• Open books and notes.

• Present your reasoning and calculations clearly. Random or inconsistent etchings will not be graded.

• Write only on the paper provided. If you run out of space for a given problem, continue on the pages at the end of the set and indicate “Continued on page X.”

• The problems are not ordered by difficulty.

• Total points: 55.

• Time: 3:00–6:00 (3.4 minutes/point)
Problem 1. (13 points)

A welding robot system

\[ G(s) = \frac{1}{s(s + 2)(s + 3)} \]

is in a feedback loop with a phase lead controller

\[ C(s) = K \frac{s + 0.5}{s + 1}. \]

Using Routh's criterion, find the range of the gain \( K \) for which the feedback system is asymptotically stable.

\[
C(s)G(s) = K \frac{s + 0.5}{s(s + 1)(s + 2)(s + 3)}
\]

Closed-loop transfer function:

\[
\frac{CG}{1 + CG} = \frac{K(s + 0.5)}{s(s + 1)(s + 2)(s + 3) + K(s + 0.5)}
\]

\[
= \frac{K(s + 0.5)}{s^4 + 6s^3 + 11s^2 + 6s + Ks + 0.5K}
\]

characteristic polynomial
Routh table

\[
\begin{array}{ccc}
S^4 & 1 & 11 & 0.5K \\
S^3 & 6 & 6+K \\
S^2 & \frac{60-K}{6} & 0.5K \\
S^1 & \frac{60-K}{60-K} (\frac{60-K}{6}(6+K) - 6 \cdot 0.5K) \\
S^0 & 0.5K \\
\end{array}
\]

Stability conditions:

\[K > 0, \quad K < 60, \quad \frac{60-K}{6}(6+K) - 6 \cdot 0.5K > 0\]

\[
\begin{align*}
& (60-K)(6+K) - 18K > 0 \\
& 360 + (54-18)K - K^2 > 0 \\
& K^2 - 36K - 360 < 0 \\
& K_{1,2} = 18 \pm \sqrt{18^2 + 360} = 18 \pm 18 \sqrt{84} \\
& K_1 \approx -8, \quad K_2 \approx 44 \\
& K_1 < K < K_2 \\
\end{align*}
\]

So \(K > 0, \, K < 60, \, K > -8, \, K < 44\) give \(0 < K < 44\)
Problem 2.(a) (10 points)

The world’s fastest elevator and the world’s largest telescope both have a qualitative model

\[ G(s) = \frac{1}{s(s^2 + 4s + 8)}. \]

Suppose that these systems are controlled with a phase lag compensator

\[ C(s) = K \frac{s + 5}{s + 2} \]

(perhaps not the best controller choice but useful for this exam). Sketch the root locus of \( C(s)G(s) \).

Problem 2.(b) (6 points)

The testing system for automotive suspension has the qualitative model

\[ G(s) = K \frac{s^2 + 4s + 8}{s^2(s + 4)}. \]

Sketch the root locus of \( G(s) \).
Part (a)

Relative degree = 3 → asymp. angles: ±60°, 180°

Asymp. center: \( \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{3} \)

\[
= \frac{0 - 2 - 2 + 2j - 2 - 2j + 5}{3} = \frac{1}{3}
\]
Relative degree \(-1\) \(\rightarrow\) asymptote angle \(= 180^\circ\)

Since arrival angles are not required to be calculated, another acceptable sketch of root locus is:
Problem 3. (13 points)

A supersonic passenger jet like Concorde has pitch rate dynamics of the form

\[ G(s) = \frac{1}{s^2 + 0.6s + 9}, \quad \omega_n = 3 \]

and is controlled by a double-phase-lead controller

\[ C(s) = 100 \frac{(s + 2)^2}{(s + 4)(s + 10)}. \]

Sketch the Bode plot of \( C(s)G(s) \).

DC gain:

\[ GC(0) = \frac{1}{9} \times 100 \times \frac{4}{40} \approx 1.1 \approx 0 \text{ dB} \]

\( \text{not to scale} \)
Actual Bode plot via Matlab:
Problem 4. (13 points)

Consider a system with a frequency response

If this system has only stable, real poles and zeros, what is the minimum number of poles and zeros sufficient to produce this frequency response (choose one; answers without justification will not receive credit even if they happen to be correct):

(a) one pole, one zero
(b) one pole, two zeros
(c) one zero, two poles
(d) one pole, three zeros
(e) one zero, three poles

\[ G(s) = K \frac{s + 2}{(s + 3)^2}, \quad p > 2 \]

Sketch a Nyquist plot based on the Bode plot.
Nyquist.

Start at $G(0) = 1$.
End at origin.
Arrival at $-90^\circ \Rightarrow \text{neg. Im axis}$.

Phase & mag. initially grow
$\Rightarrow G(j\omega)$ starts in the North-East direction.

Phase between $\pm 90^\circ \Rightarrow G(j\omega)$ in 1st and 4th quadrants.