Problem 3.29: Closed-loop step responses

For part (e) we concluded that $K < 6.50 \times 10^{-2}$ in order for $M_p <$ 20%. This is consistent with the above plots. For part (f) we found that $K \geq 1.01$ in order to have a rise time of less than 4 seconds. We actually see that our calculations is slightly off and that K can be $K \geq 0.5$, but since $K \geq 1.01$ is included in $K \geq 0.5$, our answer in part f is consistent with the above plots.

Problem 3

(a)

$$\frac{Y(s)}{R(s)} = \frac{1}{s^2 + K_1 s + K_2}$$

$$M_P = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.17 \quad \Rightarrow \quad \zeta = \sqrt{1 - \frac{1}{1 + \left(\frac{\ln M_P}{\pi}\right)^2}} = 0.4913$$

$$t_s = \frac{4.6}{\zeta\omega_n} = 3 \quad \Rightarrow \quad \omega_n = \frac{4.6}{3\zeta} = 3.1211$$

$$K_1 = 2\zeta\omega_n = 3.0667$$
$$K_2 = \omega_n^2 = 9.7415$$

(b)

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.1556$$
$$t_r = \frac{1.8}{\omega_n} = 0.5767$$