Solutions to Homework 2

3.2 (e)

$$\begin{aligned} f(t) &= \sinh t \\ &= \frac{e^t - e^{-t}}{2} \\ \mathcal{L}\{f(t)\} &= \mathcal{L}\{\frac{e^t}{2}\} - \mathcal{L}\{\frac{e^{-t}}{2}\} \\ &= \frac{1}{2}(\frac{1}{s-1}) - \frac{1}{2}(\frac{1}{s+1}) \\ &= \frac{1}{s^2-1} \end{aligned}$$

3.3 (c)

$$\begin{aligned} f(t) &= t^2 + e^{-2t}\sin 3t \\ &= \mathcal{L}\{f(t)\} = \mathcal{L}\{t^2\} + \mathcal{L}\{e^{-2t}\sin 3t\} \\ &= \frac{2!}{s^3} + \frac{3}{(s+2)^2+9} \\ &= \frac{2}{s^3} + \frac{3}{(s+2)^2+9} \end{aligned}$$

3.4 (d)

$$f(t) = t\sin 3t - 2t\cos t$$

Use the following Laplace transforms and properties

$$\begin{aligned} \mathcal{L}\{tg(t)\} &= -\frac{d}{ds}G(s) \\ \mathcal{L}\{\sin at\} &= \frac{a}{s^2+a^2} \\ \mathcal{L}\{\cos at\} &= \frac{s}{s^2+a^2} \\ \mathcal{L}\{f(t)\} &= \mathcal{L}\{t\sin 3t\} - 2\mathcal{L}\{t\cos t\} \\ &= -\frac{d}{ds}\frac{3}{s^2+9} - 2(-\frac{d}{ds}\frac{s}{s^2+1}) \\ &= \frac{-(2s*3)}{(s^2+9)^2} - 2\frac{((s^2+1)-(2s)s)}{(s^2+1)^2} \\ &= \frac{-6s}{(s^2+9)^2} + \frac{2(s^2-1)}{(s^2+1)^2} \end{aligned}$$

3.5 (a)

$$f(t) = \sin t \sin 3t$$

Use the trigonometric relation,

$$\begin{aligned}
\sin \alpha t \sin \beta t &= \frac{1}{2}\cos(|\alpha - \beta|t) - \frac{1}{2}\cos(|\alpha + \beta|t) \\
\alpha &= 1 \text{ and } \beta = 3 \\
f(t) &= \frac{1}{2}\cos(|1-3|t) - \frac{1}{2}\cos(|1+3|t) \\
&= \frac{1}{2}\cos 2t - \frac{1}{2}\sin 4t \\
\mathcal{L}\{f(t)\} &= \frac{1}{2}\mathcal{L}\{\cos 2t\} - \frac{1}{2}\mathcal{L}\{\cos 4t\} \\
&= \frac{1}{2}[\frac{s}{s^2+4} - \frac{s}{s^2+16}] \\
&= \frac{6s}{(s^2+4)(s^2+16)}
\end{aligned}$$