Homework 1 Solution

Problem 1. (FPE 3.18)
From Figure 3.43, we have

$$\begin{aligned}
X_1(s) &= \frac{1}{s}\left(-a_1X_1(s) - a_2X_2(s) - a_3X_3(s) + U(s)\right)\\
X_2(s) &= \frac{1}{s}X_1(s)\\
X_3(s) &= \frac{1}{s}X_2(s),
\end{aligned}$$

from which we have

$$X_1(s) = \frac{1}{s}\left(-a_1X_1(s) - a_2\frac{1}{s}X_1(s) - a_3\frac{1}{s^2}X_1(s) + U(s)\right).$$

This leads to

$$X_1(s) = \frac{\frac{1}{s}}{1 - \left(-a_1 - a_2\frac{1}{s} - a_3\frac{1}{s^2}\right)\frac{1}{s}}U(s) = \frac{s^2}{s^3 + a_1s^2 + a_2s + a_3}U(s),$$

from which it follows that

$$\begin{aligned}
X_2(s) &= \frac{1}{s}X_1(s) = \frac{s}{s^3 + a_1s^2 + a_2s + a_3}U(s)\\
X_3(s) &= \frac{1}{s}X_2(s) = \frac{1}{s^3 + a_1s^2 + a_2s + a_3}U(s)\\
Y(s) &= b_1X_1(s) + b_2X_2(s) + b_3X_3(s) = \frac{b_1s^2 + b_2s + b_3}{s^3 + a_1s^2 + a_2s + a_3}U(s).
\end{aligned}$$

Therefore, the transfer function from $U(s)$ to $Y(s)$ is

$$\frac{Y(s)}{U(s)} = \frac{b_1s^2 + b_2s + b_3}{s^3 + a_1s^2 + a_2s + a_3}.$$

Problem 2. (FPE 3.19)
(a) From the block diagram, we have

$$\frac{Y}{R} = \frac{G_1}{1 + G_1} + G_2.$$

(b) From the block diagram, we have

$$\frac{Y}{R} = \left(\frac{G_1}{1 + G_2G_1}\right)G_3\left(\frac{G_4}{1 + G_5G_4}\right)G_6 + G_7 = \frac{G_1G_3G_4G_6}{(1 + G_1G_2)(1 + G_4G_5)} + G_7.$$

(c) From the block diagram, we have

$$\frac{Y}{R} = \left[\left\{G_1\left(\frac{G_2}{1 + G_2}\right)G_3 + G_6\right\}\frac{G_4}{1 + G_4}\right]G_5 + G_7 = \frac{G_1G_2G_3G_4G_5}{(1 + G_2)(1 + G_4)} + \frac{G_4G_5G_6}{1 + G_4} + G_7.$$

Problem 3. The system given in Problem 3 can be reduced to the following system in Figure 1.

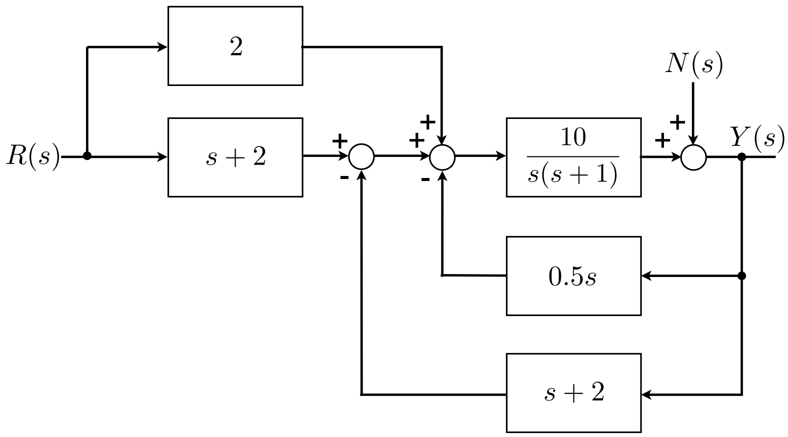

Figure 1: First reduction

The system in Figure 1 can be reduced again to the system in Figure 2.

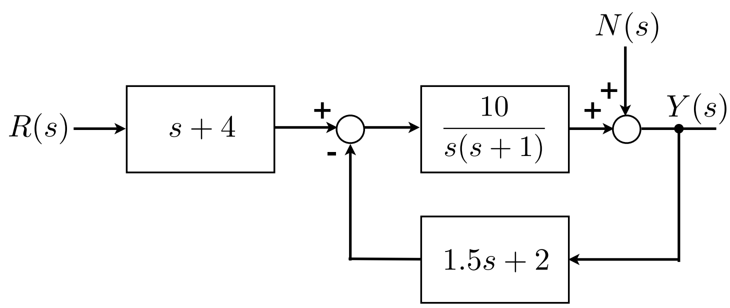

Figure 2: Second reduction

Then from the system in Figure 2, we have

$$
\begin{aligned}
Y(s) &= \frac{\frac{10}{s(s+1)}}{1+(1.5s+2)\frac{10}{s(s+1)}}(s+4)R(s) + \frac{1}{1+(1.5s+2)\frac{10}{s(s+1)}}N(s) \\
&= \frac{10(s+4)}{s^2+16s+20}R(s) + \frac{s(s+1)}{s^2+16s+20}N(s).
\end{aligned} \tag{1}
$$

(a) Letting $N(s)=0$, it follows, from (1), that

$$\frac{Y(s)}{R(s)} = \frac{10(s+4)}{s^2+16s+20}.$$

(b) Letting $N(s)=0$, it follows, from (1), that

$$\frac{Y(s)}{E(s)} = \frac{Y(s)}{R(s)-Y(s)} = \frac{\frac{Y(s)}{R(s)}}{1-\frac{Y(s)}{R(s)}} = \frac{\frac{10(s+4)}{s^2+16s+20}}{1-\frac{10(s+4)}{s^2+16s+20}} = \frac{10(s+4)}{s^2+6s-20}.$$

(c) Letting $R(s)=0$, it follows, from (1), that

$$\frac{Y(s)}{N(s)} = \frac{s(s+1)}{s^2+16s+20}.$$