

Homework 3

Problem 1. Compute the *impulse* response of the following systems:

$$(a) H(s) = e^{-5s} \frac{1}{(s+4)^2 + 2}$$

$$(b) H(s) = \frac{1}{(2s+3)^5}$$

Problem 2. Compute the *step* response of the following systems:

$$(a) H(s) = 2 \frac{s^2 + s + 1}{s^2 + 3s + 2}$$

$$(b) H(s) = \frac{1}{(s+1)^3}$$

$$(c) H(s) = \frac{2}{(s+1)(s^2+4)}$$

Problem 3. For the system with a transfer function

$$H(s) = \frac{s+1}{3s^2+7} - \frac{1}{3s+2}$$

find the differential equation governing the relationship between the input $u(t)$ and the output $y(t)$, assuming zero initial conditions.

Problem 4.

(a) Let the persistent forcing signal $u(t) = \sin(2t)1(t)$ drive the system

$$Y(s) = \frac{2s^2 + 8}{s(s^2 + 2s + 15)}U(s).$$

Does this system, despite persistent forcing, reach a steady state? If so, what is $\lim_{t \rightarrow \infty} y(t)$?

(b) Same questions as in (a) for the system

$$Y(s) = \frac{2s^2 + 8}{s(s^2 + 2s - 15)}U(s).$$

Problem 5. Using the Laplace transform, solve the following differential equations:

$$(a) \ddot{y} + 2\dot{y} + 2y = 0, \quad y(0) = 1, \quad \dot{y}(0) = 2, \quad \ddot{y}(0) = 3.$$

$$(b) \ddot{y} + 2\dot{y} + 4y = (1-t)e^{-2t}, \quad y(0) = 1, \quad \dot{y}(0) = 1.$$