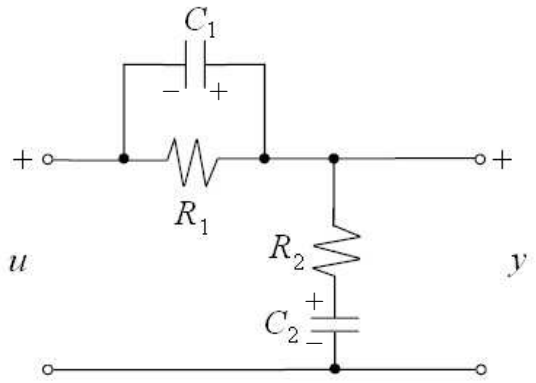
MAE 143A
SIGNALS AND SYSTEMS Prof. M. Krstic

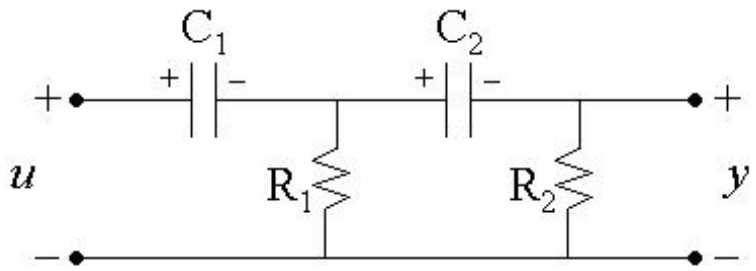
Homework 1

Problem 1. Derive the state space model for each of the following circuits, using voltages on the capacitors as the state variables:

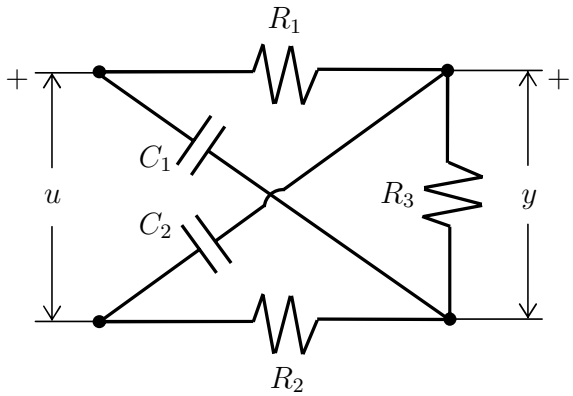
(a)



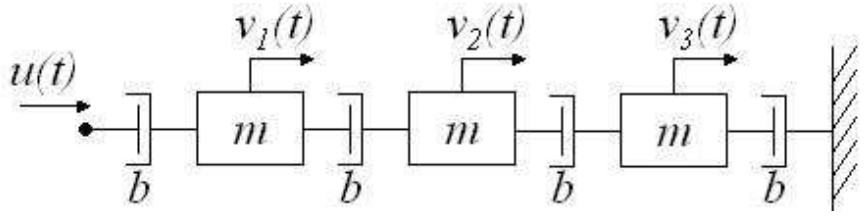
(b)



(c)



Problem 2. Consider the following mass-damper system



where $u(t)$ is a forcing velocity, m is the mass of each of the three mass elements, b is the resistance of each of the dampers, and v_1, v_2, v_3 are, respectively, the velocities of the left, middle, and right mass, in the rightward reference direction. Derive the state space model with $u(t)$ as the input, $v = [v_1, v_2, v_3]^T$ as the state, and the force acting on the middle mass (in the rightward direction) as the output y.

Problem 3. Consider the system

$$\begin{aligned} \ddot{\theta} + \theta - \theta^2 &= \sin u \\ \dot{\zeta} + \zeta &= \dot{\theta} + (\zeta + \theta)u \end{aligned}$$

(This system does not come from any physical application but its structure and its nonlinear terms mimic phenomena that arise in mechanical and bio-chemaical systems.)

(a) Treating θ as the output and u as the input, derive a state space representation of the system

(b) For $u = 0$, find all the equilibria of the system.

(c) For each equilibrium, find the linearization around that equilibrium.

Problem 4. One of the state-of-the-art HIV models is (Wodarz'99 with specific values of the parameters)

$$\begin{aligned} \dot{x} &= 2 - x - 3xy(1-u) \\ \dot{y} &= -y - zy + 3xy(1-u) \\ \dot{w} &= 2xyw - yw - \frac{1}{3}w \\ \dot{z} &= yw - z\,, \end{aligned}$$

where u is the input (drug concentration) and x, y, w, and z are concentrations of healthy cells, infected cells, memory cells, and killer cells, respectively.

For $u = 0$ (no treatment), there exist 3 equilibria in this system. One corresponds to a healthy person, another one corresponds to a person with AIDS, and the third one corresponds to a long-term non-progressor, i.e. a person with HIV who never develops AIDS.

(a) Find the three equilibria.

(b) For each equilibrium, find the linearization around that equilibrium.