Phase-Lag Compensator

Example $G(s) = \dfrac{100}{s(s+4)(s+5)}$

~~Task: Design a compensator~~

Show the Bode plots with PM=18°:

margin(100, [1, 9, 20, 0])

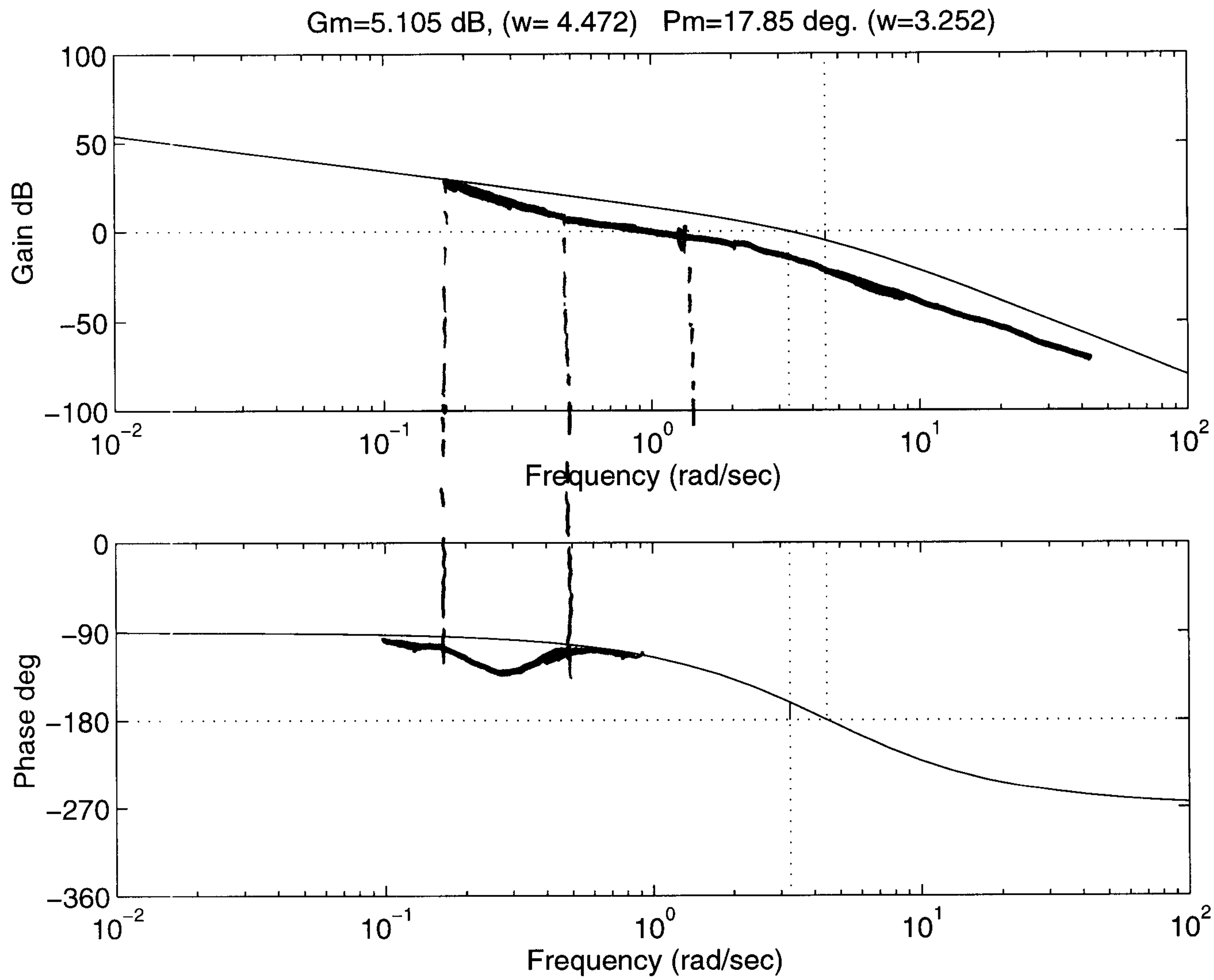
The PM of at least 40° is considered adequate

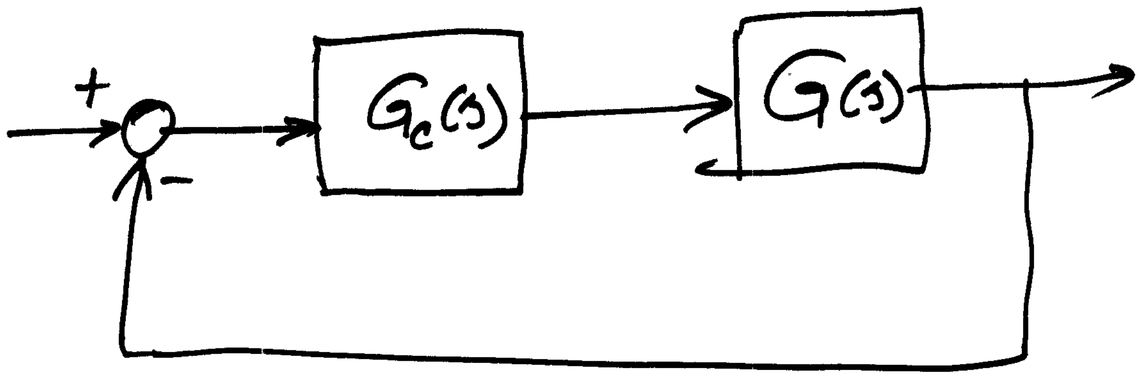
Task: design a compensator which increases the phase margin.

Observation: We want to reduce the amplitude response without affecting much the phase response (at the frequencies of interest)

Solution: pole followed by a zero at low freq.

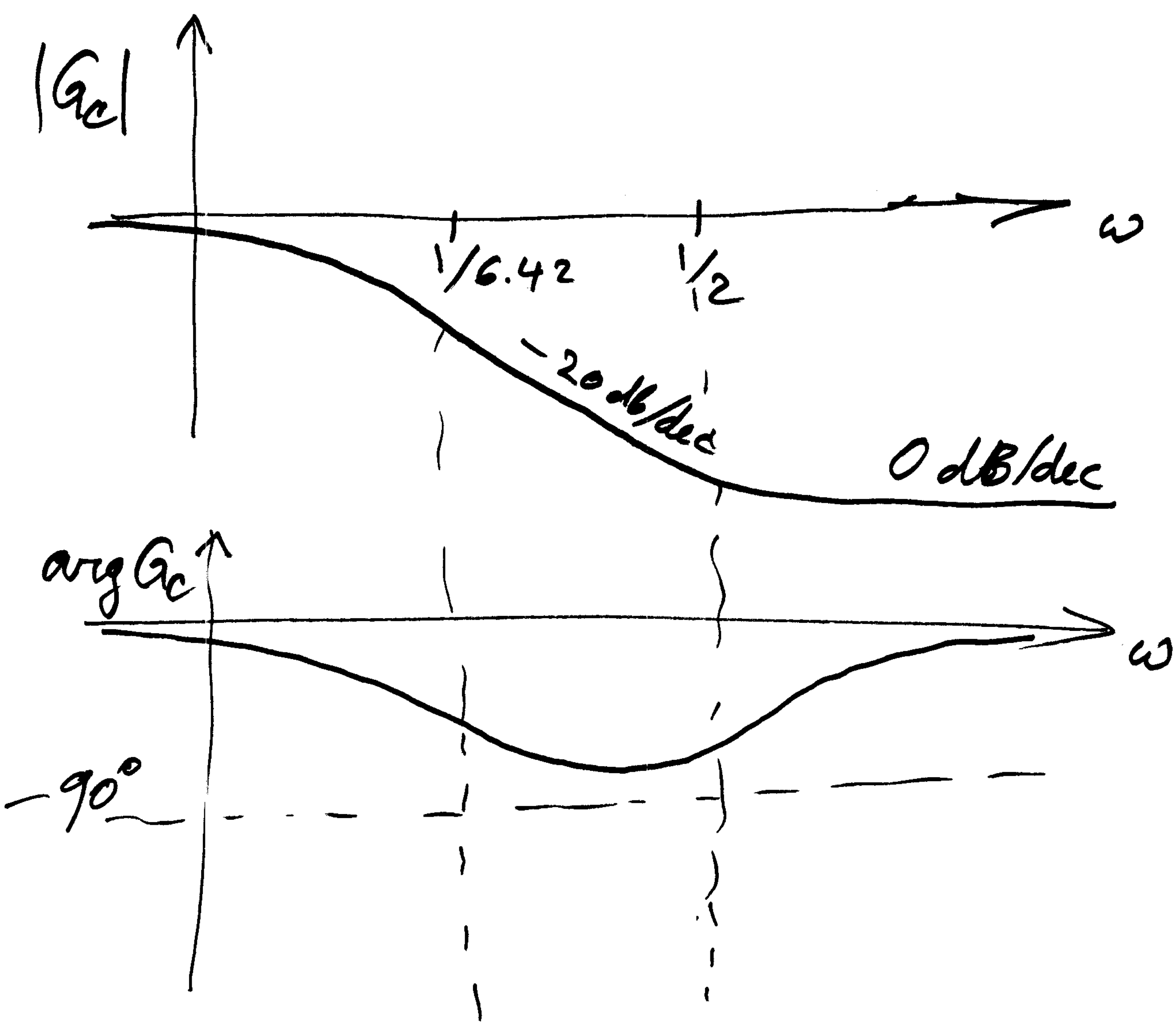
$$G(s) = \frac{100}{s(s+4)(s+5)}$$



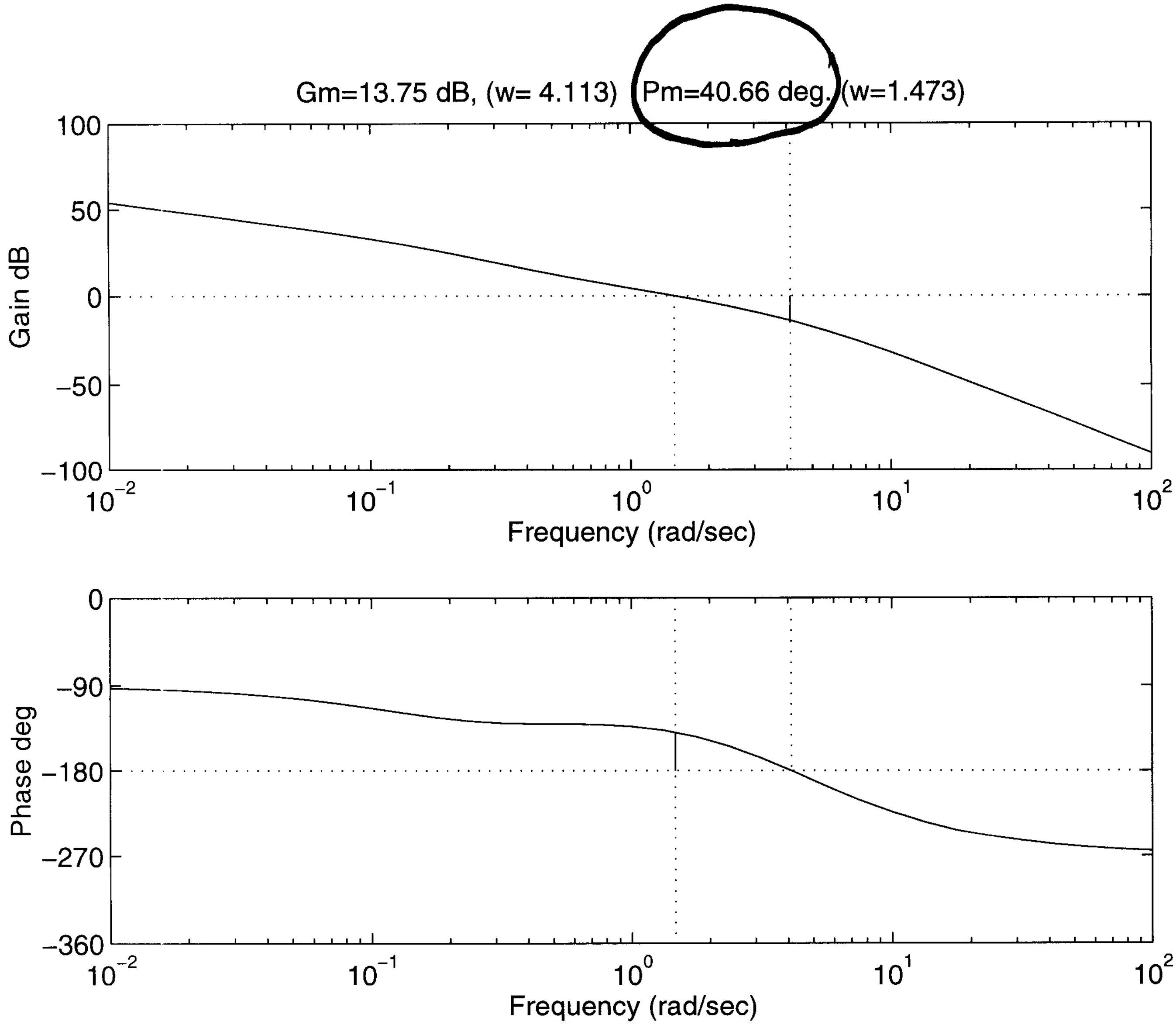


We pick

$$G_c(s) = \frac{2s+1}{6.42s+1} \qquad \underline{\text{LAG}}$$



$$G_c(s)\,G(s) = \frac{2s+1}{6.42s+1}\;\frac{100}{s(s+4)(s+5)}$$

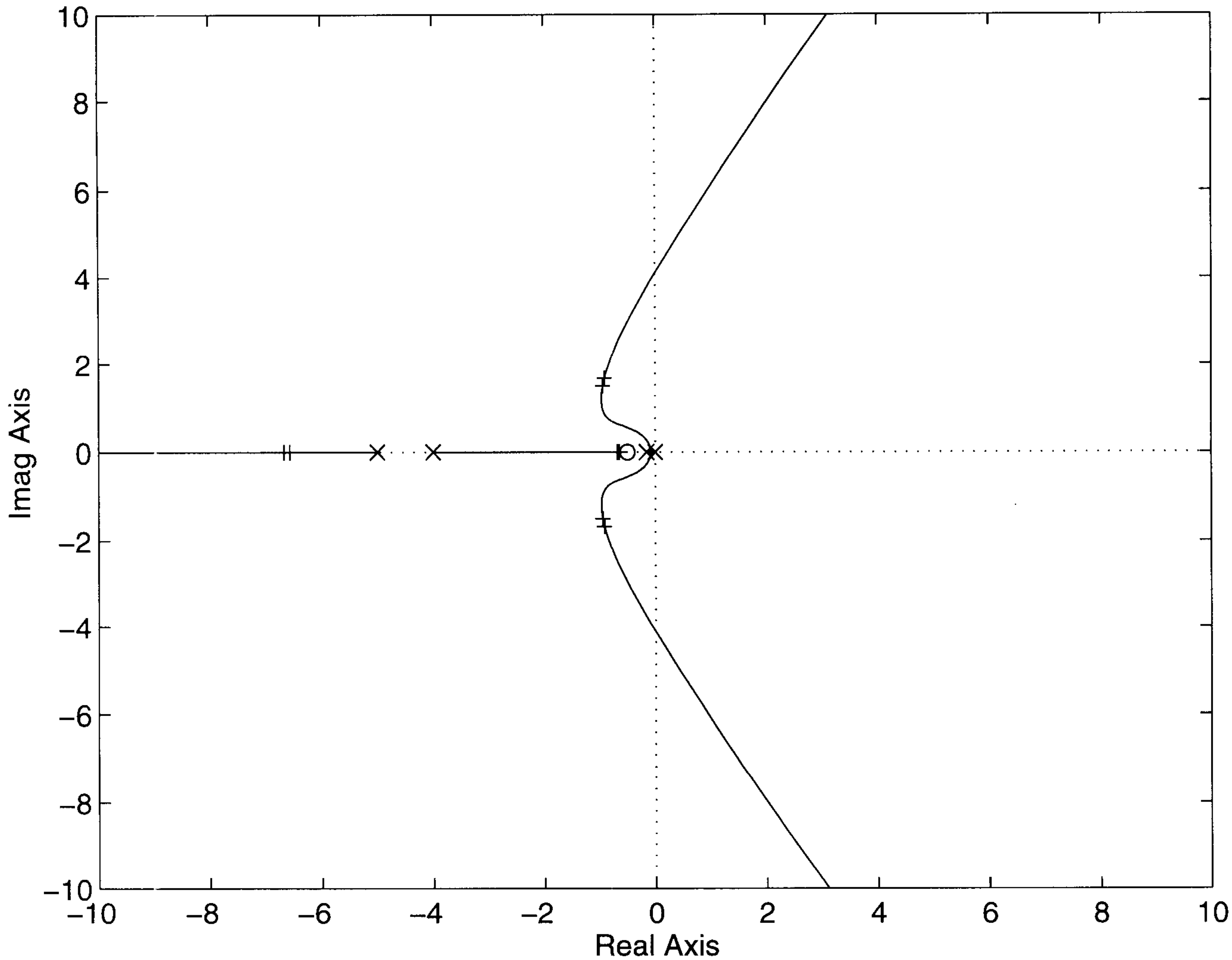



```
[n,d] = series([2 1],[6.42 1], 100, [1 9 20 0])
margin (n,d)
```

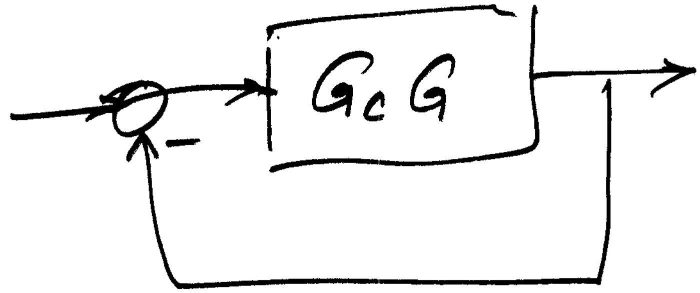
In general, phase-lag compensators have the form

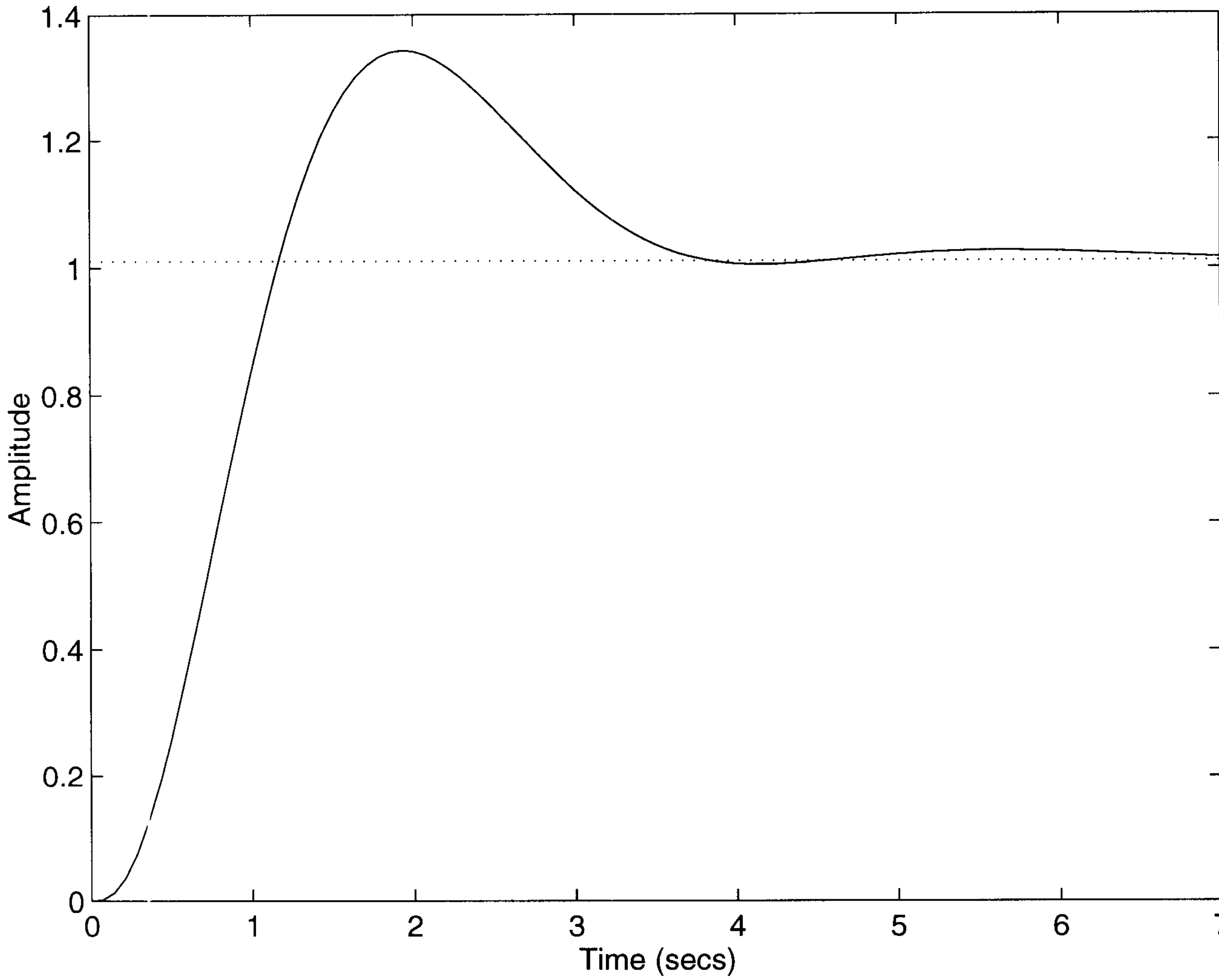
$$G_c(s) = \frac{aTs + 1}{Ts + 1}, \quad a < 1$$

Downside of phase-lag compensators: one or more closed-loop poles near zero - sluggish response!



rlocus (n,d)
rlocfind (n,d)

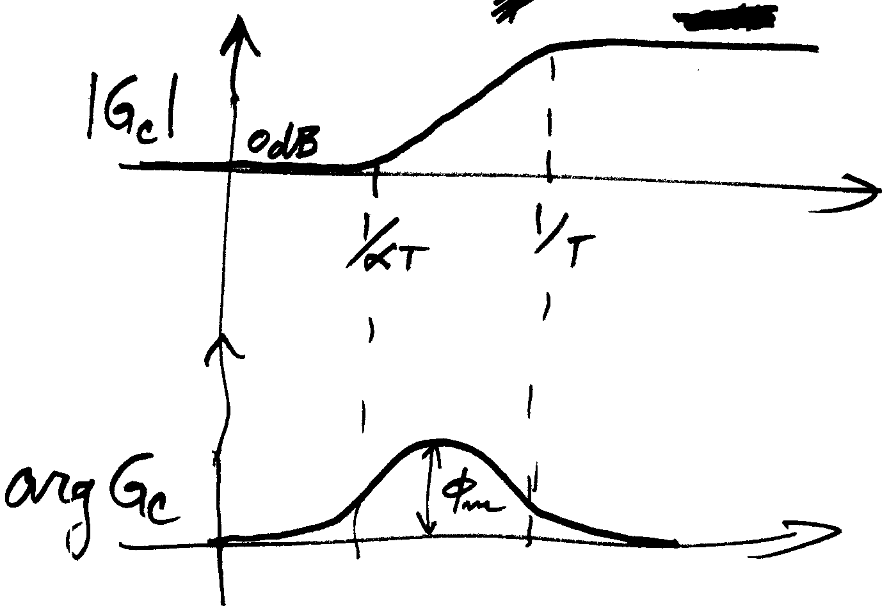





```
[nc, dc] = cloop(n,d,-1)
step(nc,dc)
```

Phase-Lead Compensator

$$G_C(s) = \frac{\alpha T s + 1}{T s + 1}, \quad \alpha > 1$$



Back to example on p. 7a

$$G(s) = \frac{100}{s(s+4)(s+5)}$$

The max. phase lead ϕ_m should be achieved at the new crossover frequency $\bar{\omega}_c$. $\bar{\omega}_c$ moves to the right of the old ω_c because of the gain increase. Pick α and T so that ω_c is between $\frac{1}{\alpha T}$ and $\frac{1}{T}$, closer to $\frac{1}{\alpha T}$.

Since $\omega_c = 3.25$, we select

$$G_c(s) = \frac{s+2}{s+20}\,\frac{20}{2}$$

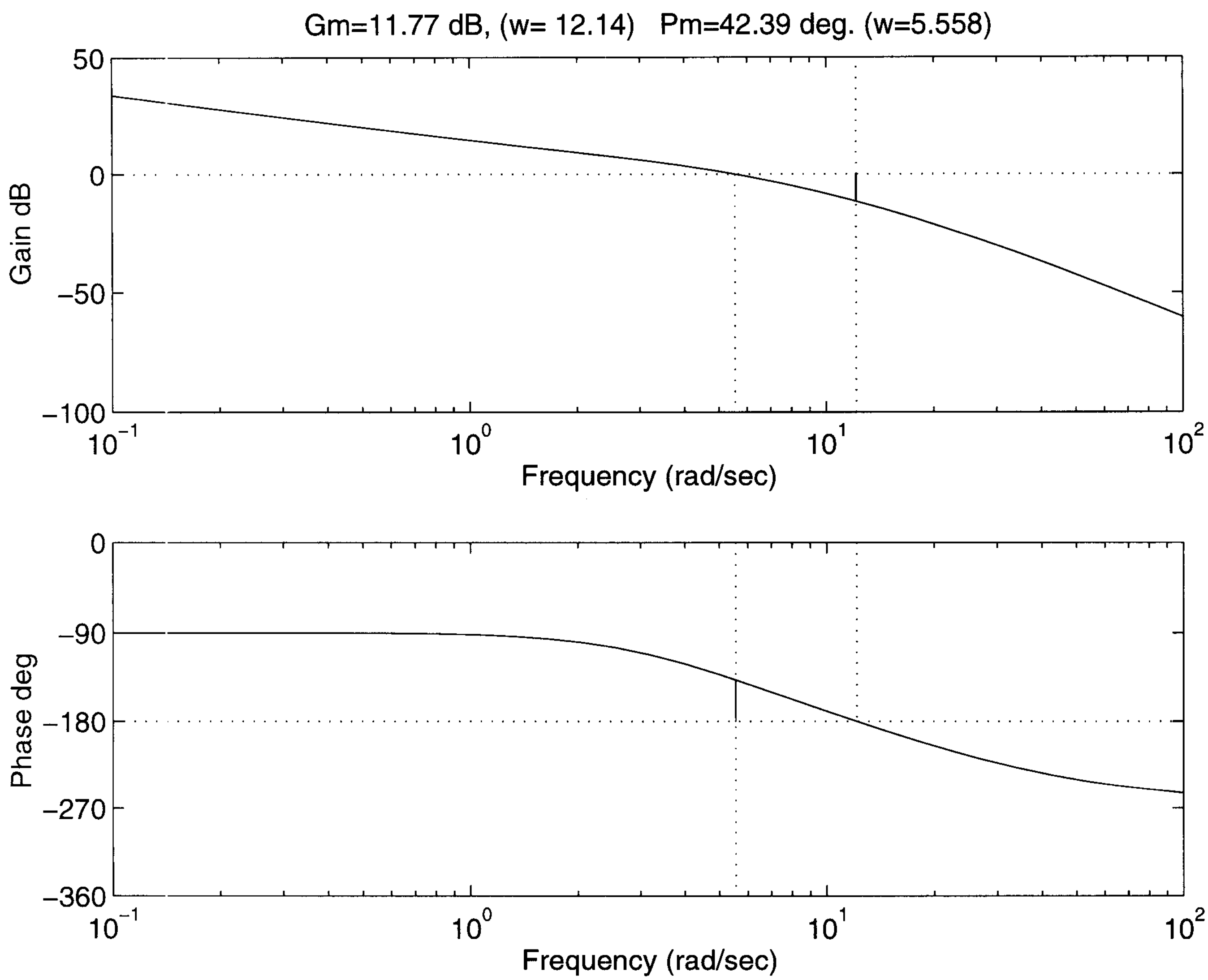
$$= \frac{0.5s+1}{0.05s+1}$$

The new $\bar{\omega}_c = 5.5$ and $PM = 42.4$

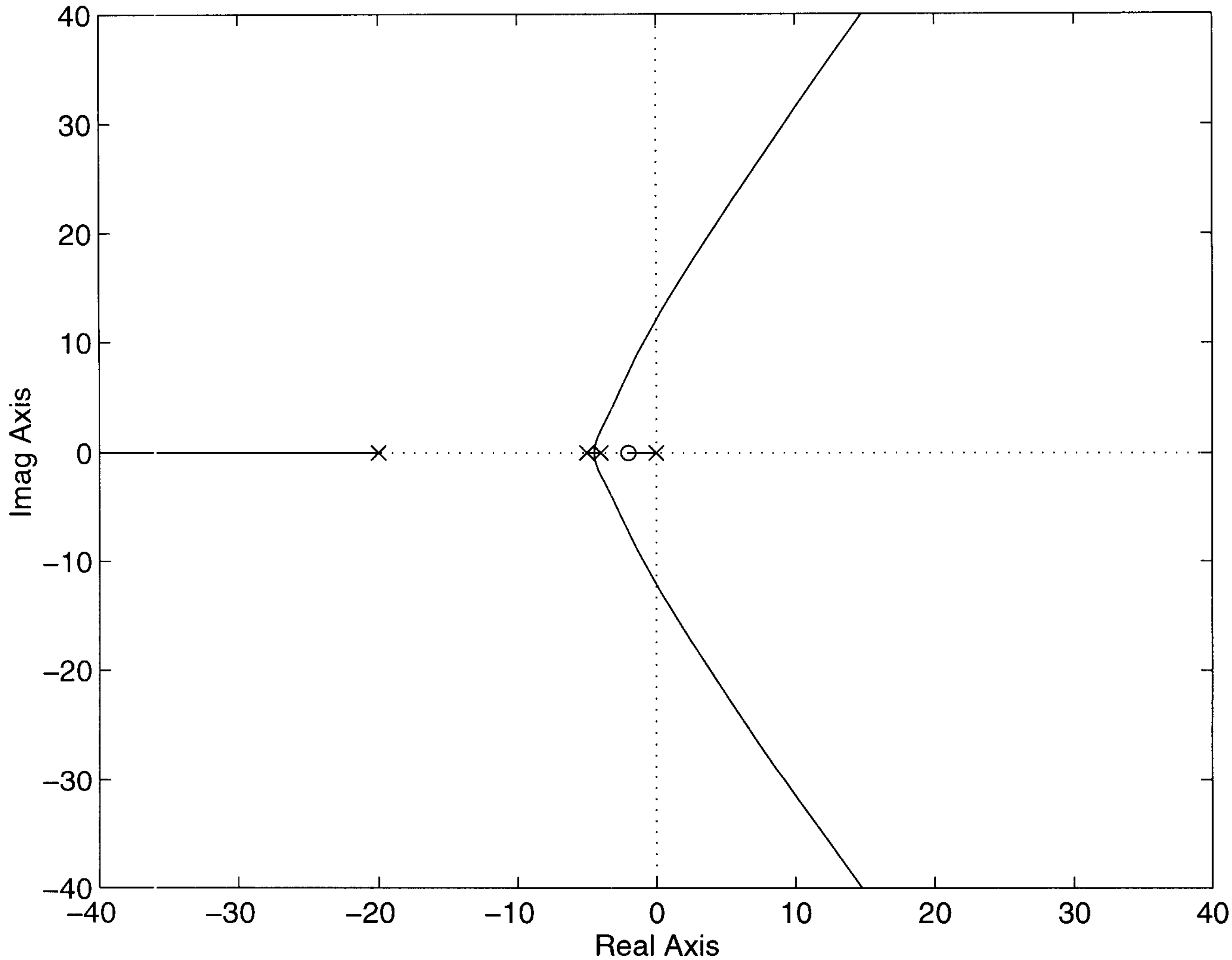
The rise time is about 5 times faster!

11a

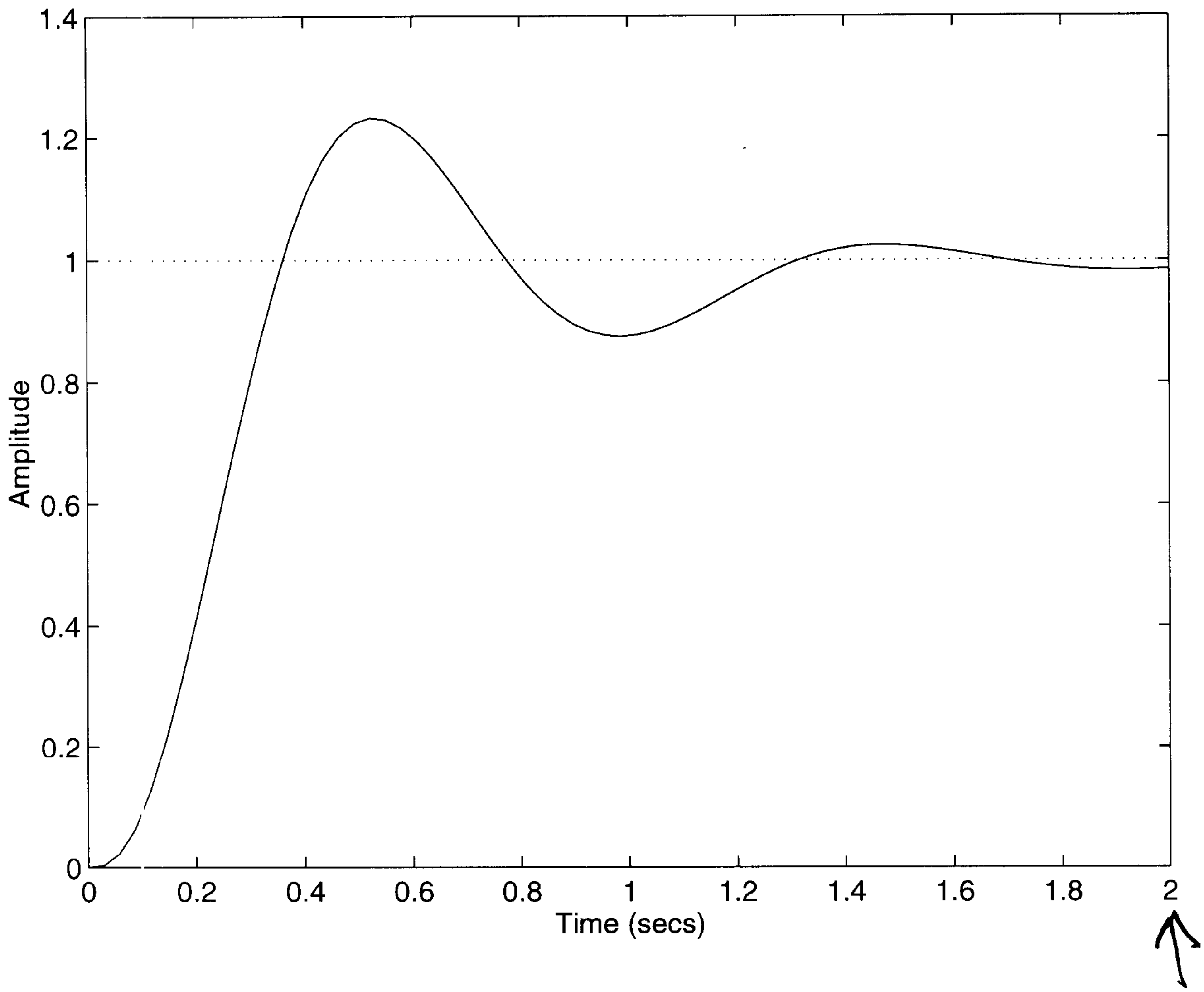
$$G_c(s)G(s) = \frac{0.5s+1}{0.05s+1}\,\frac{100}{s(s+4)(s+5)}$$



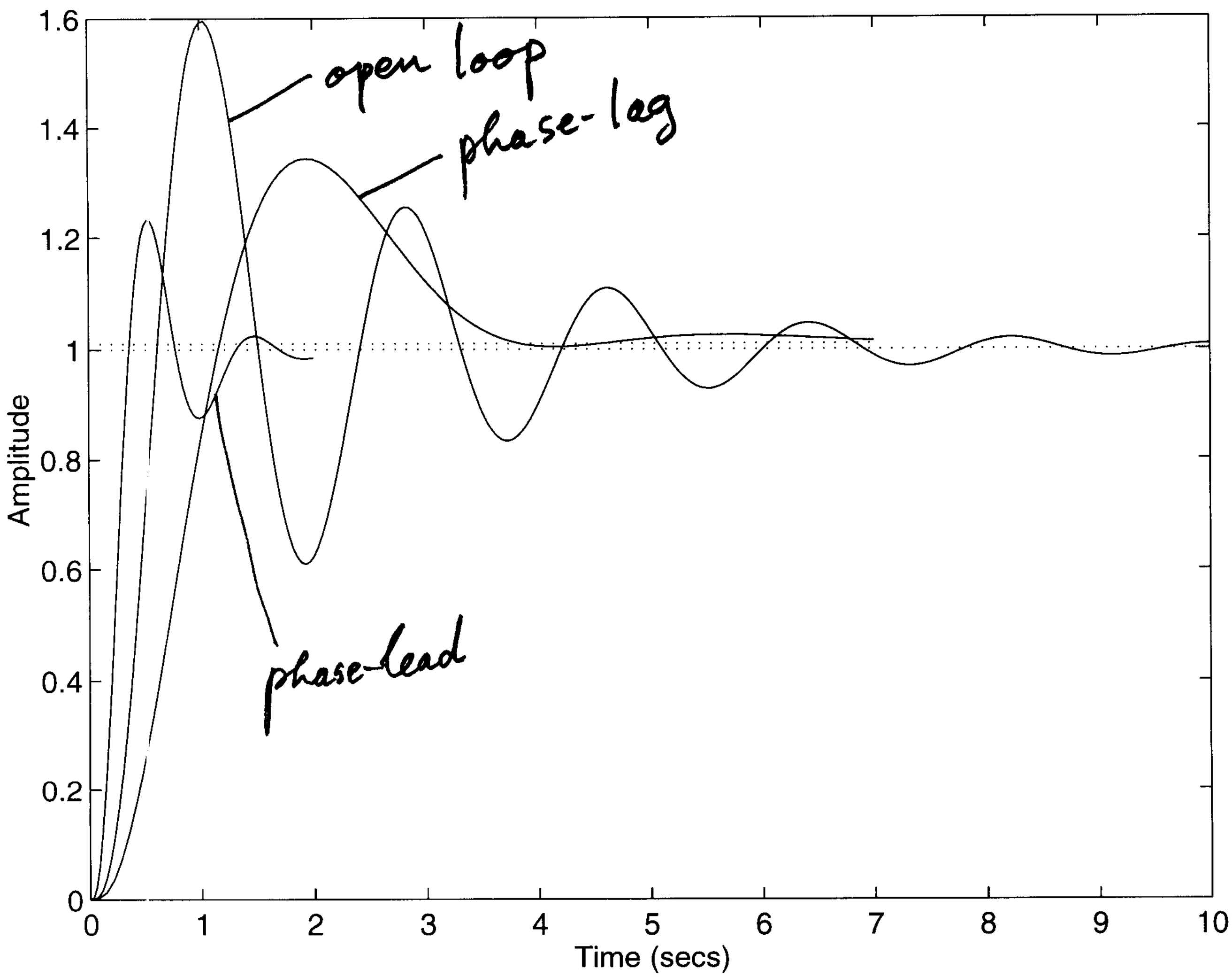
11b


40
30
20
10
0
−10
−20
−30
−40
Imag Axis
−40
−30
−20
−10
0
10
20
30
40
Real Axis

11c


1.4
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time (secs)

11d



open loop
phase-lag
phase-lead
Amplitude
Time (secs)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10