MAE 141A

LINEAR CONTROL — Prof. M. Krstic

FINAL EXAM

December 3, 2001

NAME: SOLUTIONS

- Open books and notes.
- Present your reasoning and calculations clearly. Random or inconsistent etchings will not be graded.
- Write only on the paper provided. If you run out of space for a given problem, continue on the pages at the end of the set and indicate "Continued on page X."
- The problems are *not* ordered by difficulty.
- Total points: 55.
- Time: 3:00–6:00 (3.4 minutes/point)

Problem 1. (13 points)

A welding robot system

$$G(s) = \frac{1}{s(s+2)(s+3)}$$

is in a feedback loop with a phase lead controller

$$C(s) = K\frac{s+0.5}{s+1}.$$

Using Routh's criterion, find the range of the gain K for which the feedback system is asymptotically stable.

$$C(s)G(s) = K \frac{s+0.5}{s(s+1)(s+2)(s+3)}$$

Closed-loop transfer function:

$$\frac{CG}{1+CG} = \frac{K(s+0.5)}{s(s+1)(s+2)(s+3) + K(s+0.5)}$$

$$= \frac{K(s+0.5)}{s^4 + 6s^3 + 11s^2 + 6s + Ks + 0.5K}$$

$$= \frac{K(s+0.5)}{\underbrace{s^4 + 6s^2 + 11s^2 + (6+K)s + 0.5K}_{\text{characteristic polynomial}}}$$

OVER $\longrightarrow$

Routh table

s^4	1	11	$0.5K$
s^3	6	$6+K$	
s^2	$\frac{60-K}{6}$	$0.5K$	
s^1	$\frac{6}{60-K}\left(\frac{60-K}{6}(6+K) - 6\cdot 0.5K\right)$		
s^0	$0.5K$		

Stability conditions:

$$K>0, \quad K<60, \quad \frac{60-K}{6}(6+K) - 6\cdot 0.5K > 0$$

$$(60-K)(6+K) - 18K > 0$$

$$360 + (54-18)K - K^2 > 0$$

$$K^2 - 36K - 360 < 0$$

$$K_{1,2} = 18 \pm \sqrt{18^2 + 360} = 18 \pm \sqrt{684}$$

$$K_1 \approx -8, \quad K_2 \approx 44$$

$$K_1 < K < K_2$$

So $K>0$, $K<60$, $K>-8$, $K<44$ give

$$\boxed{0<K<44}$$

Problem 2.(a) (10 points)

The world's fastest elevator and the world's largest telescope both have a qualitative model

$$G(s) = \frac{1}{s(s^2 + 4s + 8)}.$$

Suppose that these systems are controlled with a phase lag compensator

$$C(s) = K\frac{s + 5}{s + 2}$$

(perhaps not the best controller choice but useful for this exam). Sketch the root locus of $C(s)G(s)$.

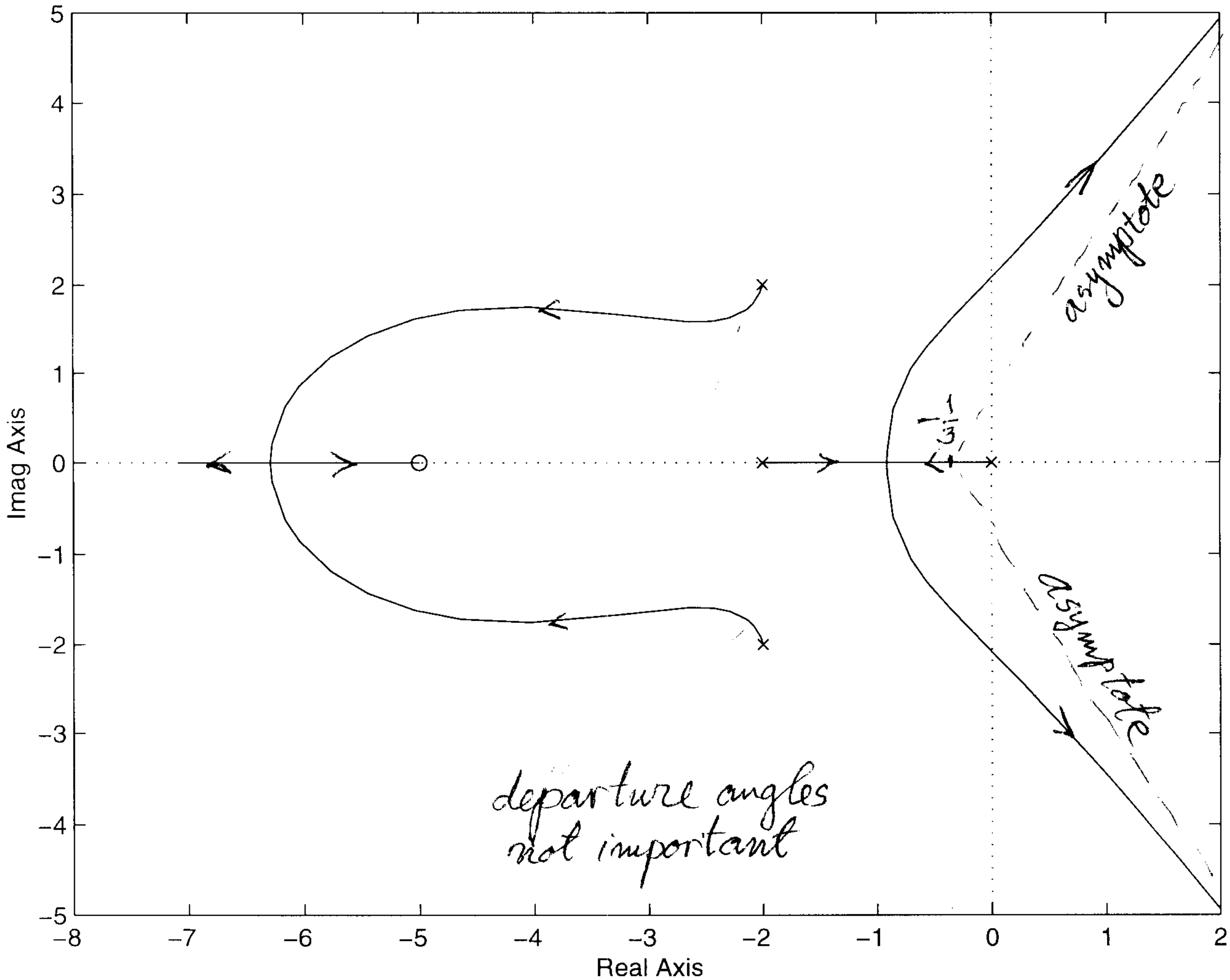
Problem 2.(b) (6 points)

The testing system for automotive suspension has the qualitative model

$$G(s) = K\frac{s^2 + 4s + 8}{s^2(s + 4)}.$$

Sketch the root locus of $G(s)$.

Part (a)



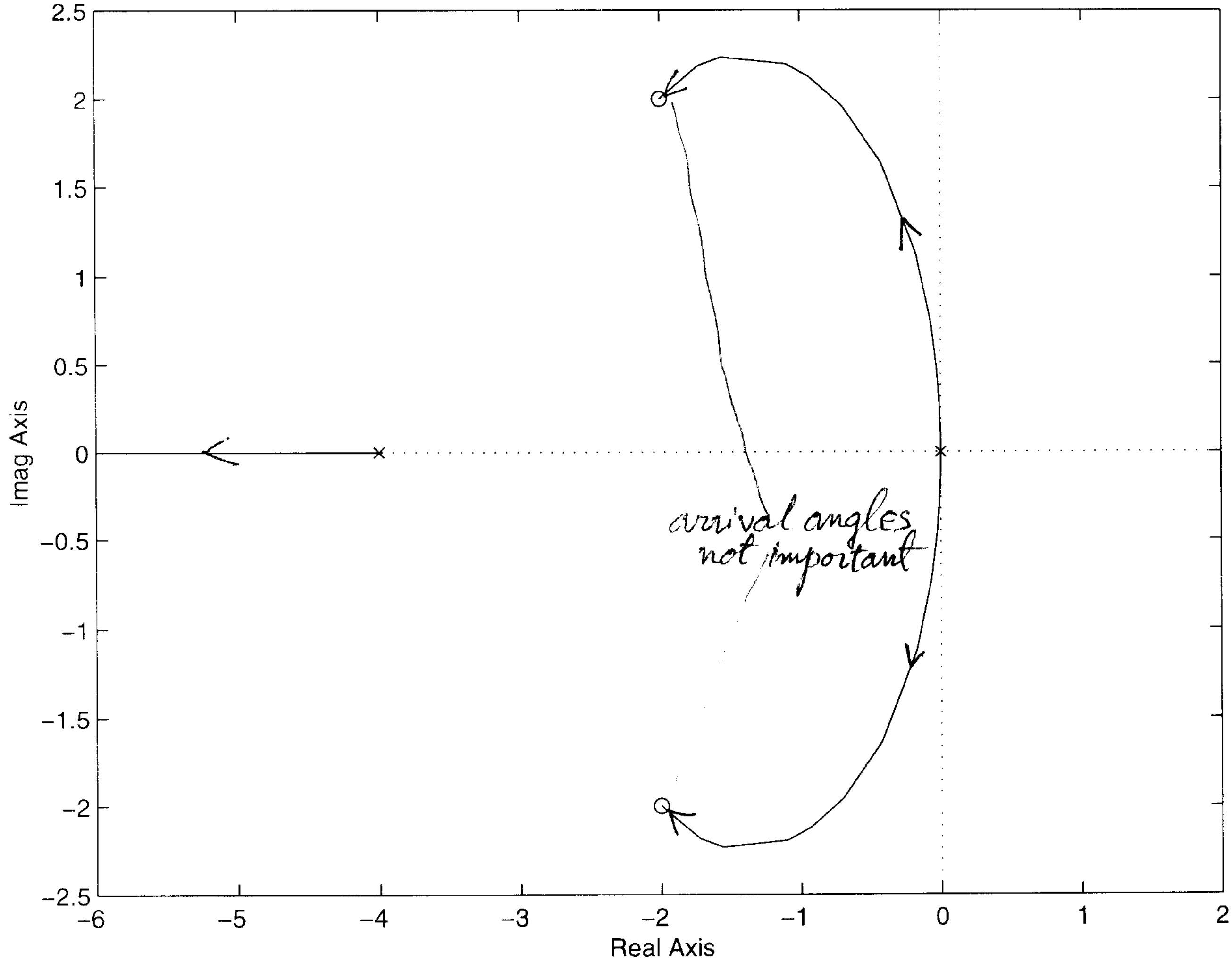
relative degree $= 3 \longrightarrow$ asymp. angles:

$\pm 60^\circ, 180^\circ$

asymp. center : $\dfrac{\Sigma \text{poles} - \Sigma \text{zeros}}{3}$

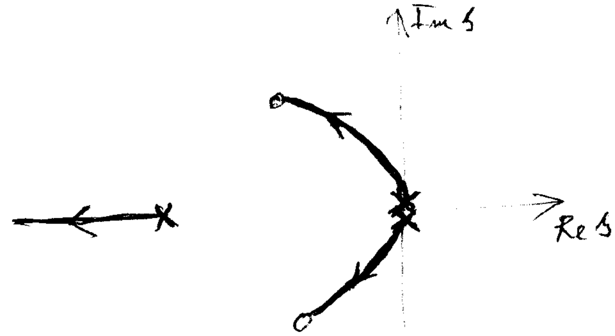
$$= \frac{0-2-2+2j-2-2j+5}{3} = -\frac{1}{3}$$

Part (b)



relative degree = 1 ⟶ asymtote angle = 180°

Since arrival angles are not required to be calculated, another acceptable sketch of root locus is:



Problem 3. (13 points)

A supersonic passenger jet like Concorde has pitch rate dynamics of the form

$$G(s) = \frac{1}{s^2 + 0.6s + 9}.$$

$\omega_n = 3$

$\zeta = 0.1$

and is controlled by a double-phase-lead controller

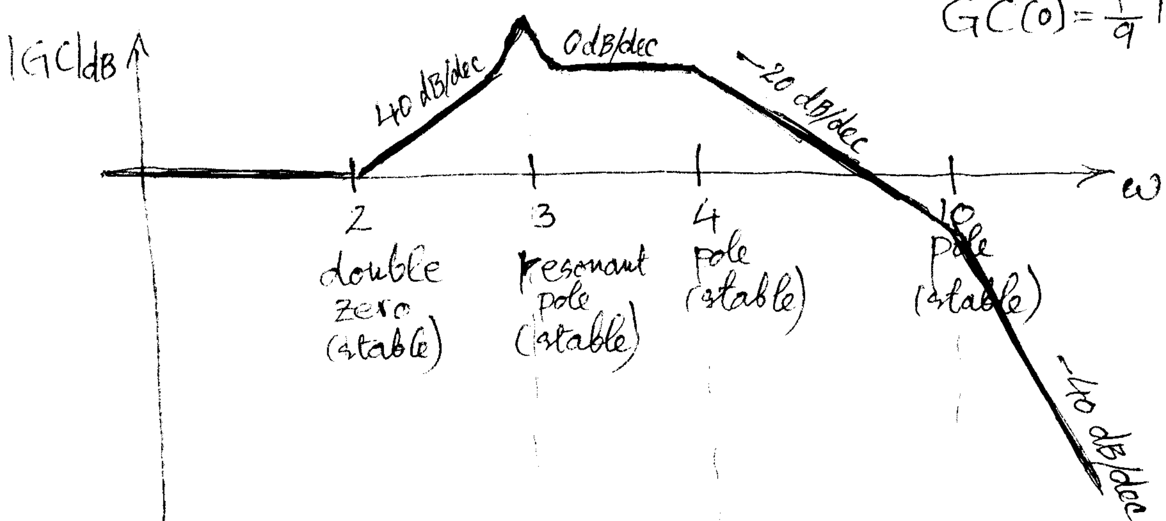
$$C(s) = 100\frac{(s+2)^2}{(s+4)(s+10)}.$$

Sketch the Bode plot of $C(s)G(s)$.

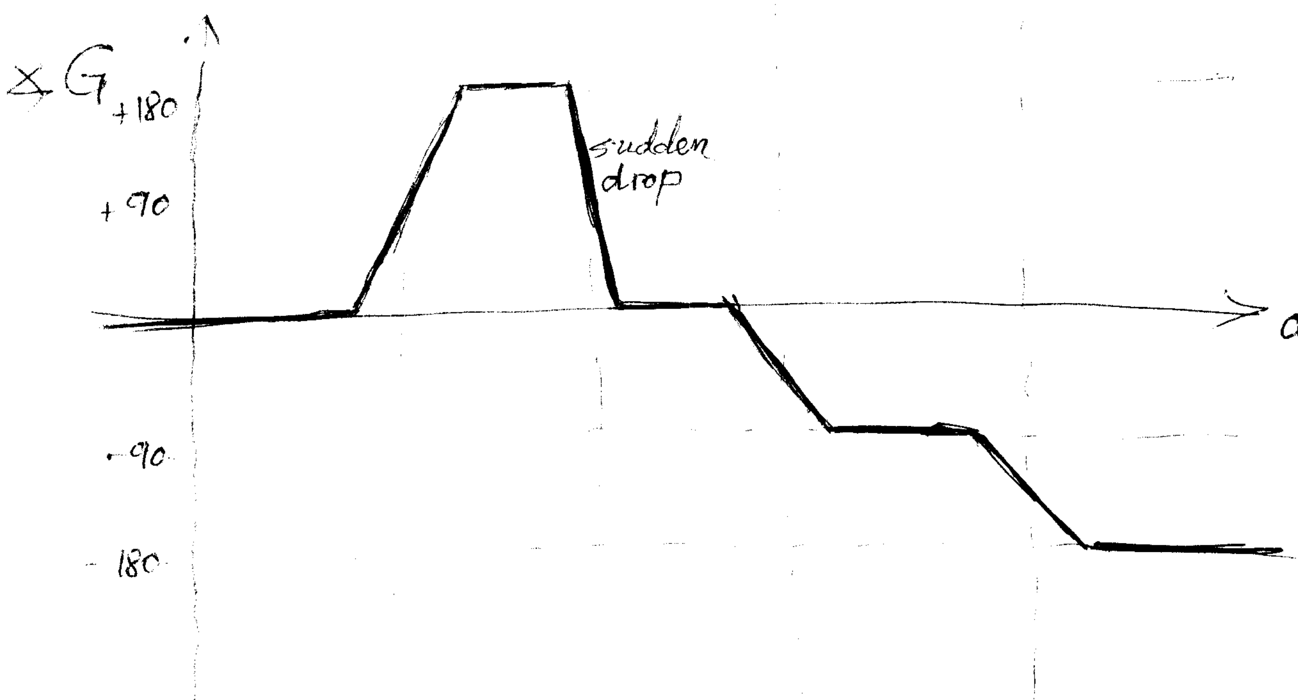
DC gain:

$GC(0) = \frac{1}{9}\,100\,\frac{4}{40} \approx 1.1$

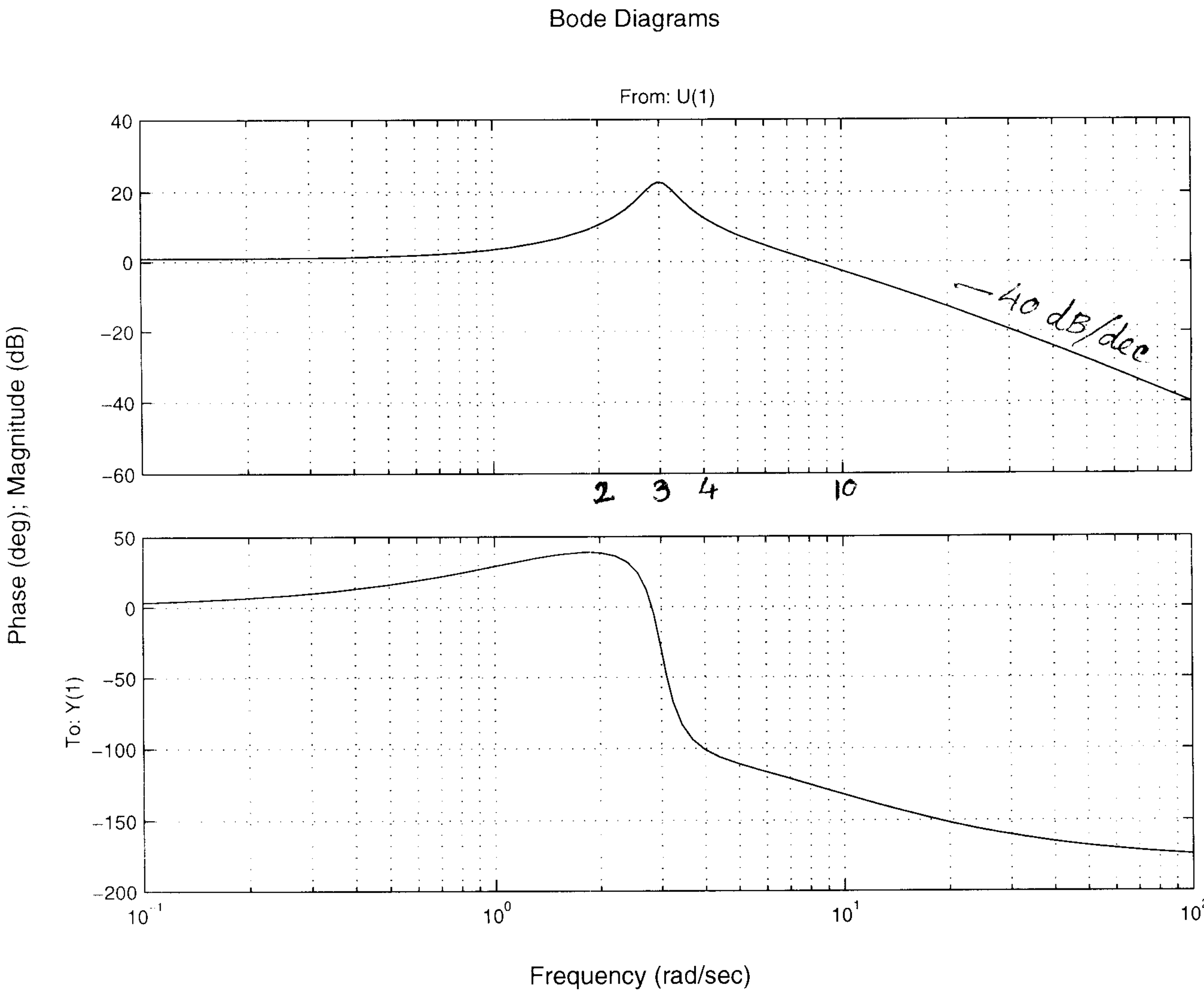
$\approx 0\,\text{dB}$



(not to scale)

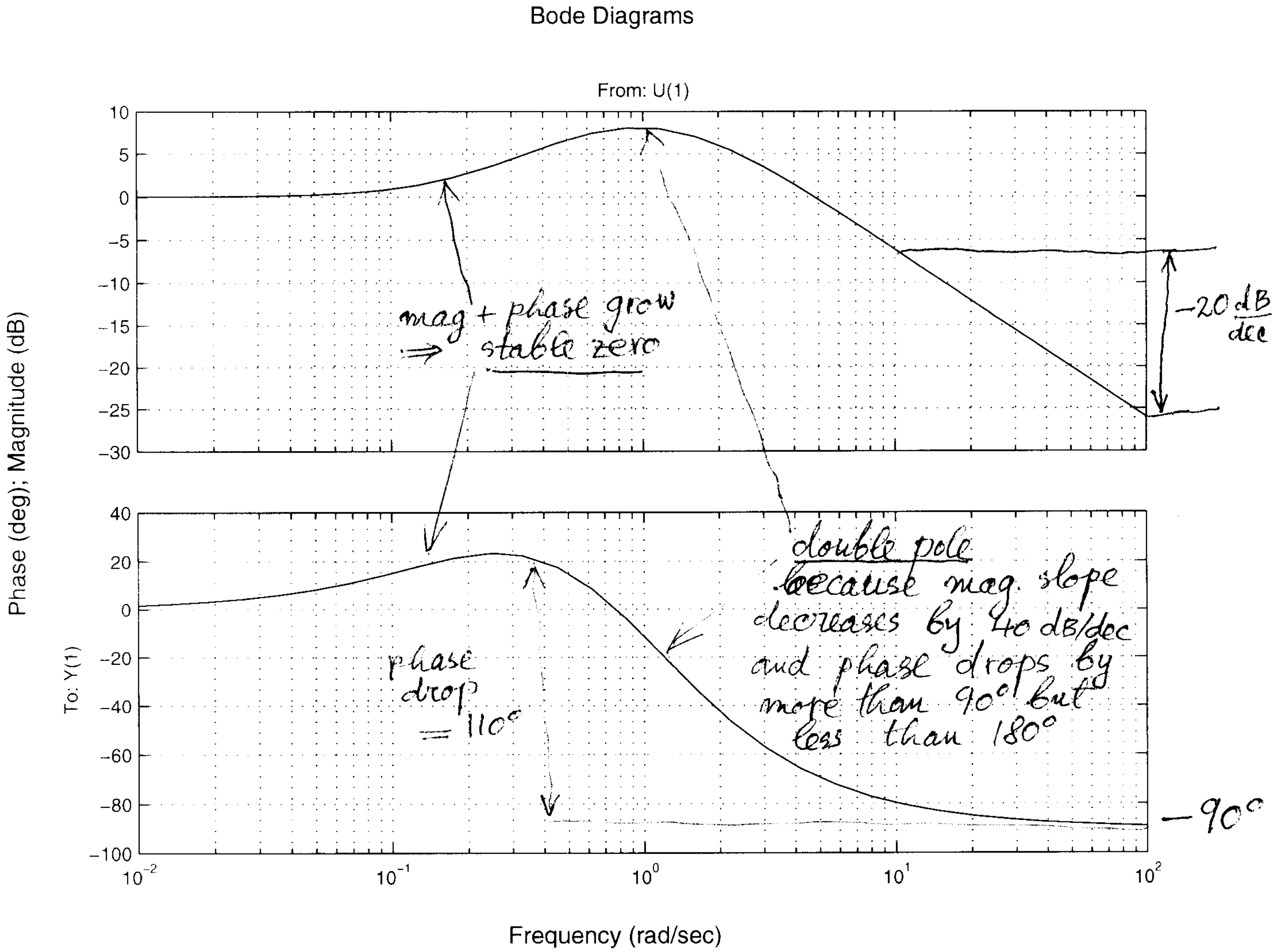


Actual Bode plot via Matlab:



Problem 4. (13 points)

Consider a system with a frequency response



If this system has only *stable*, *real* poles and zeros, what is the minimum number of poles and zeros sufficient to produce this frequency response (choose one; answers without justification will not receive credit even if they happen to be correct):

(a) one pole, one zero
(b) one pole, two zeros
(c) one zero, two poles
(d) one pole, three zeros
(e) one zero, three poles

$G(s) = K \frac{s+z}{(s+p)^2}, \quad p > z$

Sketch a Nyquist plot based on the Bode plot.

Nyquist. Start at $G(0) = 1$.
End at origin.
Arrival at $-90^\circ \Rightarrow$ neg. Im axis.
Phase & mag. initially grow
$\Rightarrow$ $G(j\omega)$ starts in the North-East direction.

Phase between $\pm 90^\circ \Rightarrow G(j\omega)$ in 1st and 4th quadrants