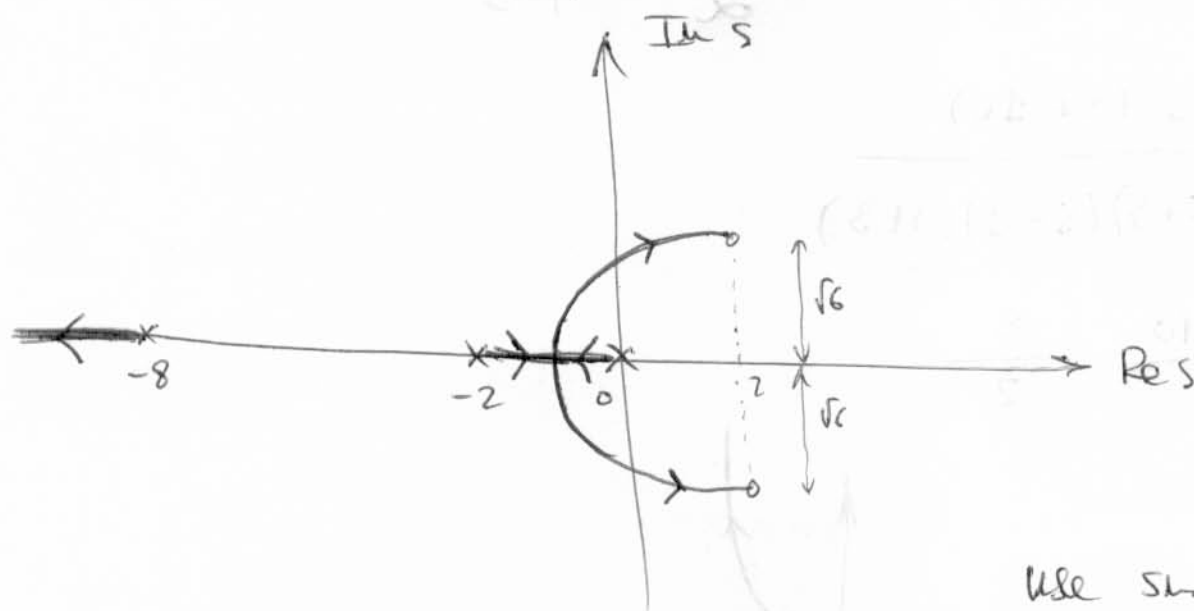


Problem 1

a) $G_1(s) = \frac{s^2 - 4s + 10}{s(s+2)(s+8)}$

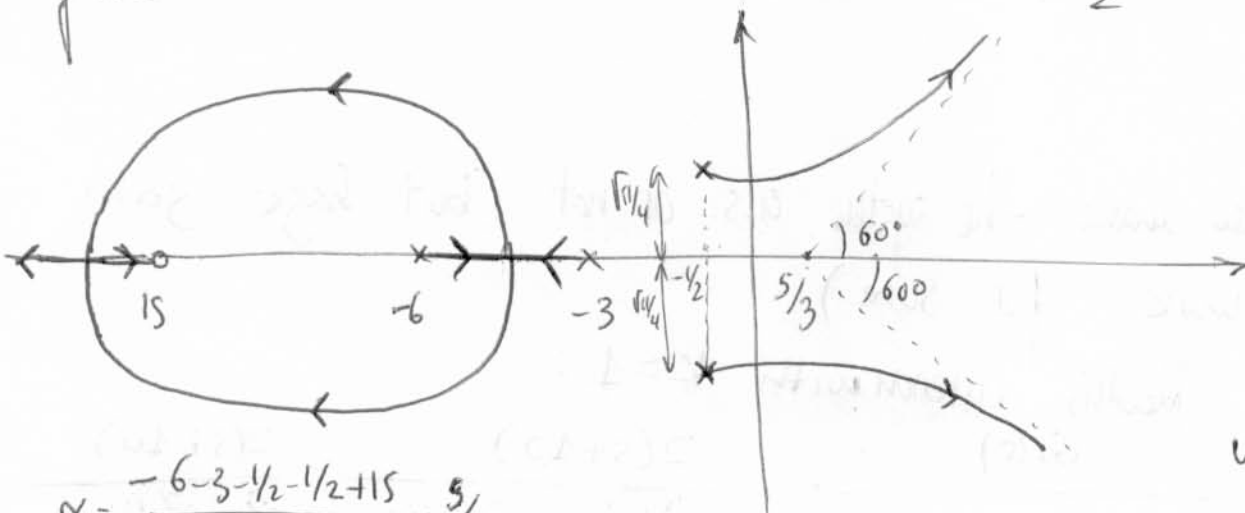
factorize the numerator: $s^2 - 4s + 10 = 0 \Leftrightarrow s = \frac{4 \pm \sqrt{16 - 40}}{2} = 2 \pm j\sqrt{6}$

$$G_1(s) = \frac{(s - 2 - j\sqrt{6})(s - 2 + j\sqrt{6})}{s(s+2)(s+8)}$$



b) $G_2(s) = \frac{s+15}{(s+6)(s+3)(s^2+s+3)}$

factorize the denominator: $s^2 + s + 3 = 0 \Leftrightarrow s = \frac{-1 \pm \sqrt{1 - 12}}{2} = -\frac{1}{2} \pm j\sqrt{11}/4$



$$\alpha = \frac{-6 - 3 - \frac{1}{2} - \frac{1}{2} + 15}{3} = \frac{5}{3}$$

use small K!

Problem 2

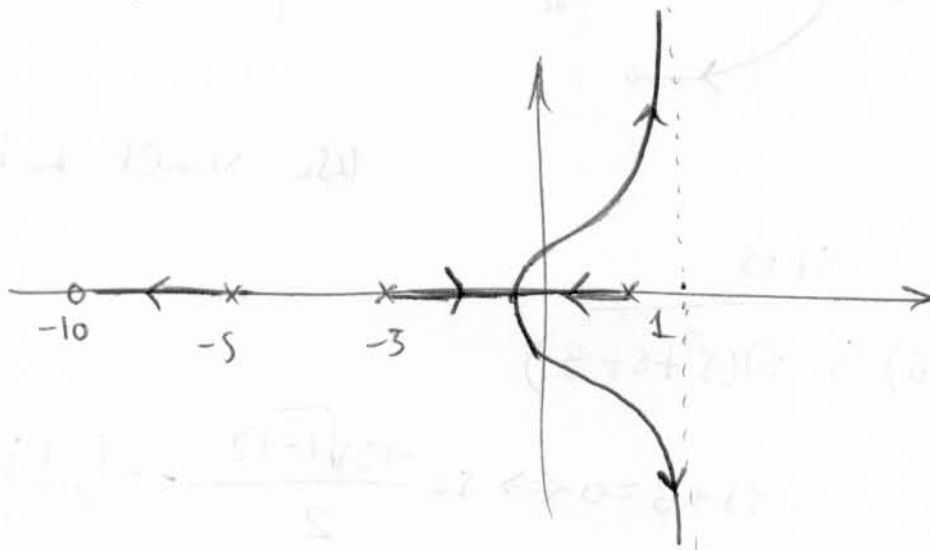
$$(a) \quad G_3(s) = \frac{2(s+10)}{(s+5)(s^2+2s-3)} = \frac{2(s+10)}{(s+5)(s-1)(s+3)}$$

factorize the denominator:

$$s^2+2s-3=0 \Rightarrow s = \frac{-2 \pm \sqrt{4+12}}{2} \quad \begin{cases} 1 \\ -3 \end{cases}$$

$$G_3(s) = \frac{2(s+10)}{(s+5)(s-1)(s+3)}$$

$$\sigma = \frac{-5+1-3+10}{2} = \frac{3}{2}$$



unity fbk can make the system a.s. or not (but large gain does not work for sure)

Check with Routh's criterion with $K=1$:

$$T(s) = \frac{G_3(s)}{1 + G_3(s)} = \frac{2(s+10)}{(s+5)(s^2+2s-3) + 2(s+10)} = \frac{2(s+10)}{s^3+7s^2+9s+5}$$

$$\begin{array}{r} s^3 \quad 1 \quad 9 \\ s^2 \quad 7 \quad 5 \\ s^1 \quad \frac{63-5}{7} \quad 0 \\ 1 \quad 5 \end{array}$$

$K=1$ works!

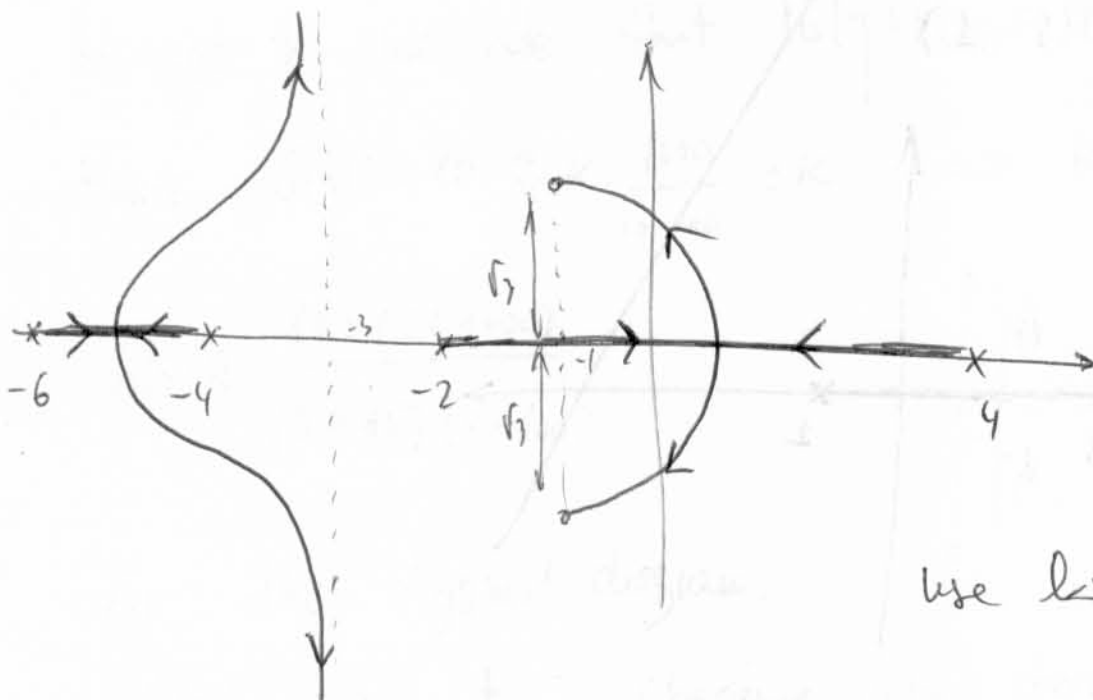
b)

$$G_Y(s) = \frac{s^2 + 2s + 4}{(s^2 - 16)(s+2)(s+6)}$$

$$s^2 + 2s + 4 = 0 \Leftrightarrow s = \frac{-2 \pm \sqrt{4-16}}{2} = -1 \pm j\sqrt{3}$$

$$G_Y(s) = \frac{(s+1-j\sqrt{3})(s+1+j\sqrt{3})}{(s+4)(s-4)(s+2)(s+6)}$$

$$\sigma = \frac{4-4-2-6+1+1}{2} = -3$$



use large gain!

Problem 3

From the Magnitude plot:

- starts at a slope of $+20\text{dB/dec}$ $\rightarrow$ zero at the origin
- at $\omega \approx 10^1$ it goes to 0dB/dec $\rightarrow$ pole at $s=10$
- at $\omega \approx 10^2$ it goes to -20dB/dec $\rightarrow$ pole at $s=100$
- at $\omega \approx 10^3$ it goes to $+0\text{dB/dec}$ $\rightarrow$ zero at $s=1000$

The phase plot verifies the above zeros and poles, moreover they are stable.

$$\text{Hence } G(s) = K s \frac{(s+1000)}{(s+10)(s+100)}$$

to get K we note that $|G(j\omega)|_{\text{dB}} = 20\text{dB}$

$$\text{Hence } |G(j\omega)| = 10 \approx K \cdot \frac{1000}{10 \cdot 100} = K \Rightarrow K \approx 10$$

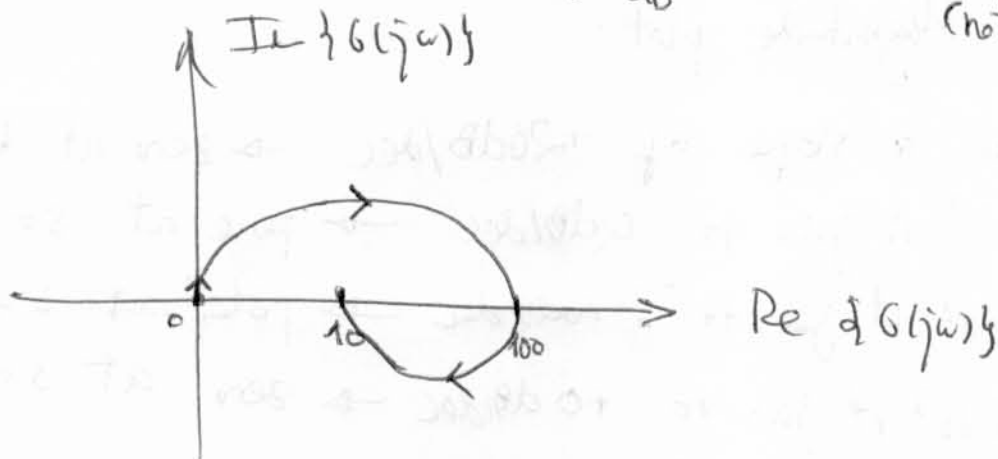
$$G(s) \approx \frac{10 s (s+1000)}{(s+10)(s+100)}$$

For the Nyquist diagram:

$$G(j\omega) = j \cdot 0 \quad \uparrow \quad (\text{because initial phase is } 90^\circ)$$

$$G(j\omega) = 20\text{dB} = 10$$

There is one cross from the 1st quadrant into the second at $\omega \approx 30$ with $|G(j\omega)| \approx 40 \text{ dB} \rightarrow$ cross at ≈ 100 (not exactly)



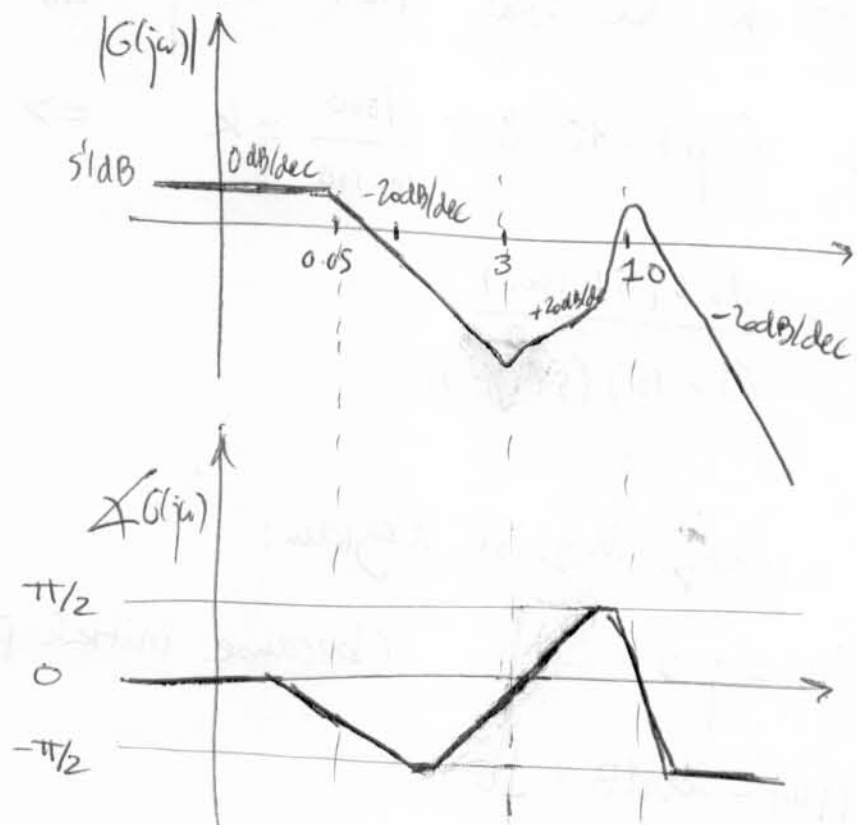
Problem 4.

$$P(s) = \frac{s^2 + 3s + 9}{(s + 0.05)(s^2 + 2s + 100)}$$

resonant zero: $\omega_n = 3$
 $\zeta = 0.05$

resonant pole: $\omega_n = 10$
 $\zeta = 0.1$

$$|P(j\omega)|_{\text{dB}} = |9/s|_{\text{dB}} = 5 \text{ dB}$$



Problem 5

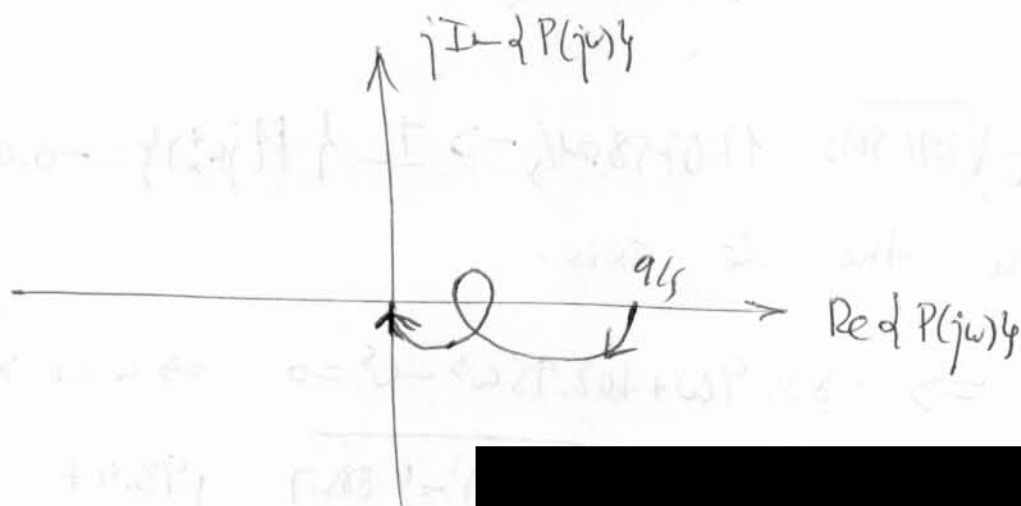
From the previous problem:

$$P(j\omega) = \frac{9}{s}$$

$$P(j\omega) = -j\omega$$

There are two intersections when crossing from the 2nd quadrant into the 1st and back.

Hence:



One can find the crossing points:

$$P(j\omega) = \frac{9 - \omega^2 + 3j\omega}{(j\omega + 0.05)(100 - \omega^2 + 2j\omega)}$$

$$\text{call } \Delta = (0.05^2 - \omega^2)((100 - \omega^2)^2 + 4\omega^2)$$

$$\text{then } P(j\omega) = \frac{((9 - \omega^2) + 3j\omega)(-j\omega + 0.05)(100 - \omega^2 - 2j\omega)}{\Delta}$$

$$= \frac{[(0.05(9 - \omega^2) + 3\omega^2) + j(3\omega \cdot 0.05 - \omega(9 - \omega^2))](100 - \omega^2 - 2j\omega)}{\Delta}$$

Δ

Finally

$$P(j\omega) = \frac{45 + 276.85\omega^2 - 0.95\omega^4}{\Delta} + j \frac{-885.9\omega + 102.95\omega^3 - \omega^5}{\Delta}$$

Crosses with the Im axis:

$$\text{Re}\{P(j\omega)\} = 0 \Rightarrow 45 + 276.85\omega^2 - 0.95\omega^4 = 0$$

$$\Rightarrow \omega^2 = \frac{-276.85 \pm \sqrt{(276.85)^2 + 0.95 \cdot 4 \cdot 45}}{-2 \cdot 0.95}$$

$$= \begin{cases} 291.5835 \\ -0.1625 \times \end{cases}$$

crosses at $\omega_* = \sqrt{291.5835} = 17.0758 \text{ rad/s} \Rightarrow \text{Im}\{P(j\omega_*)\} = -0.0864$

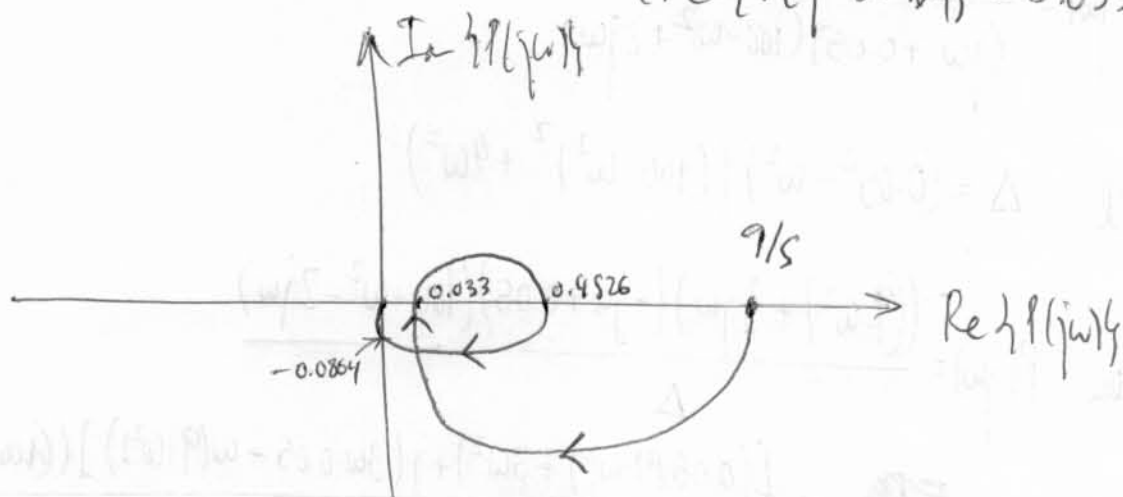
Crosses with the Re axis:

$$\text{Im}\{P(j\omega)\} = 0 \Rightarrow -885.9\omega + 102.95\omega^3 - \omega^5 = 0 \Rightarrow \omega = 0 \times$$

$$\omega^2 = \frac{-102.95 \pm \sqrt{(102.95)^2 - 4 \cdot 885.9}}{-2} = \begin{cases} 93.47 \\ 9.47 \end{cases}$$

crosses at $\omega_* = \begin{cases} \sqrt{93.47} = 9.6681 \text{ rad/s} \\ \sqrt{9.47} = 3.0786 \text{ rad/s} \end{cases} \Rightarrow \begin{cases} \text{Re}\{P(j \cdot 9.6681)\} = 0.4526 \\ \text{Re}\{P(j \cdot 3.0786)\} = 0.033 \end{cases}$

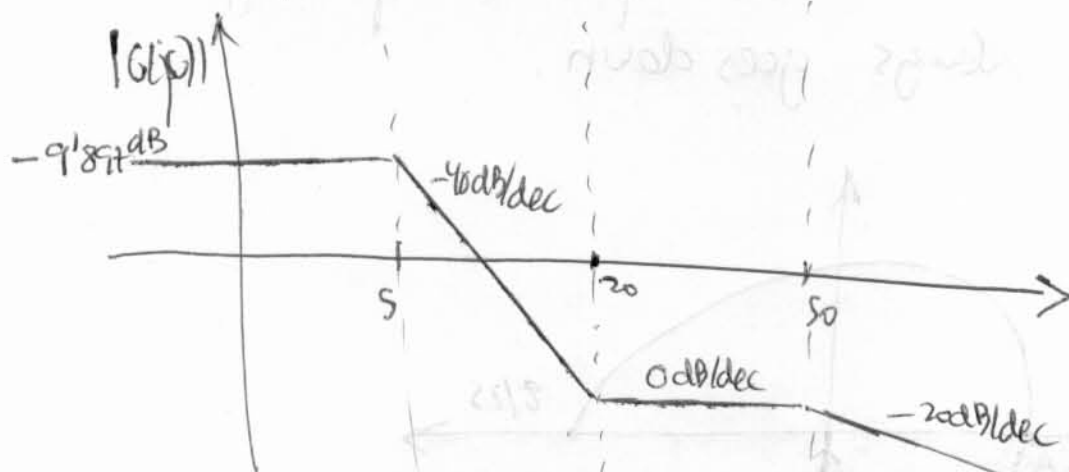
more exact:



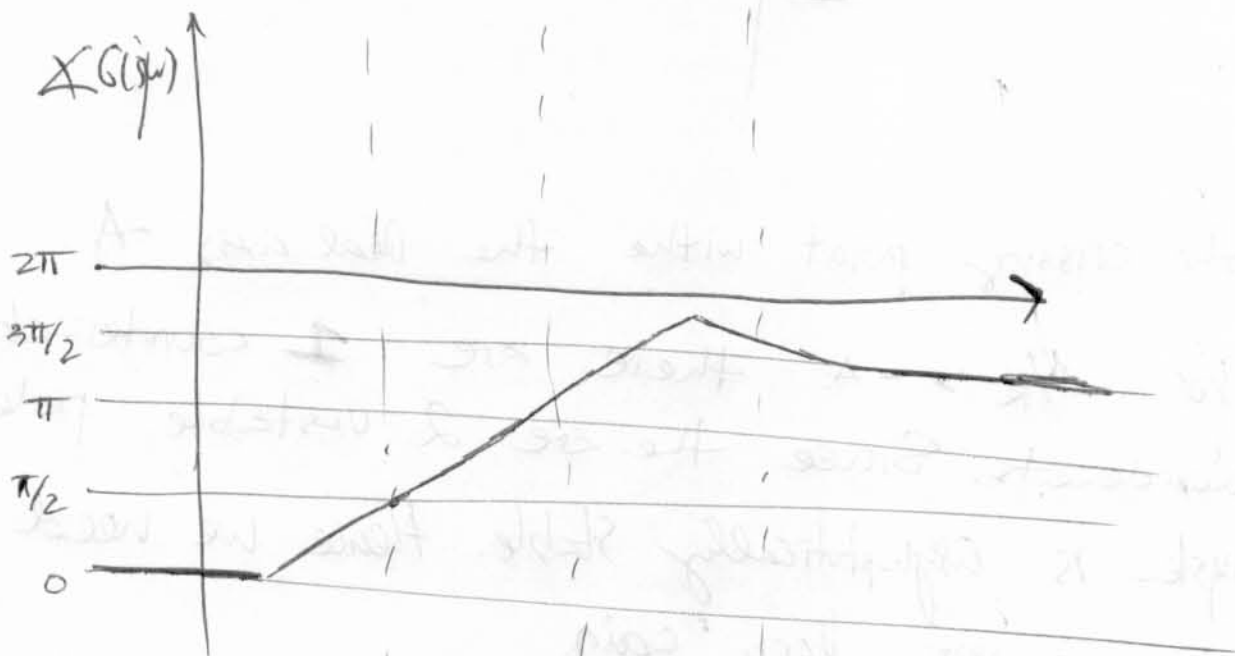
Problem 6

$$G(s) = \frac{(s+20)^2}{(s-5)^2(s+50)}$$

note there are unstable poles! they raise the phase instead of lowering it.



$$\begin{aligned} |G(j\omega)| &= \frac{20^2}{5^2 \cdot 50} = \frac{8}{25} \\ &= -9.87 \text{ dB} \end{aligned}$$



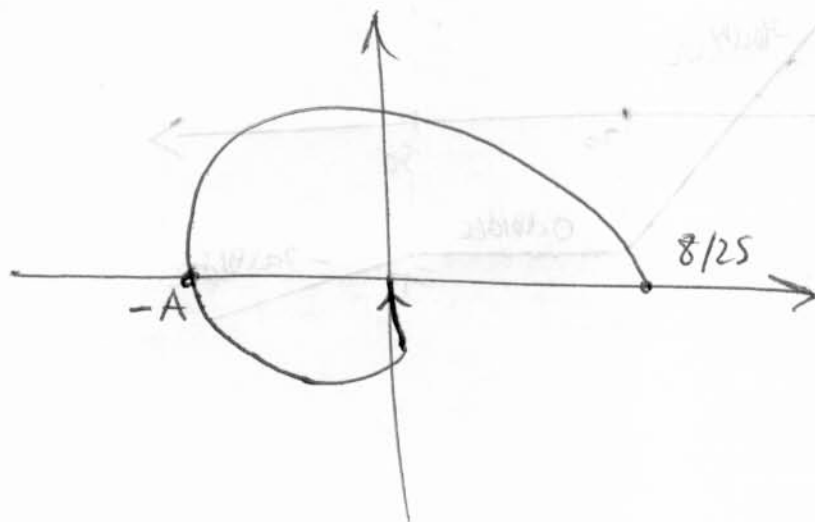
Problem 7

From the Bode:

$$G(j\cdot 0) = 8/25$$

$$G(j\cdot \infty) = -j\cdot 0$$

Goes from the 1st to the 4th quadrant finishing at the $-j$ axis. Magnitude always goes down.



Call the crossing point with the Real axis $-A$.

• Then for $-1/k > -A$ there are **1** counter-clockwise encirclements. Since there are 2 unstable poles

$\Rightarrow$ system is asymptotically stable. Hence we need

$$k > A \Rightarrow \text{very large gain}$$

• if $-1/k < -A$ ($k < A$) no encirclements $\Rightarrow$ system is unstable.

We can solve problem 7 using the transfer function.

$$G(j\omega) = \frac{(j\omega + 20)^2}{(j\omega - 5)^2(j\omega + 50)}$$

call $\Delta = (\omega^2 + 25)^2(\omega^2 + 50^2)$

Then $G(j\omega) = \frac{(j\omega + 20)^2(-j\omega - 5)^2(-j\omega + 50)}{\Delta}$

$$= \frac{500.000 - 36250\omega^2}{\Delta} + j \frac{240.000\omega - 1675\omega^3 - \omega^5}{\Delta}$$

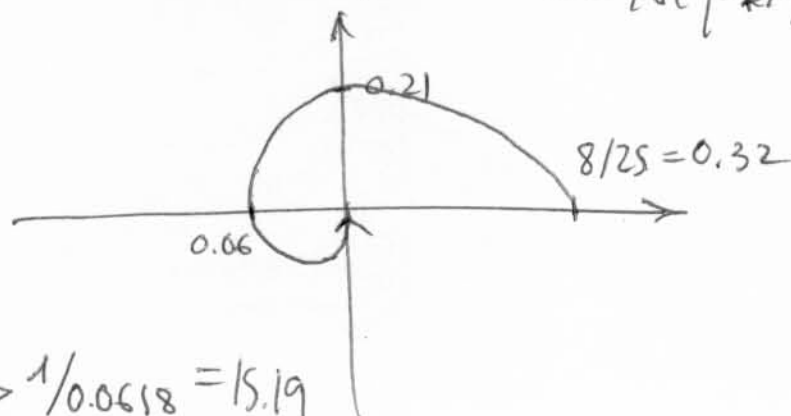
$$\text{Re}\{G(j\omega_*)\} = 0 \Rightarrow \omega_* = \sqrt{\frac{500.000}{3625}} = 3.7139 \text{ rad/s} \Rightarrow \text{Im}\{G(j\omega_*)\} = 0.2127$$

$$\text{Im}\{G(j\omega_*)\} = 0 \Rightarrow \omega = 0 \times$$

$$\text{or } \omega^4 + 1675\omega^2 - 240.000 = 0$$

$$\Rightarrow \omega^2 = \frac{-1675 \pm \sqrt{(1675)^2 + 4 \cdot 240.000}}{2} = \begin{cases} -1807.8 \times \\ 132.76 \end{cases}$$

$$\omega_* = \sqrt{132.76} = 11.522 \text{ rad/s} \Rightarrow \text{Re}\{G(j\omega_*)\} = -0.0658$$



Nyquist criterion: $\underline{k > 1/0.0658 = 15.19}$