MAE 143B

LINEAR CONTROL

Prof. M. Krstic

FINAL

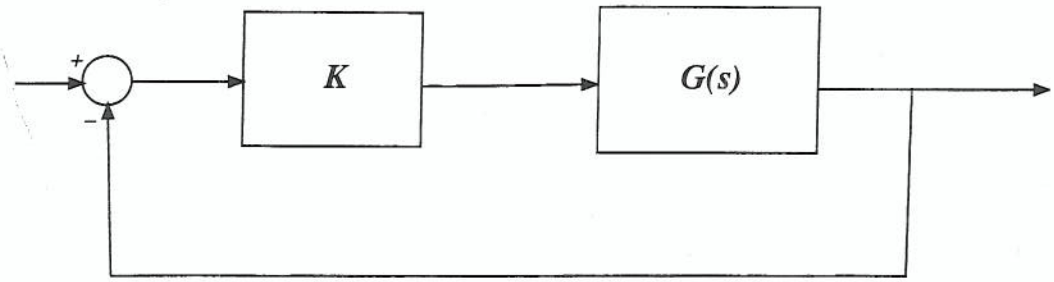
June 14, 2006

NAME: SOLUTIONS

- Open books and notes.
- Graphing calculators are not allowed. No borrowing of calculators.
- Present your reasoning and calculations clearly. Random or inconsistent etchings will not be graded.
- Write only on the paper provided. If you run out of space for a given problem, continue on the pages at the end of the set and indicate "Continued on page X."
- The problems are *not* ordered by difficulty.
- Total points: 60.
- Time: 3.00–6.00 pm.

Problem 1. (7 points)

Consider the feedback system



Which of the following plants is more likely to be stable in feedback with the gain K when K is very large, and why?

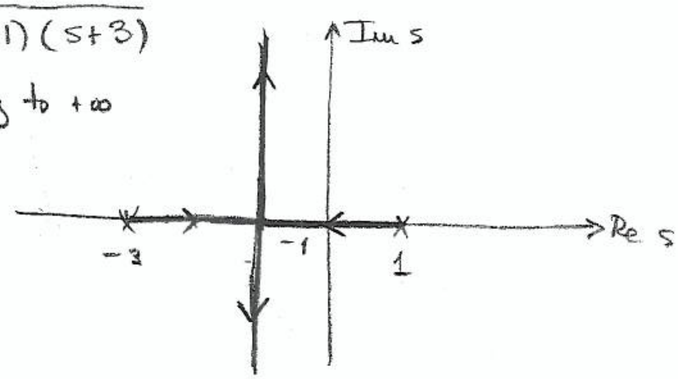
(a) (3.5 points) $G_1(s) = \dfrac{1}{s^2 + 2s - 3}$

(b) (3.5 points) $G_2(s) = \dfrac{1}{(s+1)^3}$

(Sketch the root locus for both G_1 and G_2.)

(a) $G_1(s) = \dfrac{1}{(s-1)(s+3)}$

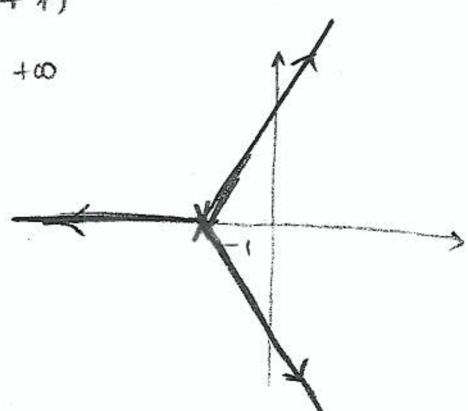
- 2 branches diverging to $+\infty$ at $\pm 90°$
- double pole at -1



will be stable w/. high gain (will have 2 complex conjugate poles with -1 real part)

b) $G_2(s) = \dfrac{1}{(s+1)^3}$

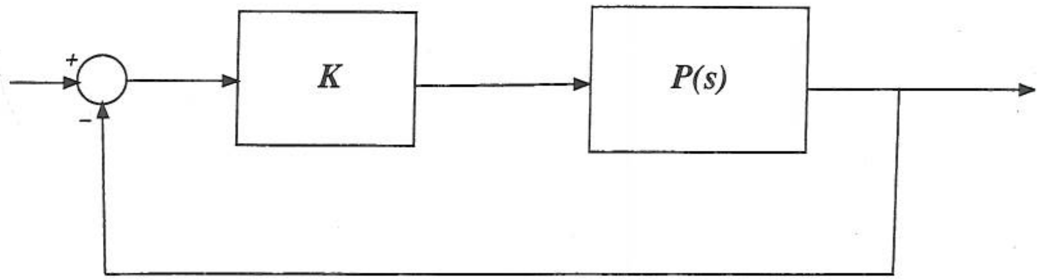
- 3 branches diverging to $+\infty$ at $60°, 180°, 300°$



will be UNSTABLE with high gain (with 2 complex conjugate poles with positive real part)

Problem 2. (8 points)

Consider the feedback system



Suppose that as a feedback designer you are allowed to use only either very large K or very small (positive) K. Which would you choose for each of the following two plants?

(a) (4 points) $P_1(s) = \dfrac{s^2 - 2s + 2}{s(s+1)(s+3)}$

(b) (4 points) $P_2(s) = \dfrac{s(s+1)}{(s^2 - 2s + 2)(s+3)}$

(Sketch the root locus for both P_1 and P_2.)

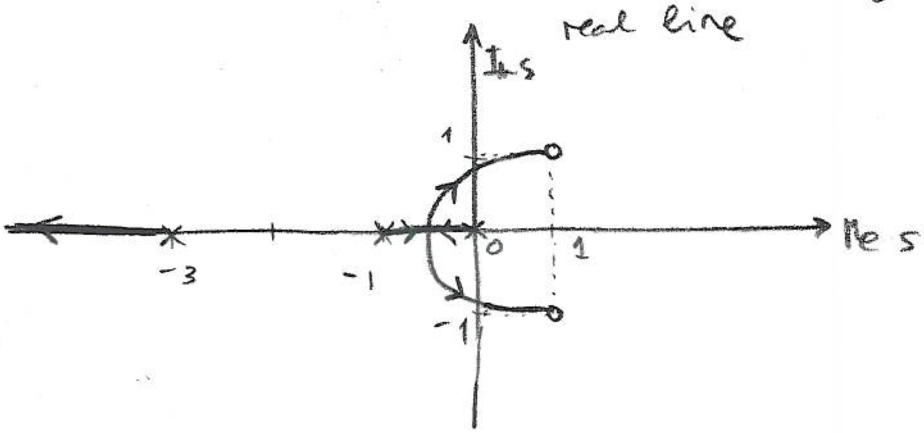
a)

$P_1(s) = \dfrac{(s-1+j)(s-1-j)}{s(s+1)(s+3)}$

$m = 2$ zeros

$n = 3$ poles

one branch diverges to ∞ at $-180°$ through the real line



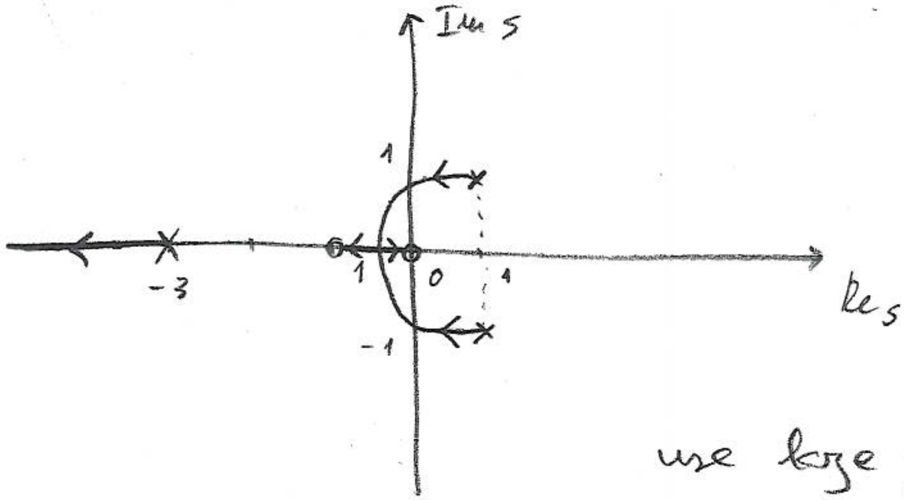
Use small K so the poles are in the LHP

b)

$m = 2$ zeros
$n = 3$ poles

$$P_2(s) = \frac{s(s+1)}{(s-1-j)(s-1+j)(s+3)}$$

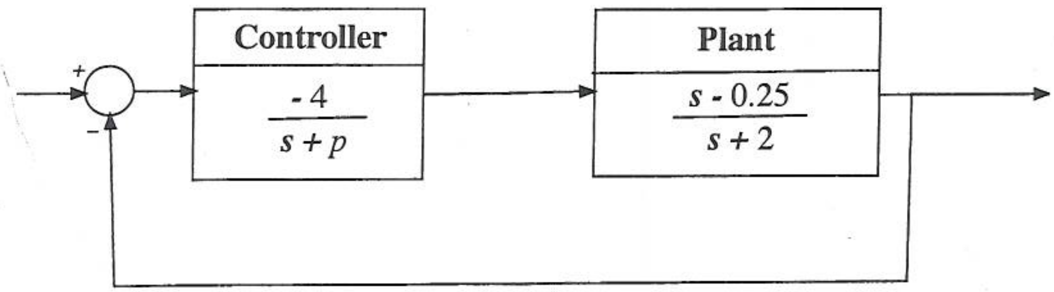
similar to a)



use large gain so the poles move to the LHP!

Problem 3. (8 points)

Consider the following feedback system



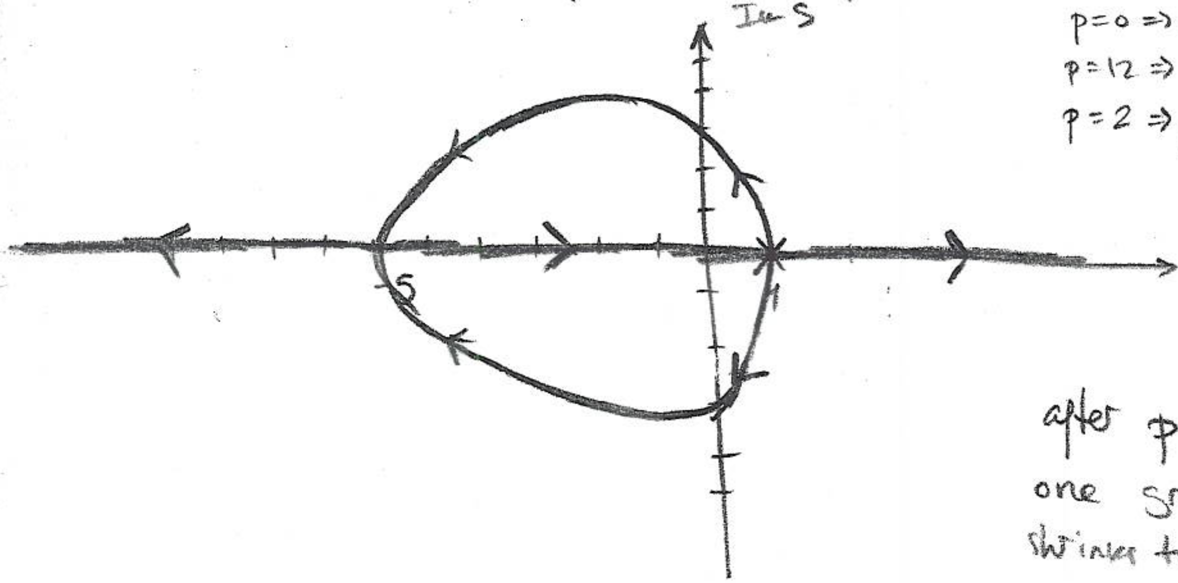
Using the idea from the beginning of the lecture notes on Root Locus, sketch the variation of the closed-loop poles as the controller pole p varies from 0 to $+\infty$.

closed loop transfer function $T(s) = \dfrac{\frac{-4}{s+p}\,\frac{s-0.25}{s+2}}{1+\frac{-4}{s+p}\,\frac{s-0.25}{s+2}} = \dfrac{-4(s-0.25)}{(s+p)(s+2)-4(s-0}$

closed loop poles are the roots of $(s+p)(s+2)-4(s-0.25) = s^2+s(p-2)+2$

roots of $s^2+s(p-2)+2p+1$ are $\dfrac{2-p \pm \sqrt{(p-2)^2-4(2p+1)}}{2} = 1-\frac{p}{2} \pm \dfrac{\sqrt{p(p-12)}}{2}$

2 roots; real when $p \geq 12$ or $p=0$. Double when $p=0, 12$



$p=0 \Rightarrow$ at 1

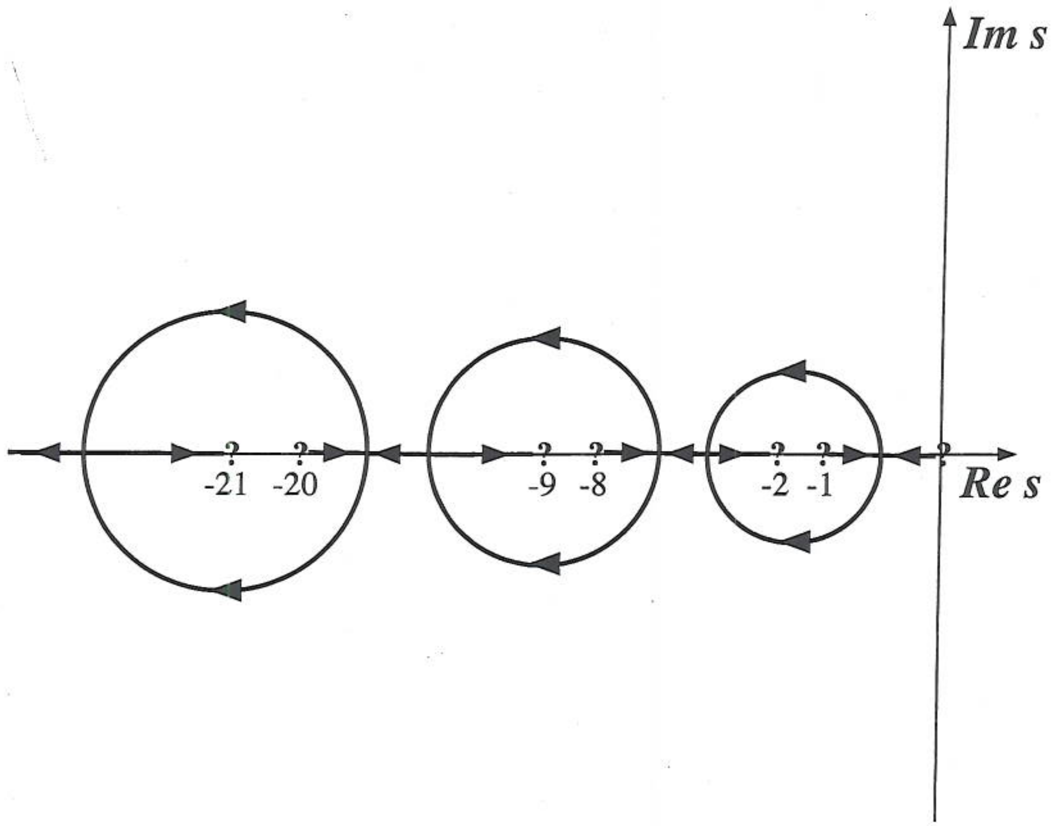
$p=12 \Rightarrow$ at -5

$p=2 \Rightarrow$ pure imaginary at $\pm j\sqrt{10}$ 22 3

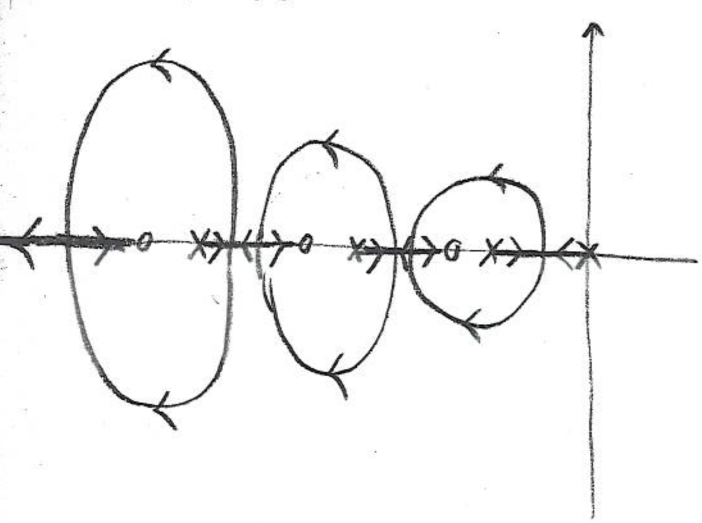
after $p=12$: both real, one grows to $+\infty$ and the other shrinks to $-\infty$

Problem 4. (7 points)

The root locus of a system $G(s)$ looks like



where the symbols "?" represent open-loop poles and zeros. Replace the ?s by symbols "x" and "o" so that the root locus makes sense. Then, write the expression for $G(s)$. You must justify your answers.



place x where arrows are leaving
place o where arrows are arriving

$$G(s) = \frac{(s+21)(s+9)(s+2)}{s(s+20)(s+8)(s+1)}$$

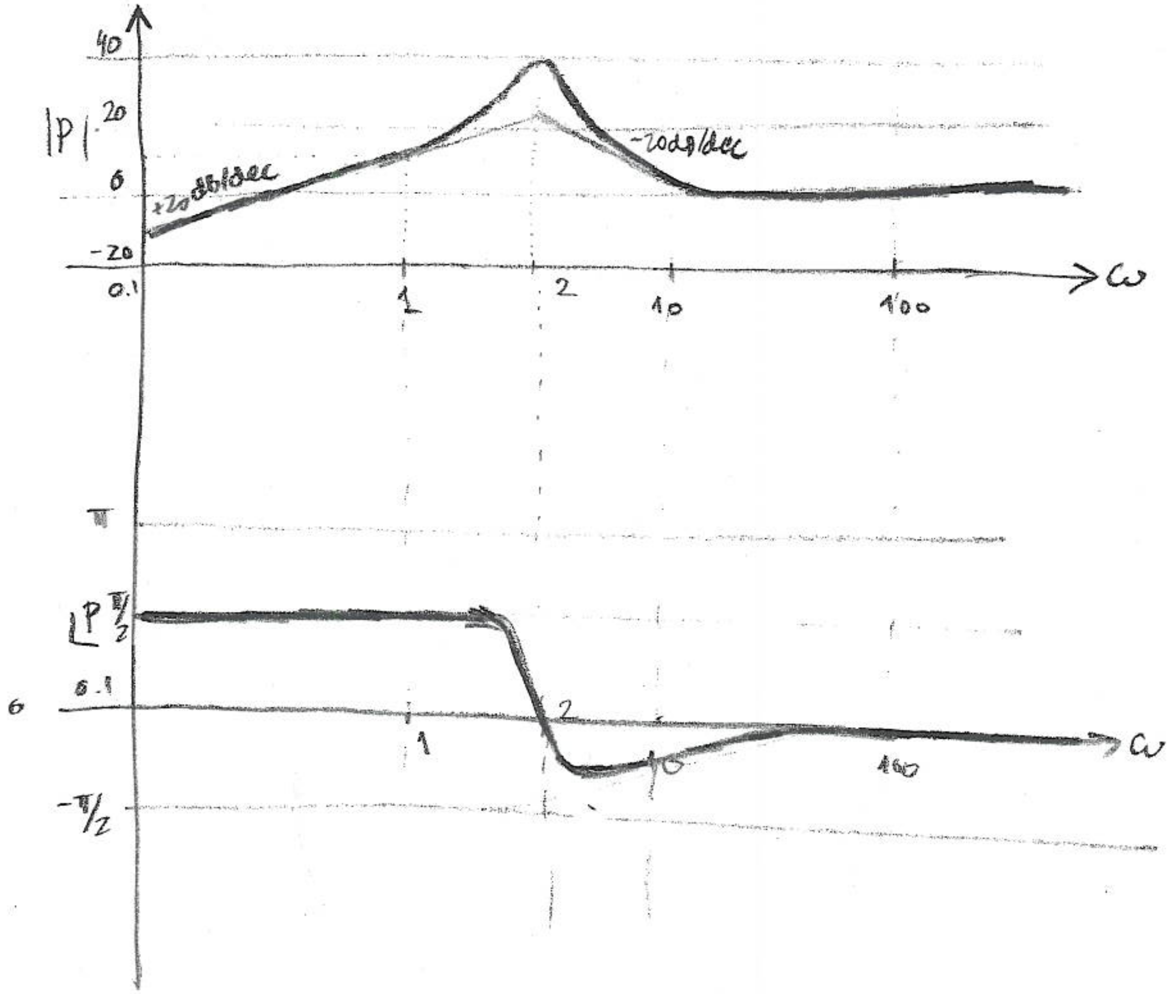
poles: $-20, -8, -1, 0$

zeros: $-21, -9, -2$

Problem 5. (7 points)

Sketch the Bode plots for the system

$$P(s) = \frac{s(s+10)}{s^2 + 0.4s + 4}$$



zeros at $s = 0$, $s = -10$

resonant poles, $\omega_n = 2$, $\zeta = 0.1$

initial phase $+\pi/2$ (from zero at $s = 0$)

initial slope $+20$dB/dec (" " " ")

$P(j \cdot 1) \approx 10/4 = 2.5$

final slope 0dB/dec

final value 0 dB

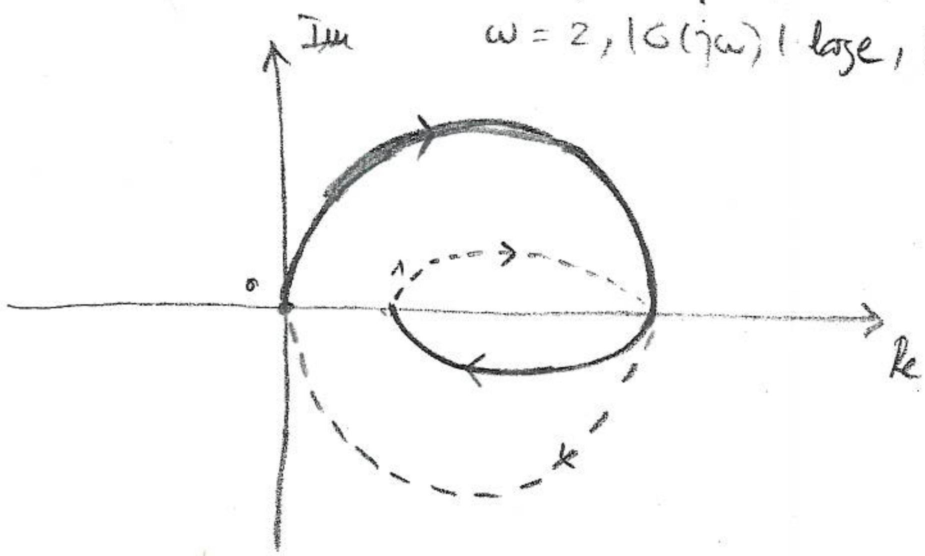
Problem 6. (7 points)

Sketch the Nyquist diagram for the system from Problem 5.

at $\omega = 0$, $|G(j\omega)| = 0$, $\angle G(j\omega) = \pi/2$

$\omega = +\infty$, $|G(j\omega)| = 1$, $\angle G(j\omega) = 0$

$\omega = 2$, $|G(j\omega)|$ large, $\angle G(j\omega) \approx 0$



$\omega \in (0, 2)$ → comes from 0 at $\pi/2$ degrees, magnitude grows and angle decreases till it hits the Re axis

$\omega \in (2, +\infty)$ → negative angle, decreasing magnitude till it hits 1.

Problem 7. (8 points)

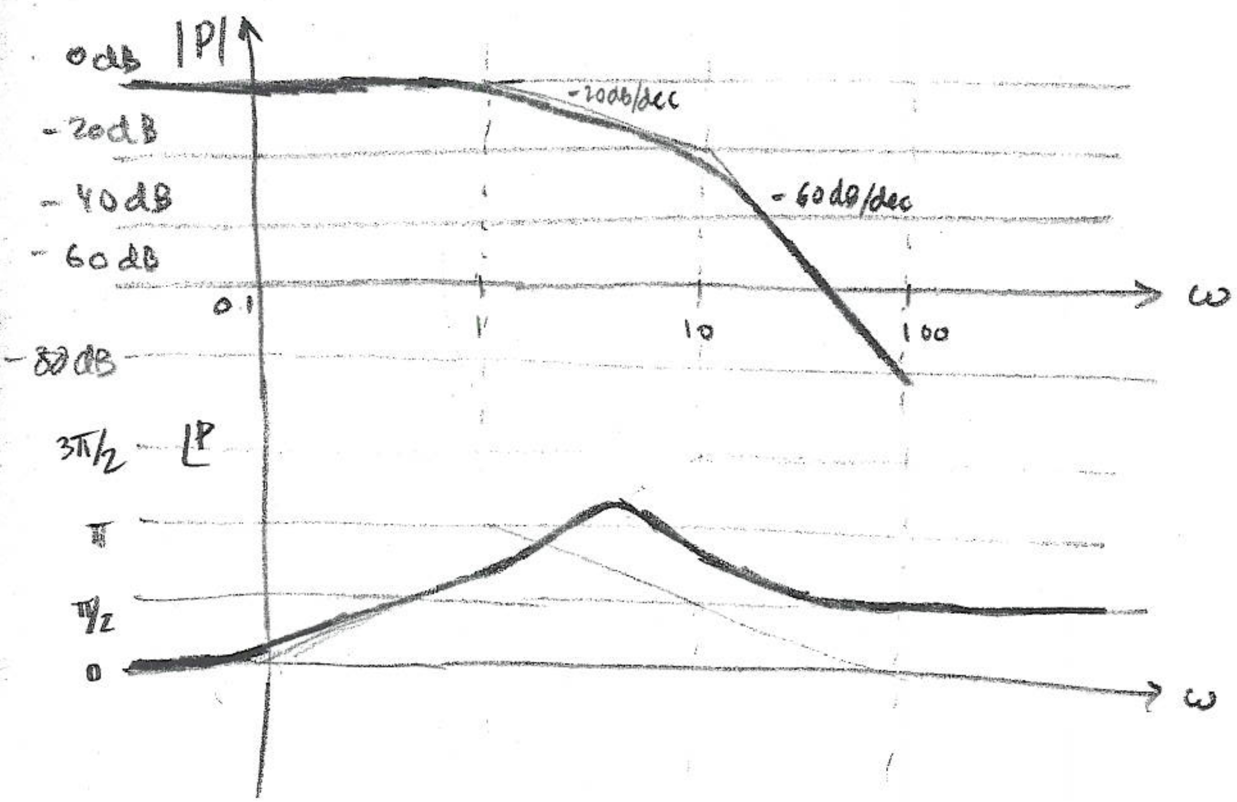
Sketch the Bode plots for the system

$$P(s) = \frac{s+1}{(s-1)^2(1+0.1s)^2}$$

$\frac{s+1}{(s-1)^2}$ produces a slope change of -20 dB/dec and phase change of $+3\pi/2$ around $\omega = 1$

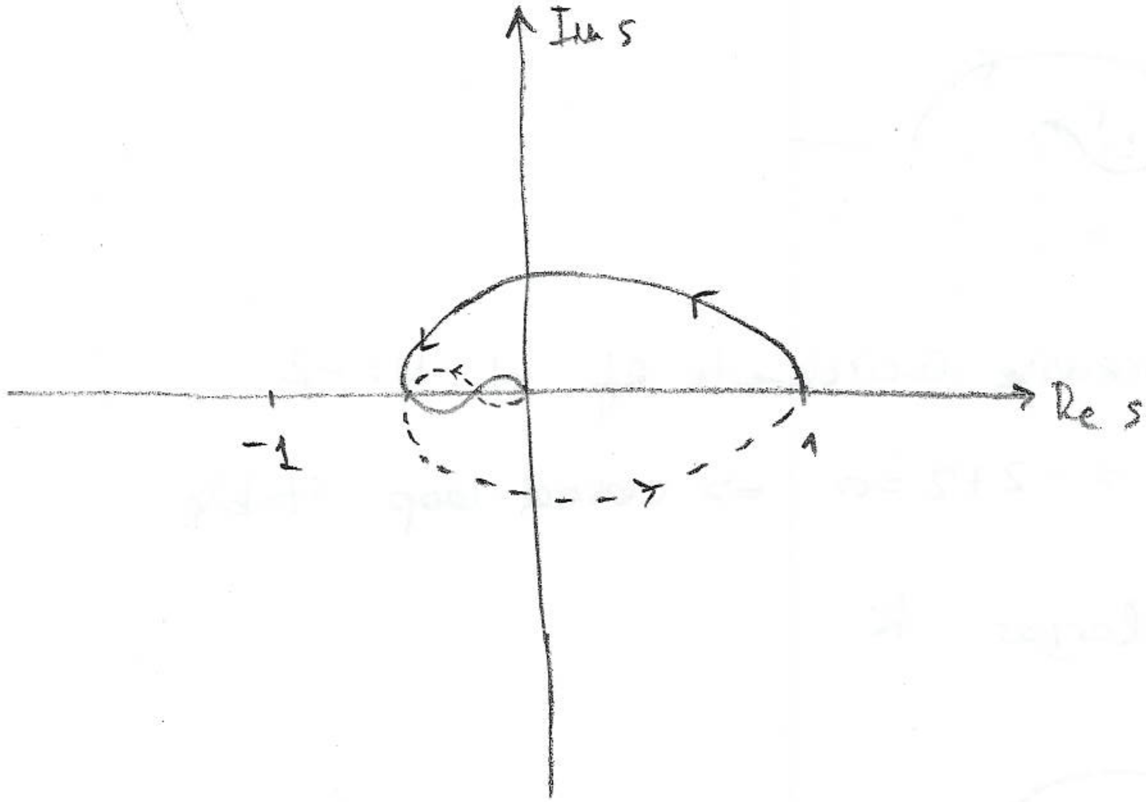
$|P(j\omega)|_{\omega=0} = 1$, no poles/zeros at the origin

$\Rightarrow$ the Bode at low frequencies is constant at 0 dB, 0°.



Problem 8. (8 points)

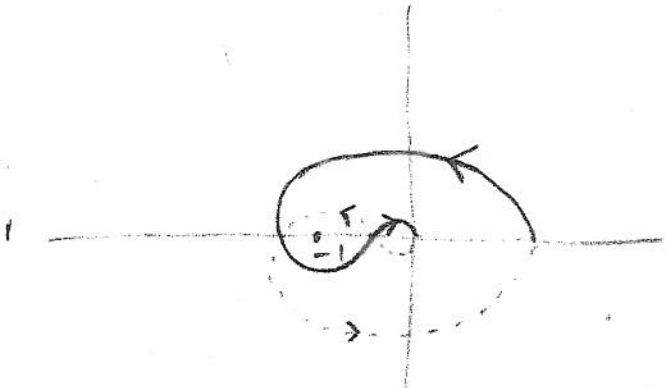
Sketch the Nyquist diagram for the system from Problem 7. Then, consider the problem of stability for this system in feedback with a gain K. It turns out that, as K is varied from 0 to $+\infty$, stability character changes at some $K_1 > 0$, and then again at some $K_2 > K_1$. Is the loop stable for $K \in (K_1, K_2)$ or for $K \in (0, K_1) \cup (K_2, \infty)$?



- starts at 1, magnitude always decreasing, as $\omega \to \infty$ goes to the origin, crosses π two times.

- $N = 0$ (no encirclements), $P = 2$ (2 unstable open-loop poles)
hence $Z = 2$ for $K = 1$ (or smaller) $\Rightarrow$ unstable closed-loop system

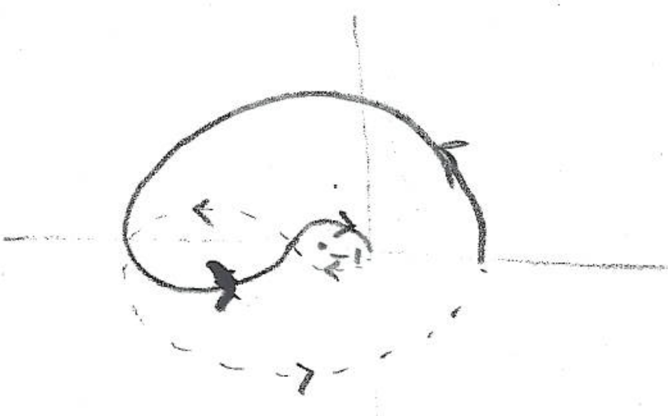
for larger K, we will have



2 counterclockwise encirclements of -1: $N=-2$

hence $Z=-2+2=0 \Rightarrow$ closed-loop stable

For even larger K



1 clockwise and one counterclockwise encirclement of -1

$N=-1+1=0 \Rightarrow Z=2$, closed-loop unstable.

Hence it will be closed-loop stable for $K\in(k_1,k_2)$