Extremum Seeking for Motion Optimization: From Bacteria to Nonholonomic Vehicles

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Based on contributions by:

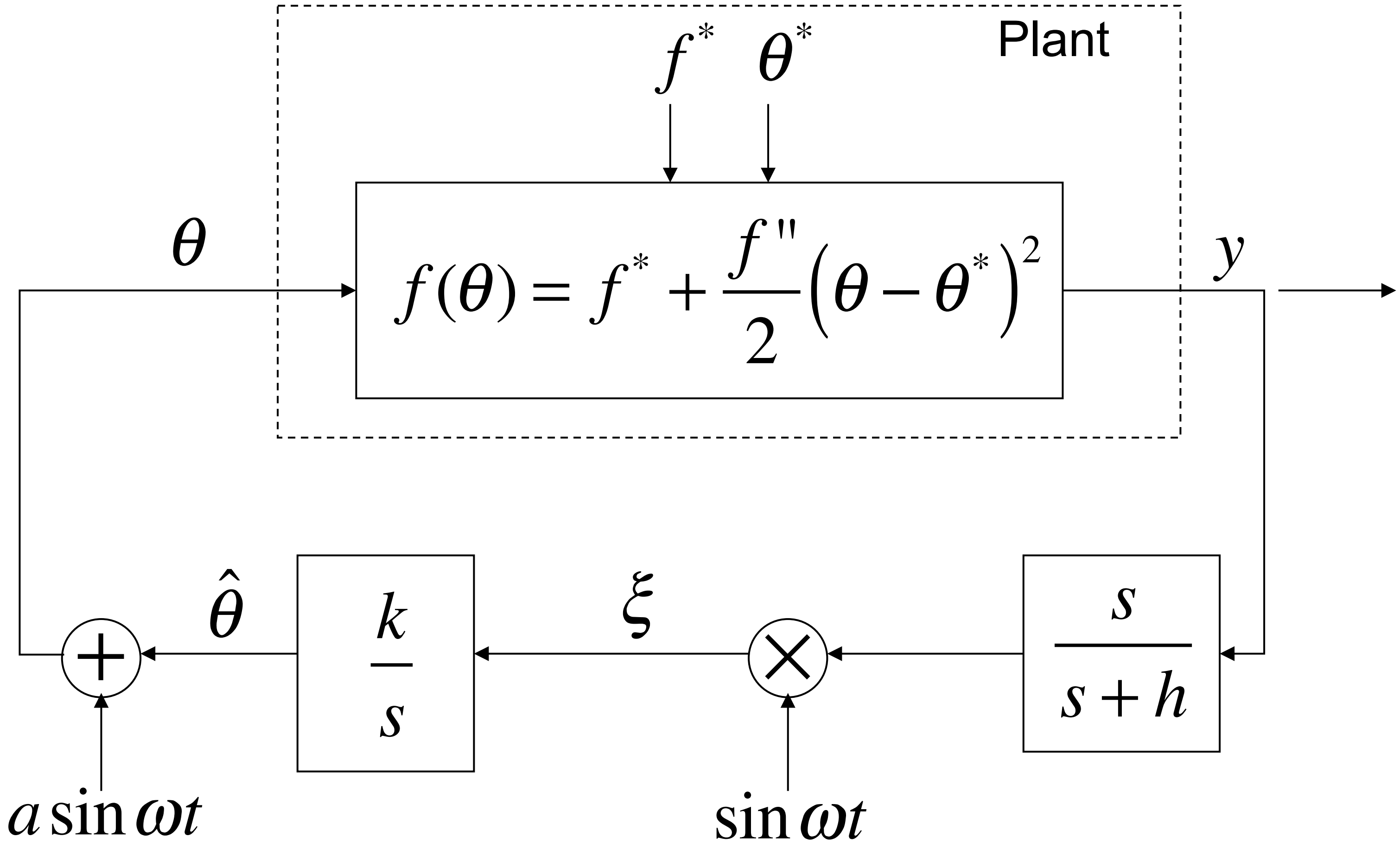
Jennie Cochran, Dan Arnold, Nima Ghods, Chunlei Zhang, Antranik Siranosian, and Chris Manzie

Outline

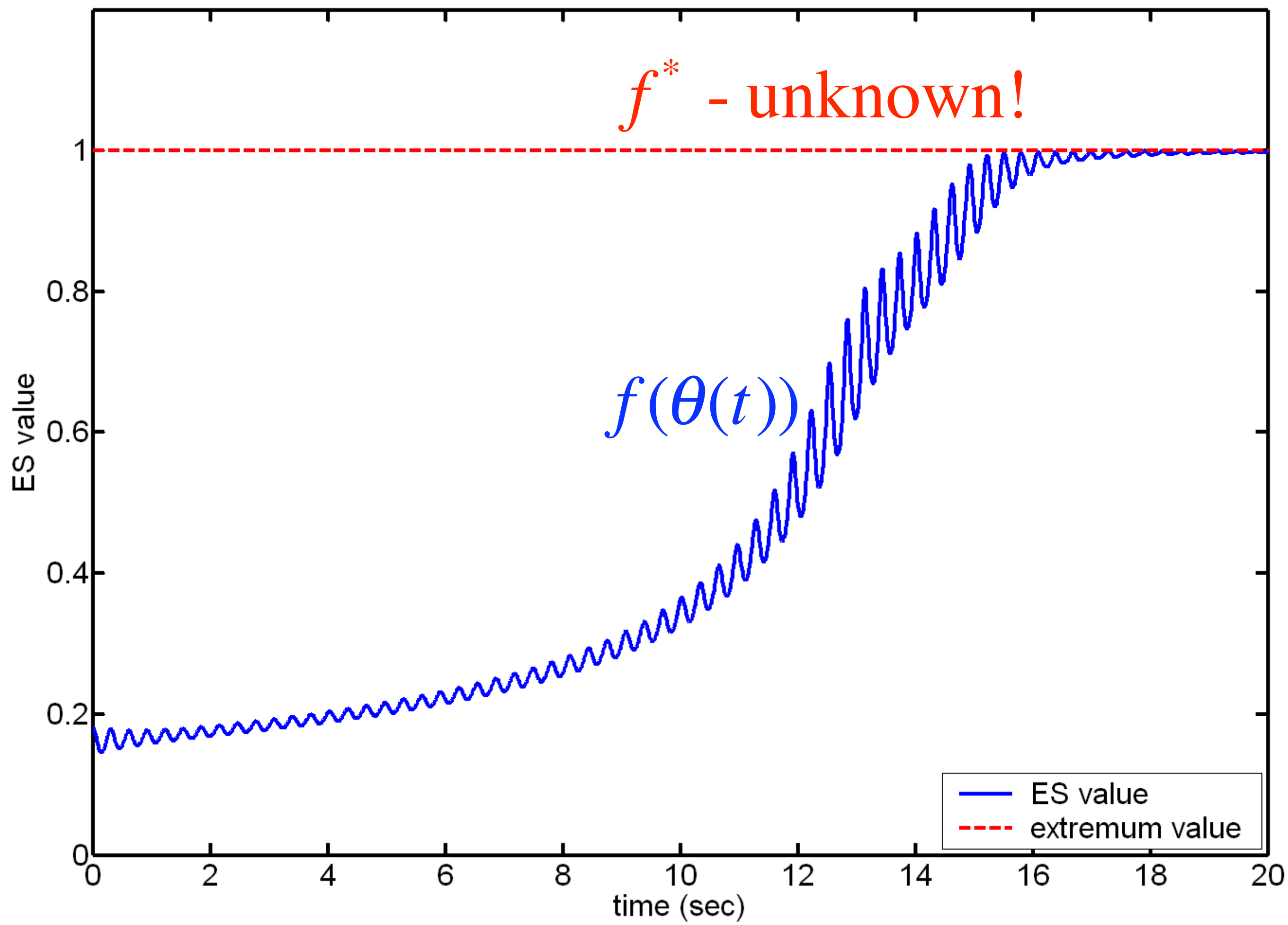
- **Extremum seeking - the basics**
- **Source seeking**
  - 2D - full actuation
  - 2D - nonholonomic vehicle, tuning of speed
  - 2D - nonholonomic vehicle, tuning of heading
  - 3D
  - Fish locomotion models
  - Stochastic source seeking - bacterial locomotion

Extremum Seeking - Basics

Example of Single-Parameter Maximum Seeking



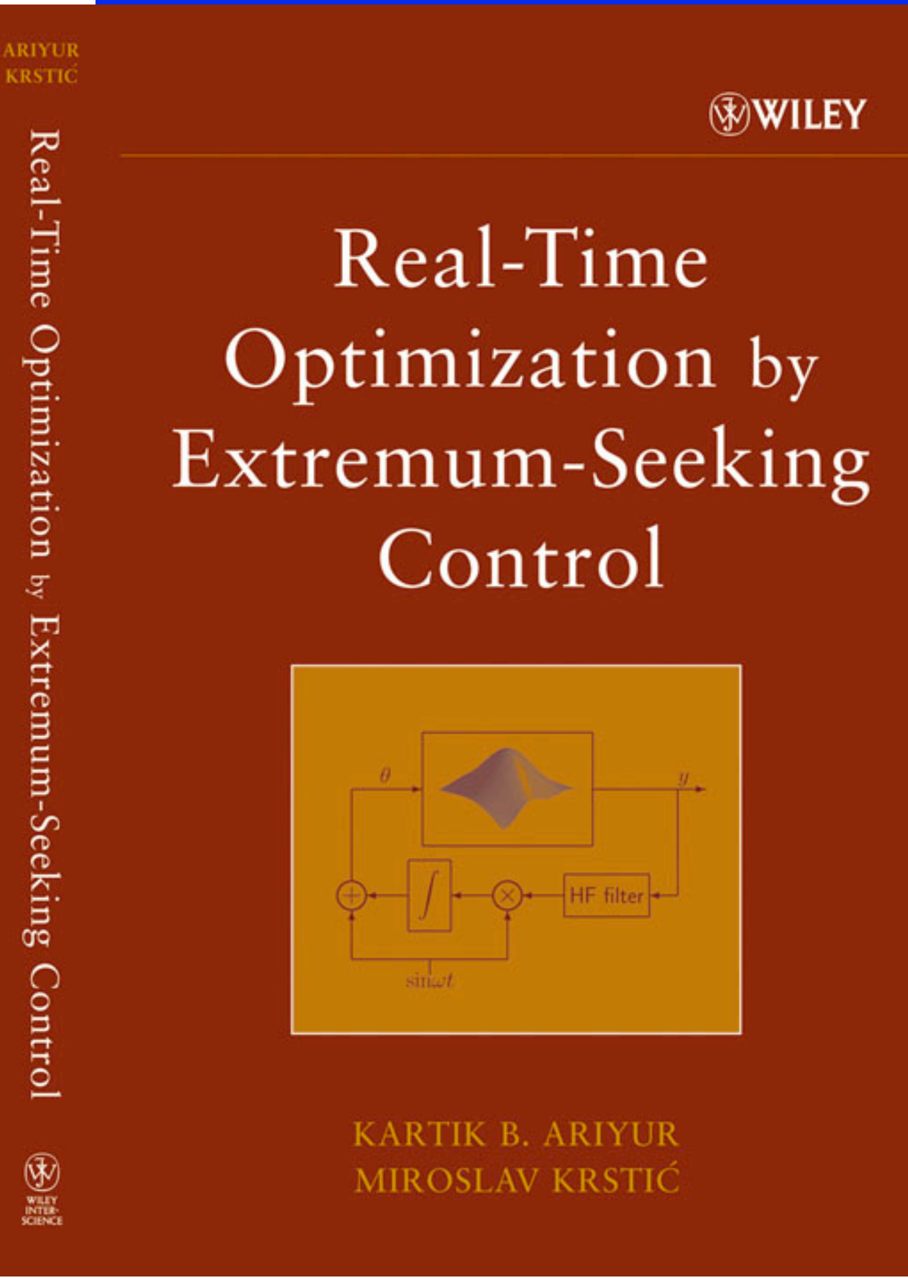
Example of Single-Parameter Maximum Seeking



History

- **Leblanc (1922)** - electric railways
- **Early Russian literature (1940's)** - many papers
- **Drapper and Li (1951)** - application to IC engine spark timing tuning
- **Tsien (1954)** - a chapter in his book on Engineering Cybernetics
- **Feldbaum (1959)** - book *Computers in Automatic Control Systems*
- **Blackman (1962 chap. in book by Westcott)** - nice intuitive presentation of ES
- **Meerkov (1967, 1968)** - papers with averaging analysis
- **Sternby (1980)** - survey
- **Astrom and Wittenmark (1995 book)** - rates ES as one of the most promising areas for adaptive control

Developments Over Last Decade

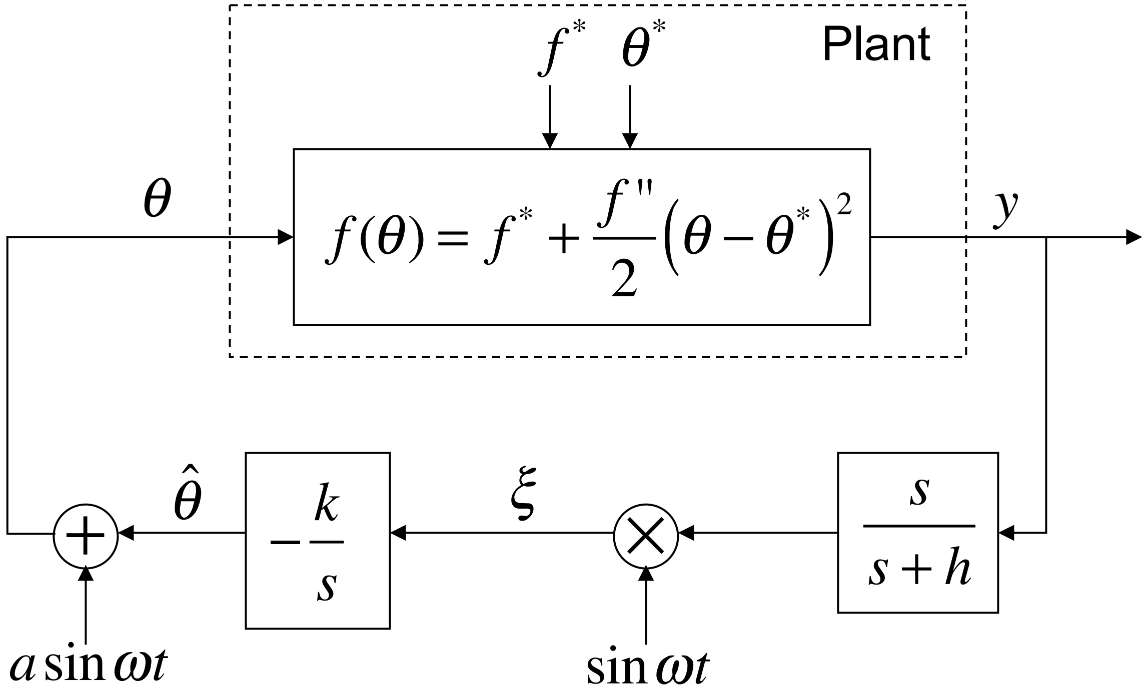


- **Krstic and Wang** - stability proof for single-parameter general dynamic nonlinear plants
- **Krstic, Ariyur, Choi, Wang** - discrete-time, limit cycle minimization, IMC for parameter tracking, dynamic compensators, etc.
- **Rotea; Walsh; Ariyur-Krstic** - multi-parameter ES
- **Ariyur, Krstic** - slope seeking
- **Tan, Nesic, Mareels (2005)** - semi-global stability of ES

Recent Applications

- Compressor instabilities in jet engines
- Combustion instabilities and thermoacoustic coolers
- Formation flight
- Fusion reflected RF power
- Beam matching in particle accelerators
- Flow control: diffusers, airfoils, “minivan” shaped bluff bodies,” cavity flow
- Internal combustion (HCCI) engine fuel consumption minimization
- PID tuning

Basic Extremum Seeking - Static Map



$y =$ output to be minimized

$f^* =$ minimum of the map

$f'' =$ second derivative (positive - $f(\theta)$ has a min.)

$\theta^* =$ unknown parameter

$\hat{\theta} =$ estimate of θ^*

$k =$ adaptation gain (positive) of the integrator $\frac{1}{s}$

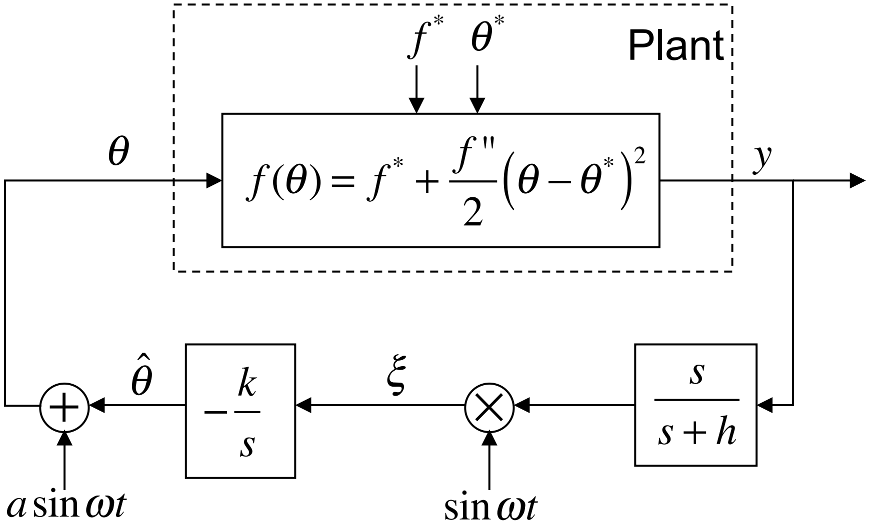
$a =$ amplitude of the probing signal

$\omega =$ frequency of the probing signal

$h =$ cut-off frequency of the "washout filter" $\frac{s}{s+h}$

$+/\times =$ modulation/demodulation

Stability Analysis by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

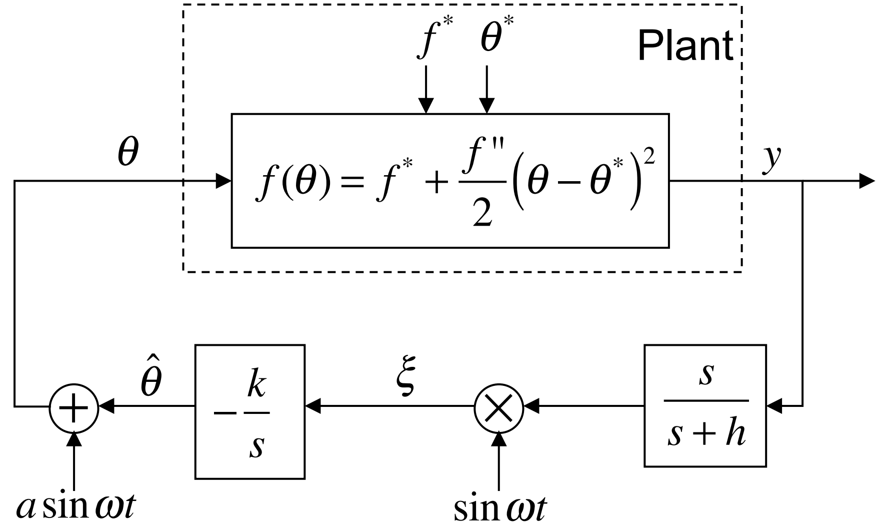
$$\tau = \omega t$$

Full nonlinear time-varying model:

$$\frac{d}{d\tau}\tilde{\theta} = \frac{k}{\omega}\left(\frac{f''}{2}\left(\tilde{\theta} - a\sin\tau\right)^2 - e\right)\sin\tau$$

$$\frac{d}{d\tau}e = \frac{h}{\omega}\left(-e - \frac{f''}{2}\left(\tilde{\theta} - a\sin\tau\right)^2\right)$$

Stability Analysis by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

$$\tau = \omega t$$

Average system:

$$\frac{d}{d\tau}\tilde{\theta}_{\text{av}} = -\frac{kaf''}{2\omega}\tilde{\theta}_{\text{av}}$$

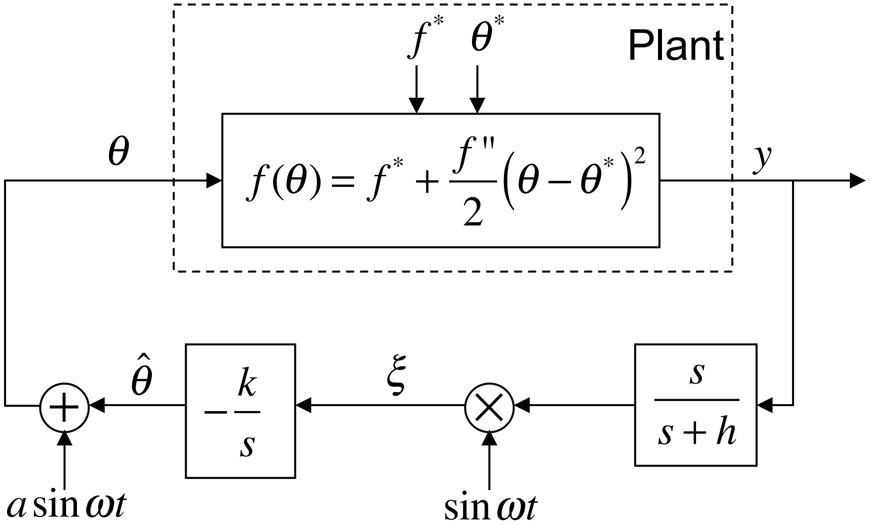
$$\frac{d}{d\tau}e_{\text{av}} = \frac{h}{\omega}\left(-e_{\text{av}} - \frac{f''}{2}\left(\tilde{\theta}_{\text{av}}^2 + \frac{a^2}{2}\right)\right)$$

Average equilibrium:

$$\tilde{\theta}_{\text{av}} = 0$$

$$e_{\text{av}} = -\frac{a^2 f''}{4}$$

Stability Analysis by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

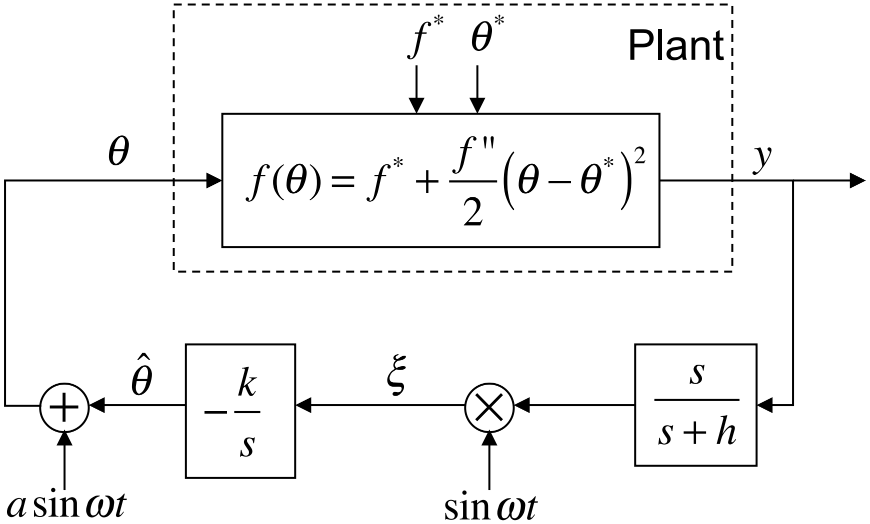
$$e = f^* - \frac{h}{s+h}[y]$$

$$\tau = \omega t$$

Jacobian of the average system:

$$J_{\mathrm{av}} = \begin{bmatrix} -\frac{kaf''}{2\omega} & 0 \\ 0 & -\frac{h}{\omega} \end{bmatrix}$$

Stability Analysis by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

$$\tau = \omega t$$

Theorem. *For sufficiently large* ω *there exists a unique exponentially stable periodic solution of period* $2\pi/\omega$ *and it satisfies*

$$\left|\tilde{\theta}_{2\pi/\omega}(t)\right| + \left|e_{2\pi/\omega}(t) - \frac{a^2 f''}{4}\right| \le O\left(\frac{1}{\omega}\right), \qquad \forall t \ge 0$$

Speed of convergence proportional to $1/\omega$, a^2, k, f''

Source Seeking

for Autonomous Vehicles Without Position Measurement

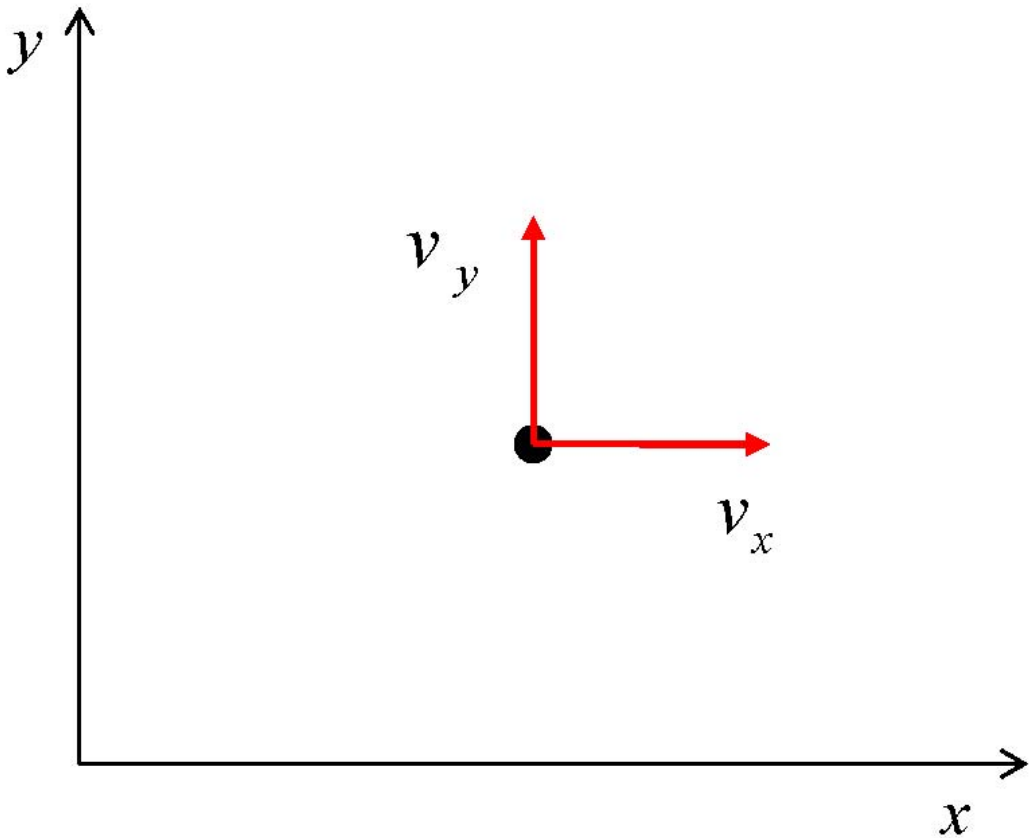
Source Seeking - Introduction

- Motivation
  - Control a vehicle to locate the source of measurable signal with unknown spatial distribution, **without position measurement**
  - Think: *pursuit-evasion* while blindfolded and left to rely on the nose (with one "nostril")

- Related work (only partial autonomy)
  - Porat and Nehorai - vehicle has *position information*
  - Ogren, Fiorelli and Leonard - "group" gradient estim. w/ communication
  - Justh and Krishnaprasad - w/ distance information
  - Klein and Morgansen - tracking of slow ground vehicle w/ fast UAV
  - Marshal, Broucke and Francis - cyclic pursuit problem

Introductory Example:
Fully Actuated Point Mass

Model - fully actuated point mass



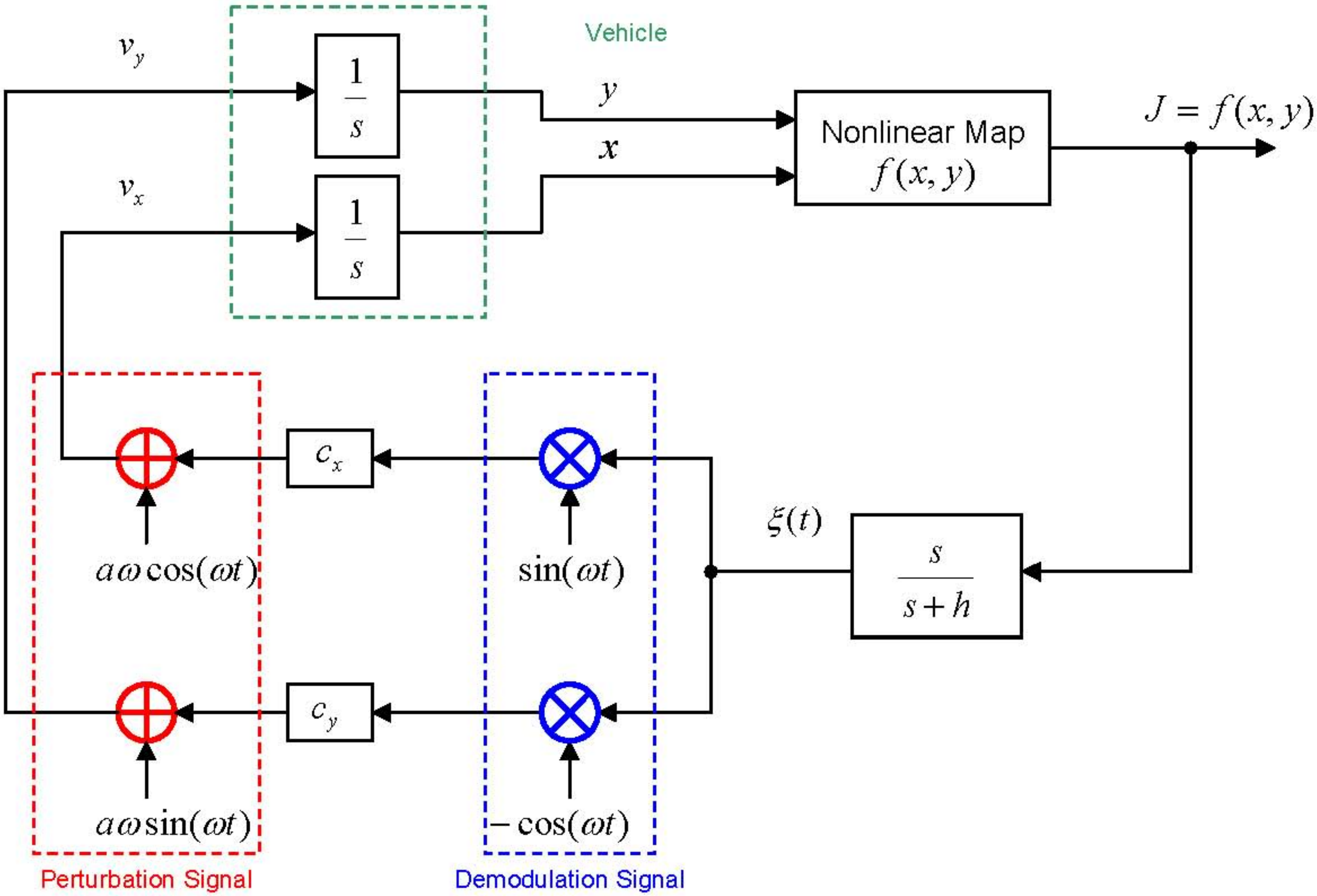
Dynamics

$$\dot{x} = v_x$$
$$\dot{y} = v_y$$

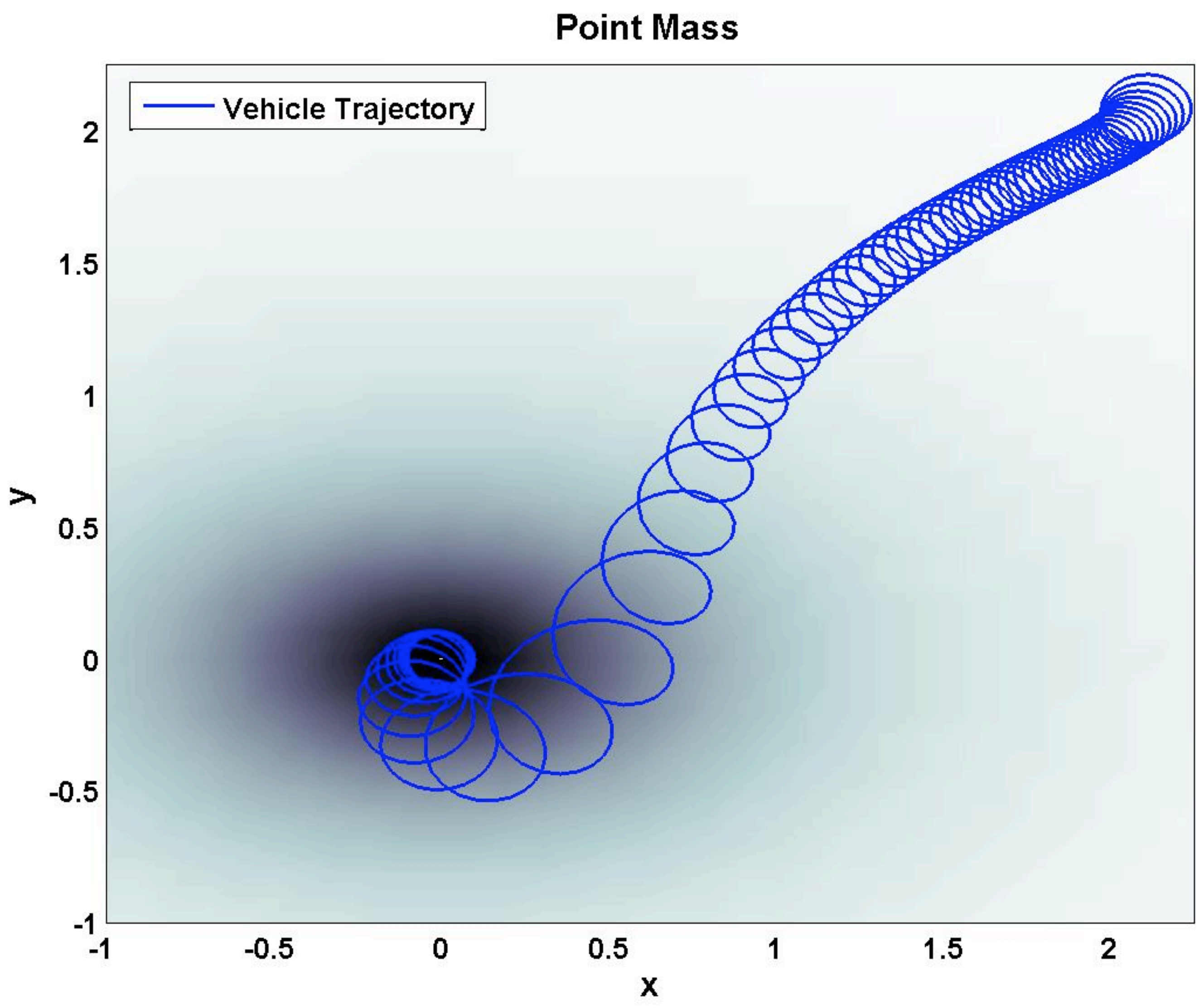
Inputs

$$v_x, v_y$$

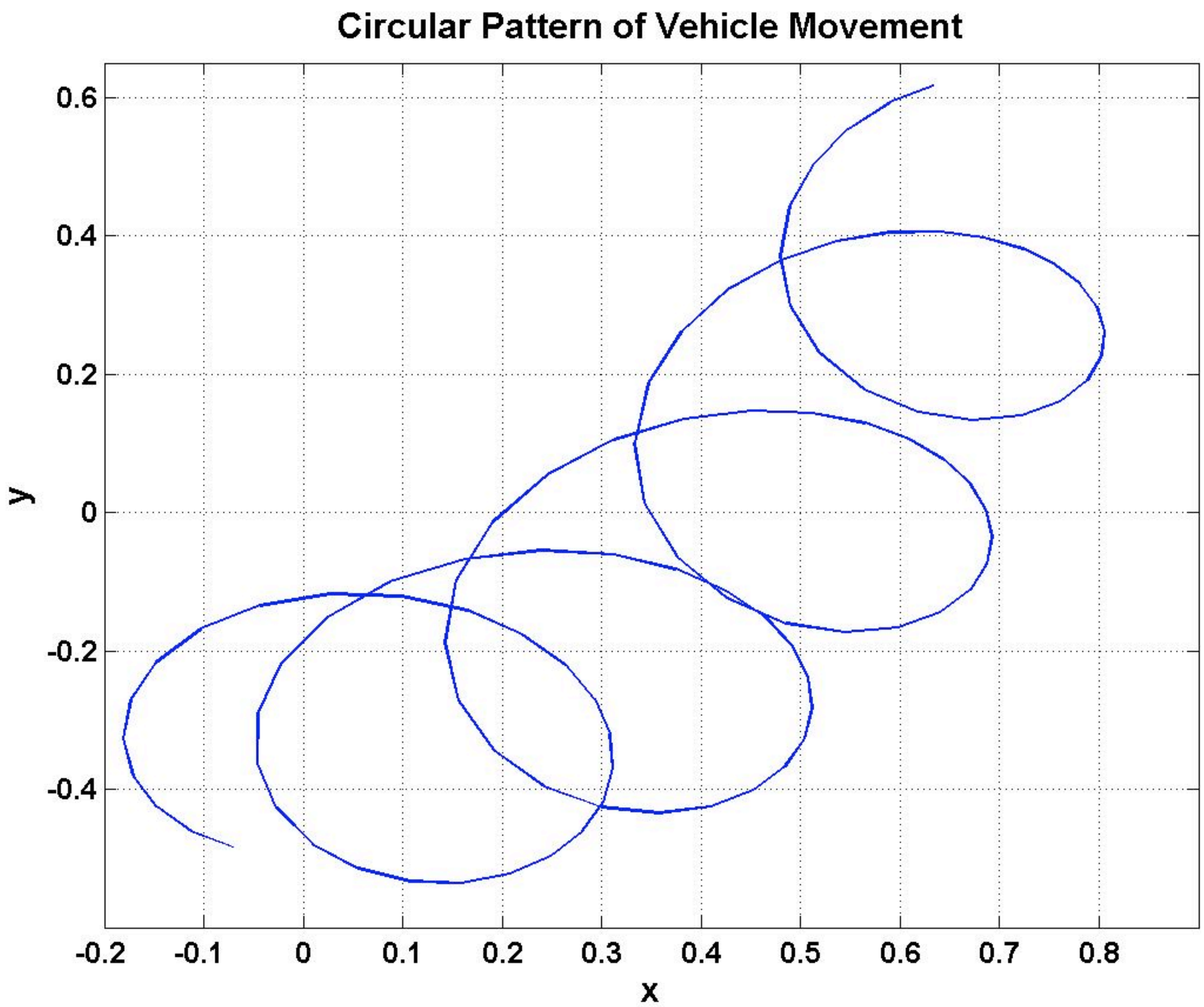
ES - block diagram



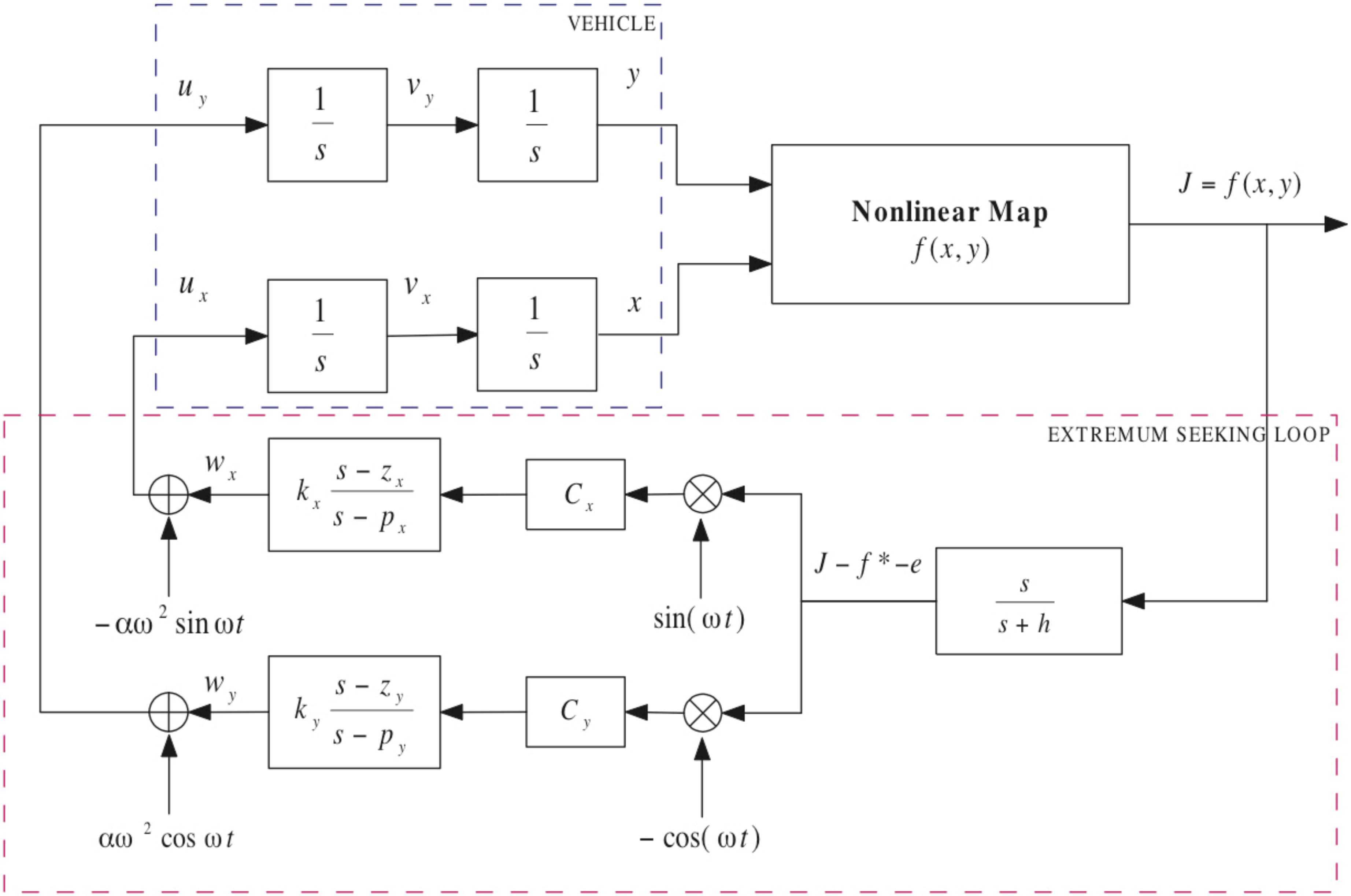
Simulation Results



Simulation Results



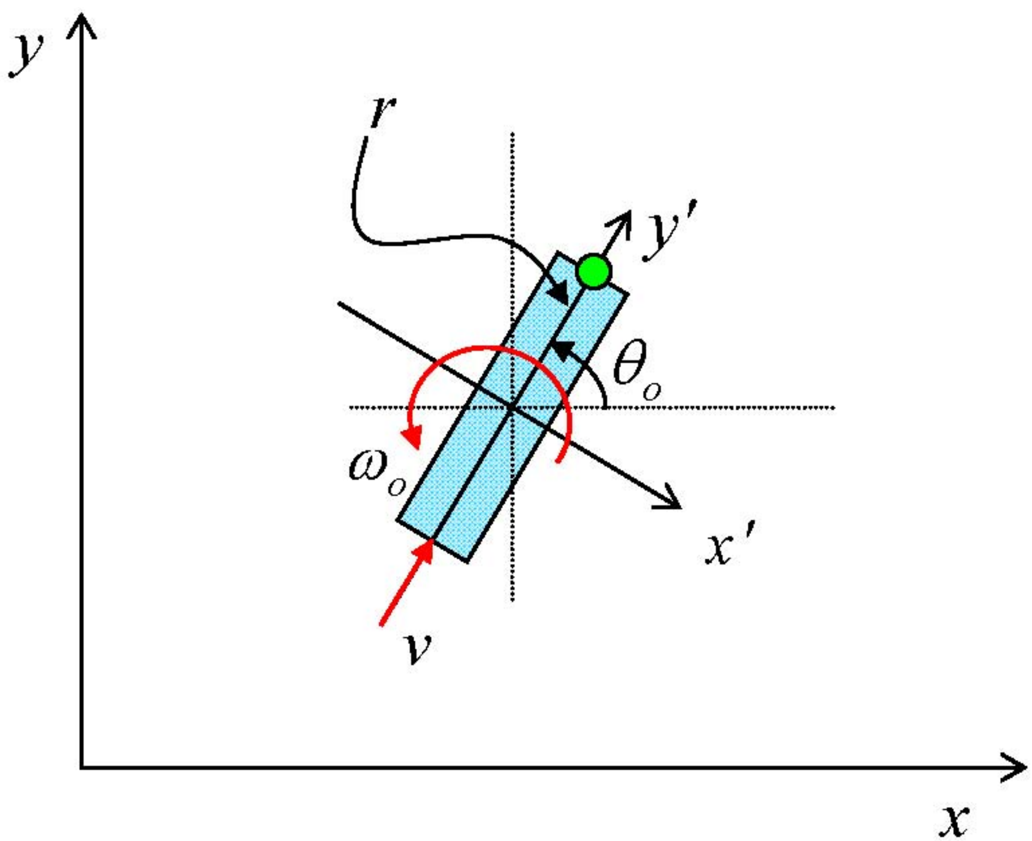
Force-Actuated Point Mass



Nonholonomic “Unicycle”

- Kinematically constrained, underactuated
- Control only steering or only speed, not both

Unicycle Model



Sensor Dynamics

$$\dot{x}_s = v\cos\theta_o - r\dot{\theta}_o \sin\theta_o$$

$$\dot{y}_s = v\sin\theta_o + r\dot{\theta}_o \cos\theta_o$$

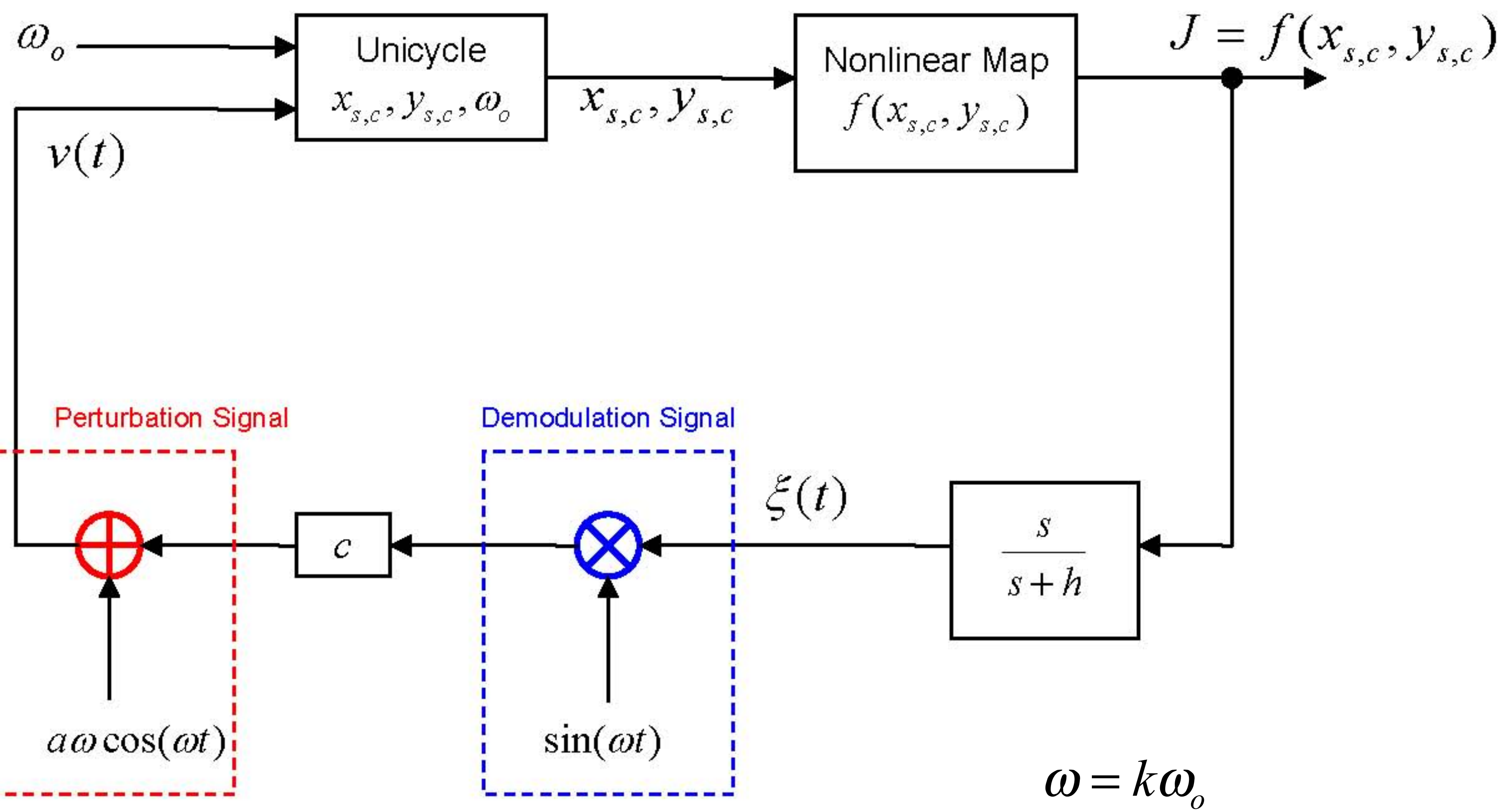
$$\dot{\theta}_o = \omega_o = \frac{d}{dt}\theta_o$$

Inputs

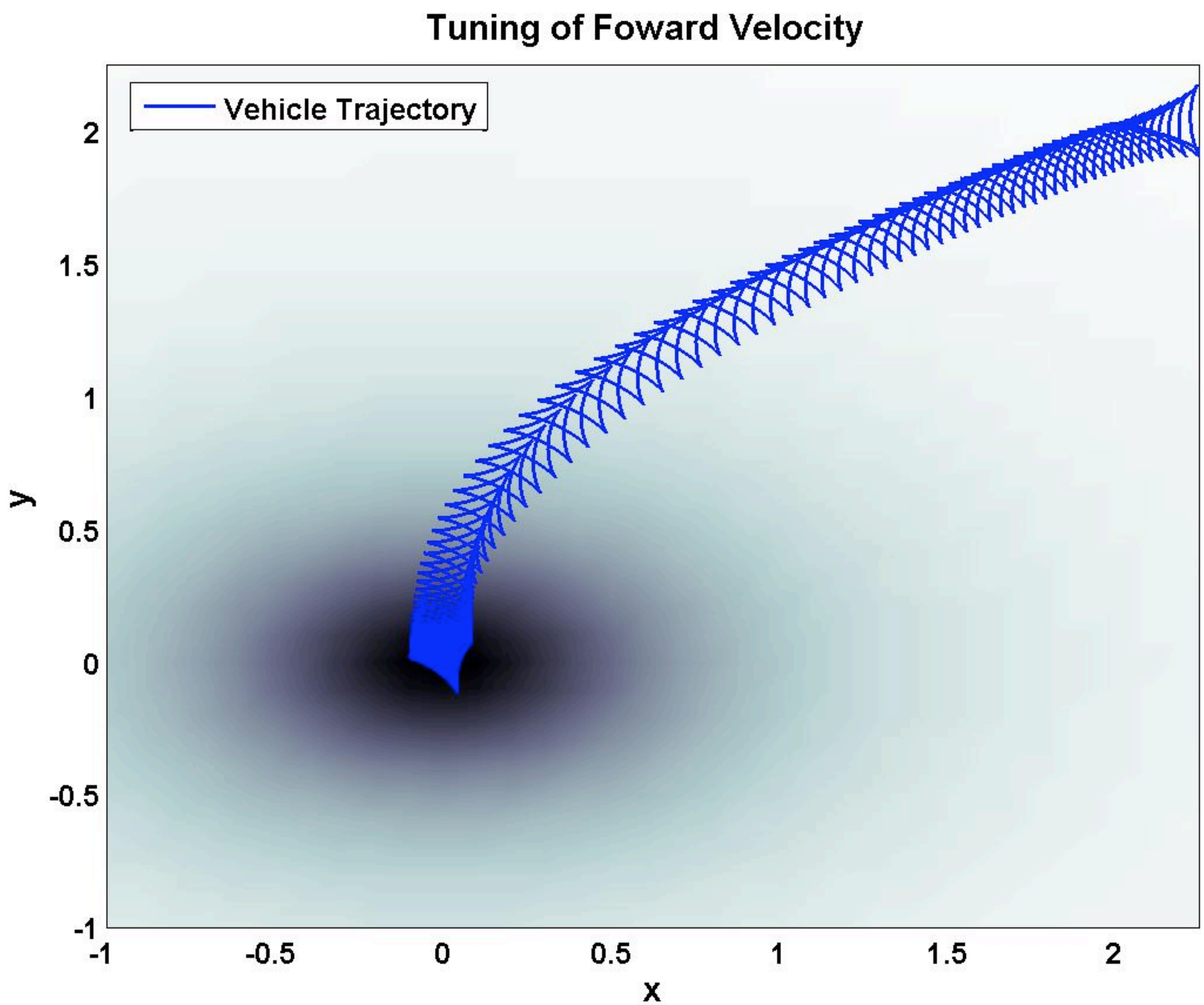
$$v, \omega_o$$

System is linearly **uncontrollable** (from inputs v, ω_o)
and **unobservable** (from the output $f(x,y)$ at its peak)

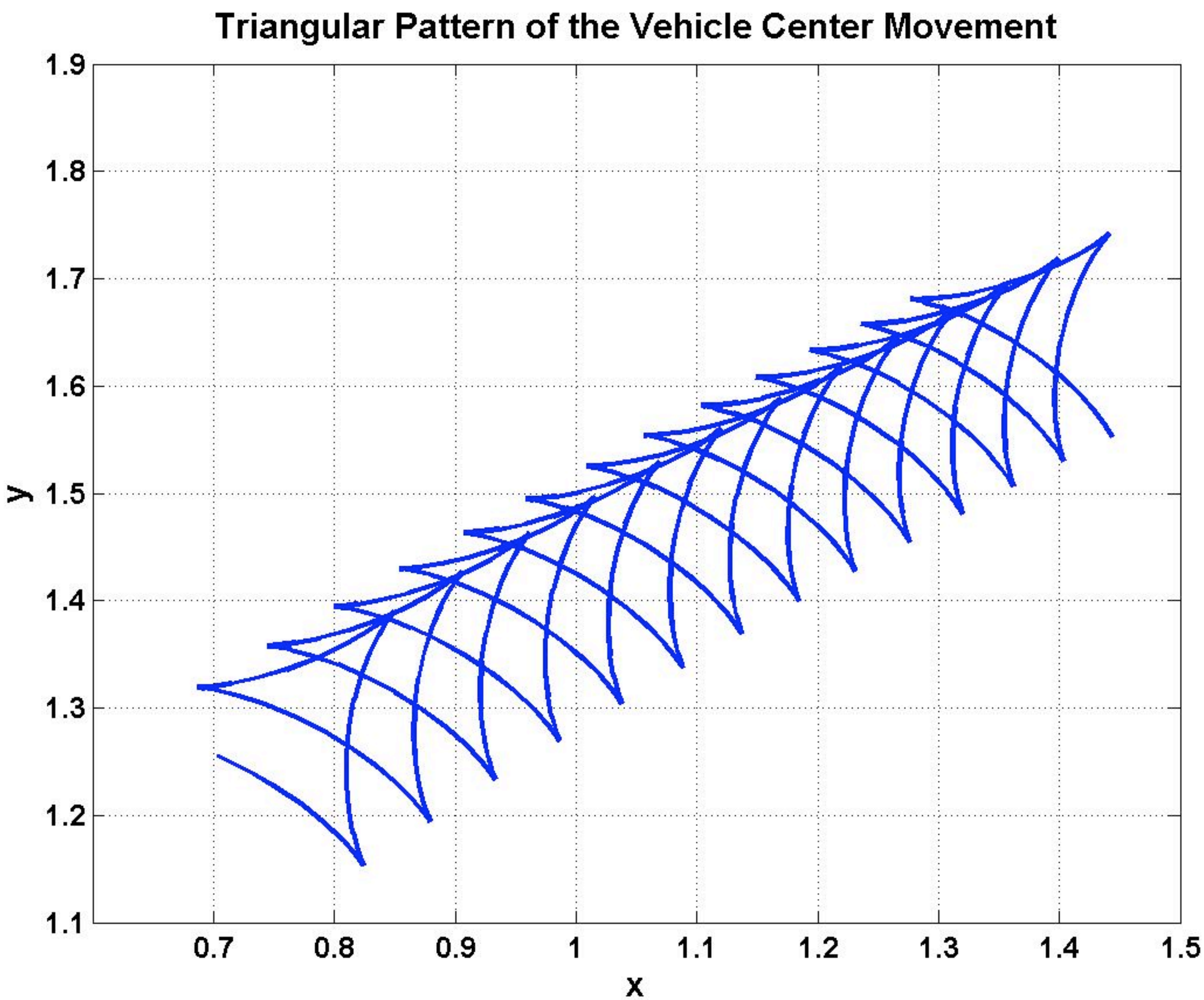
Tuning Speed Only; Heading Const.



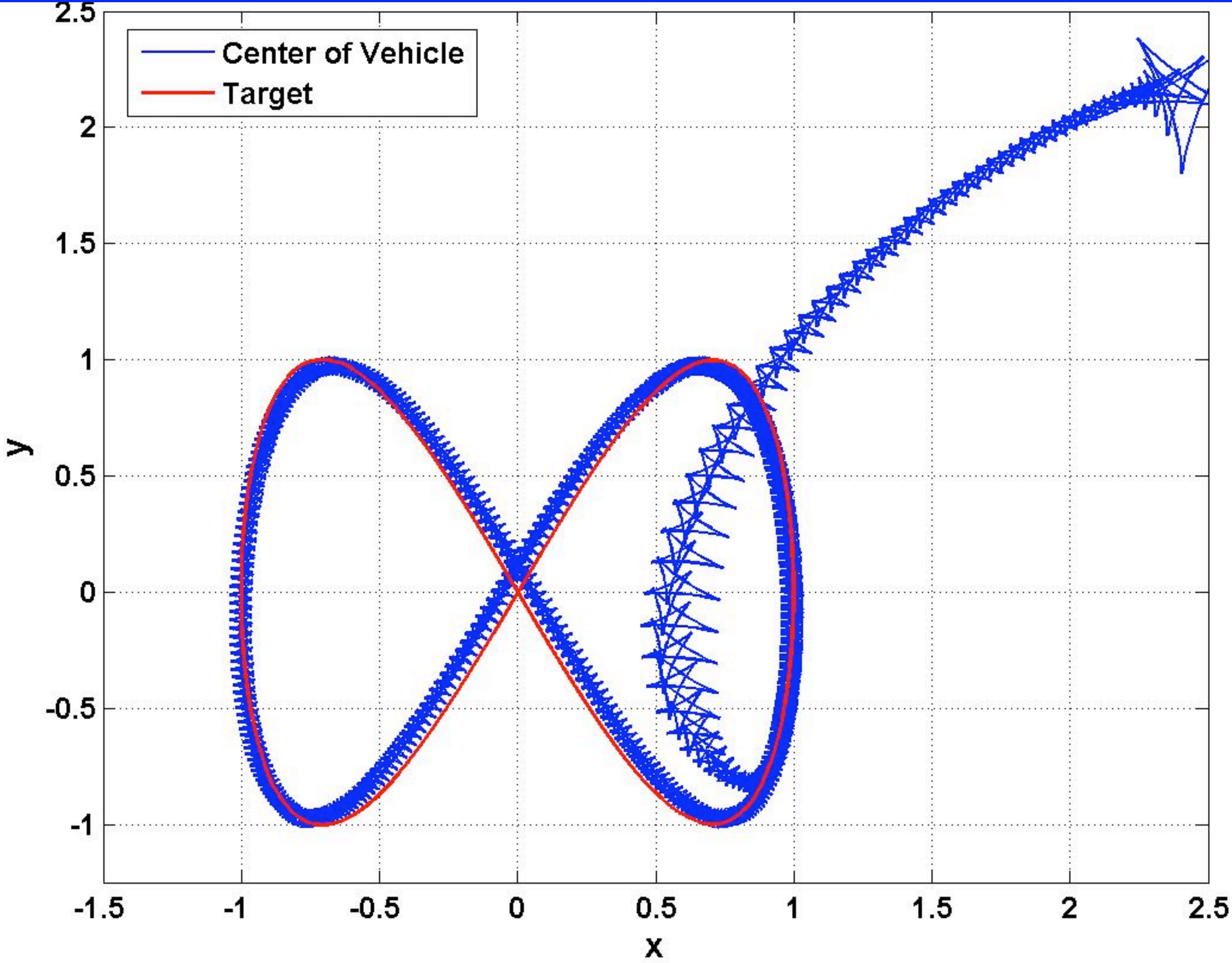
Simulation Results



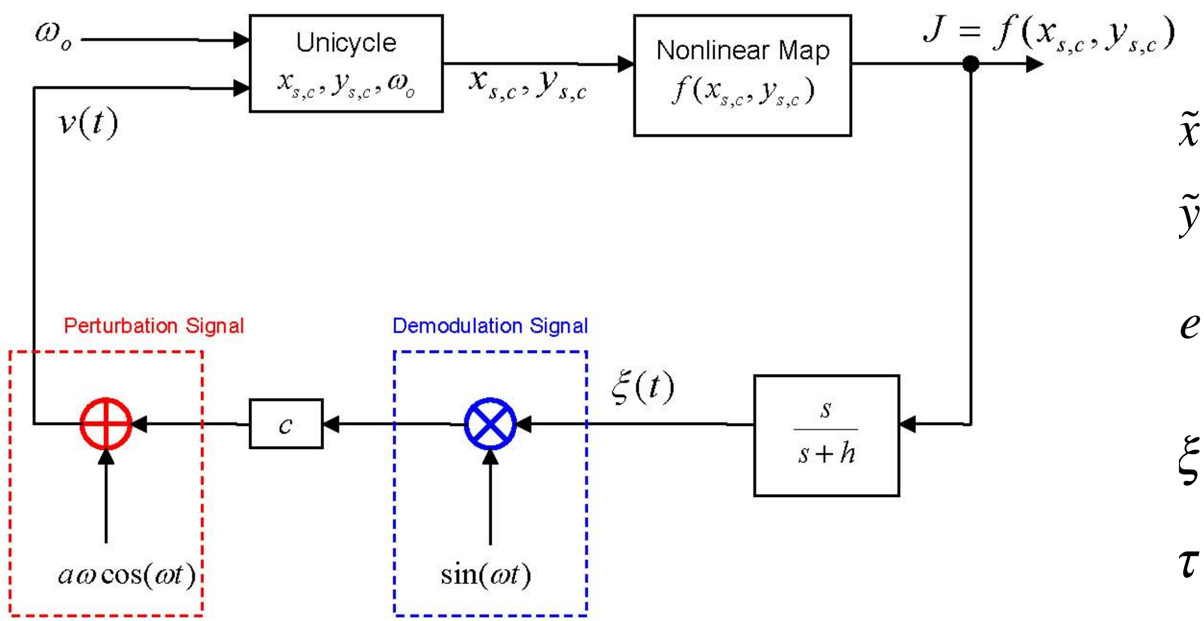
Simulation Results



Simulation Results - Moving Target



Stability Proof by Averaging



$$\tilde{x} = x_c - x^* - a\sin(\omega t)\cos(\omega_o t)$$

$$\tilde{y} = y_c - y^* - a\sin(\omega t)\sin(\omega_o t)$$

$$e = \frac{h}{s+h}[J] - f^*$$

$$\xi = \frac{s}{s+h}[J] = J - f^* - e$$

$$\tau = \omega t$$

$$\frac{d}{d\tau}\tilde{x} = \frac{1}{\omega}\left[c\sin\tau\cos\left(\frac{\tau}{k}\right)\xi + a\omega_o\sin\tau\sin\left(\frac{\tau}{k}\right)\right]$$

$$\frac{d}{d\tau}\tilde{y} = \frac{1}{\omega}\left[c\sin\tau\sin\left(\frac{\tau}{k}\right)\xi - a\omega_o\sin\tau\cos\left(\frac{\tau}{k}\right)\right]$$

$$\frac{d}{d\tau}e = \frac{h}{\omega}\xi$$

where

$$\xi = -q_x\left(\tilde{x} + a\sin\tau\cos\left(\frac{\tau}{k}\right)\right)^2 - q_y\left(\tilde{y} + a\sin\tau\sin\left(\frac{\tau}{k}\right)\right)^2 - e$$

Stability Proof by Averaging

Theorem:

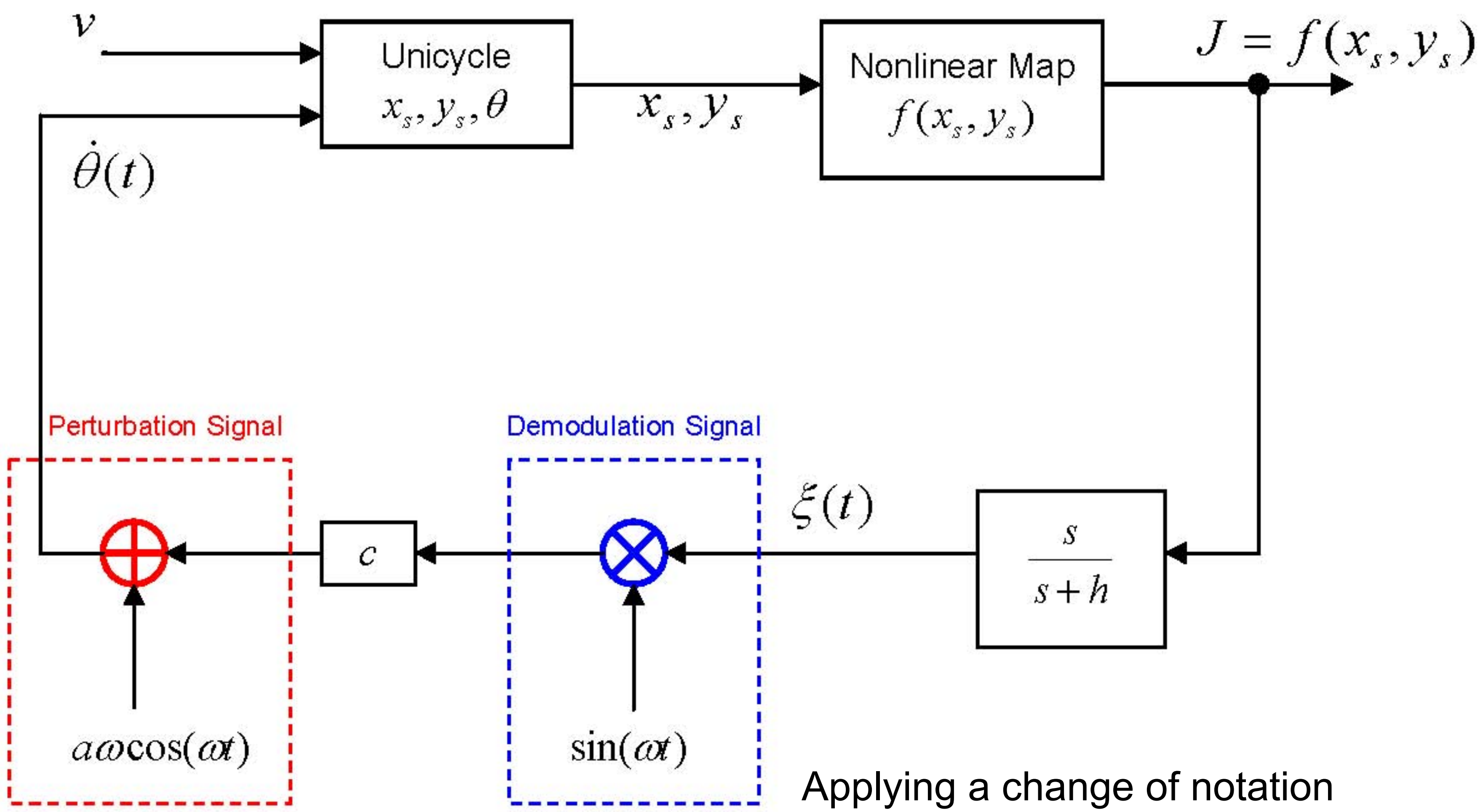
For sufficiently large w *there exists a unique* ***exponentially stable periodic solution*** *of period* 2π/w *and it satisfies*

$$\left\| \begin{bmatrix} \tilde{x}^{\frac{2\pi}{\omega}} \\ \tilde{y}^{\frac{2\pi}{\omega}} \\ e^{\frac{2\pi}{\omega}} + \frac{a^2}{4}\left(q_x + q_y\right) \end{bmatrix} \right\| \leq O(1/\omega), \quad \forall \ t \geq 0$$

Speed of convergence proportional to $1/\omega$, a^2, c, q_x, q_y

Alternative Design:
Tuning of *Heading Rate* Only

Tuning the Angular Velocity



Applying a change of notation

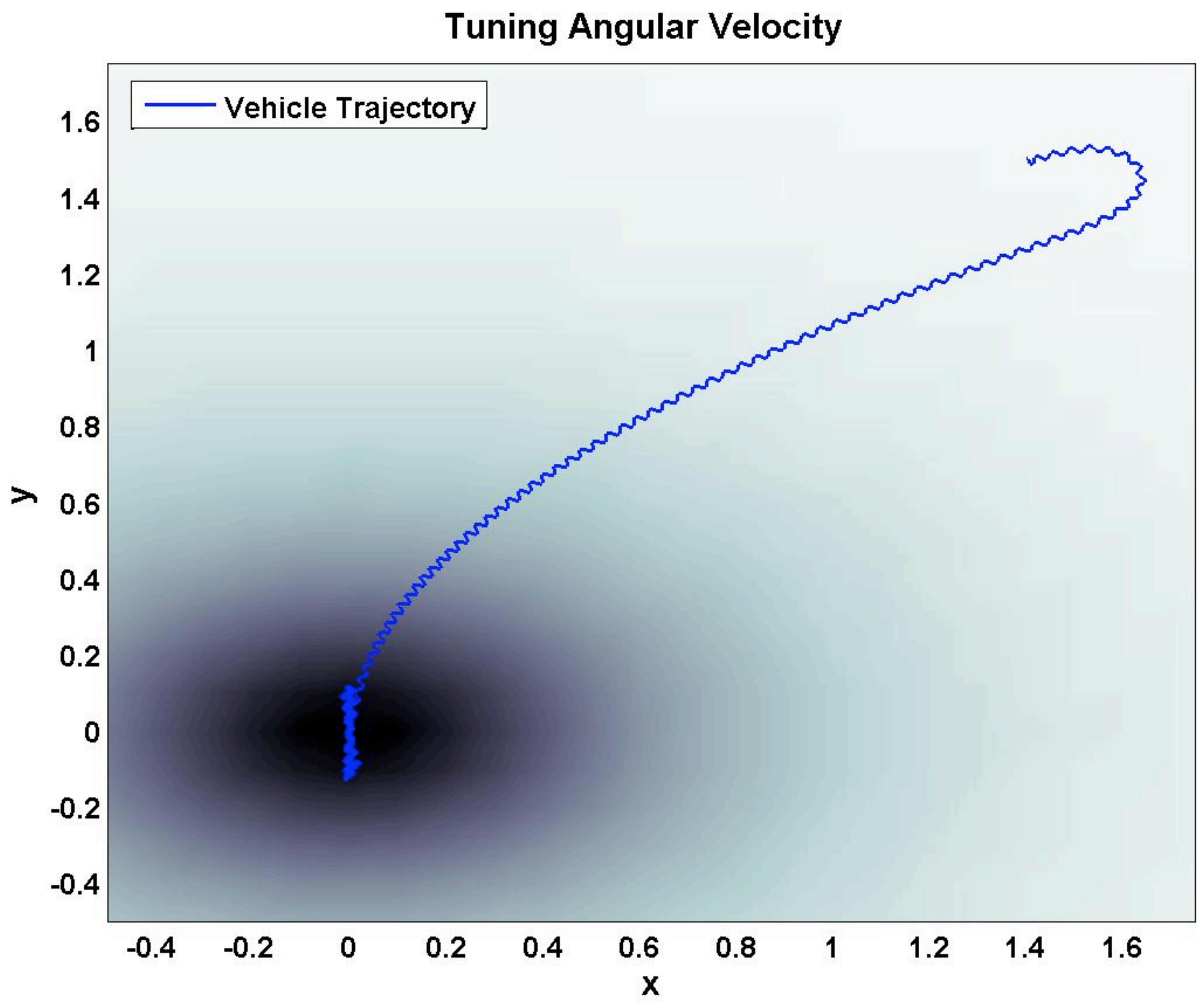
$$\theta = \theta_o$$

The Algorithm

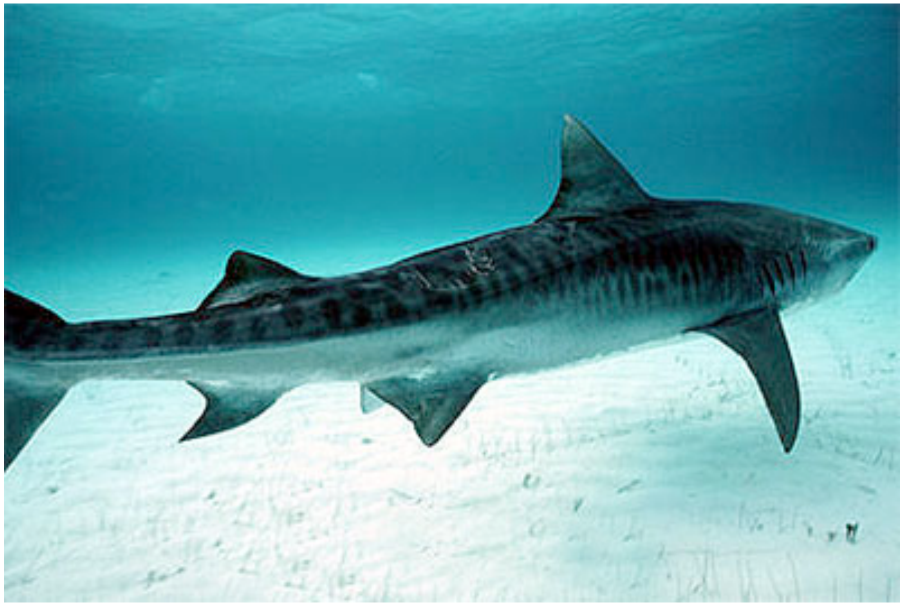
$$\dot{\theta}(t) = a\omega \cos(\omega t) + c\frac{s}{s+h}[J(t)]\sin(\omega t)$$

Linear combination of cosine and sine
but non-constant coefficients!

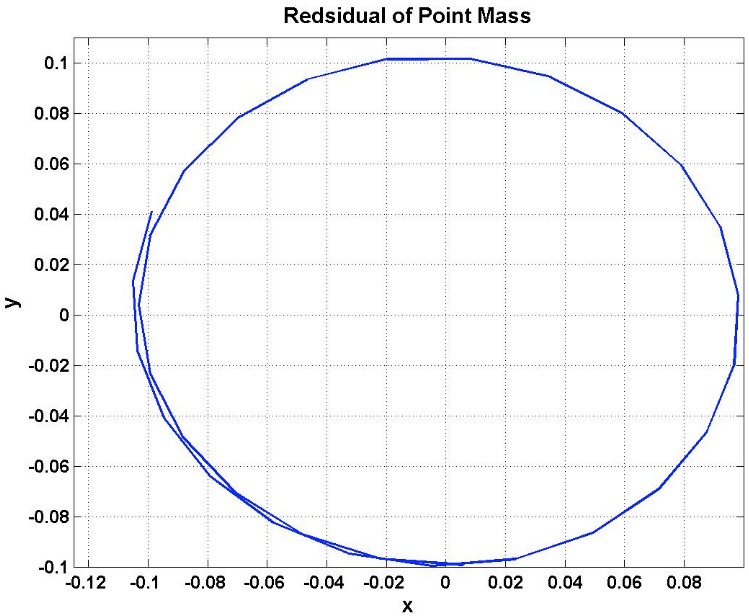
Simulation Results

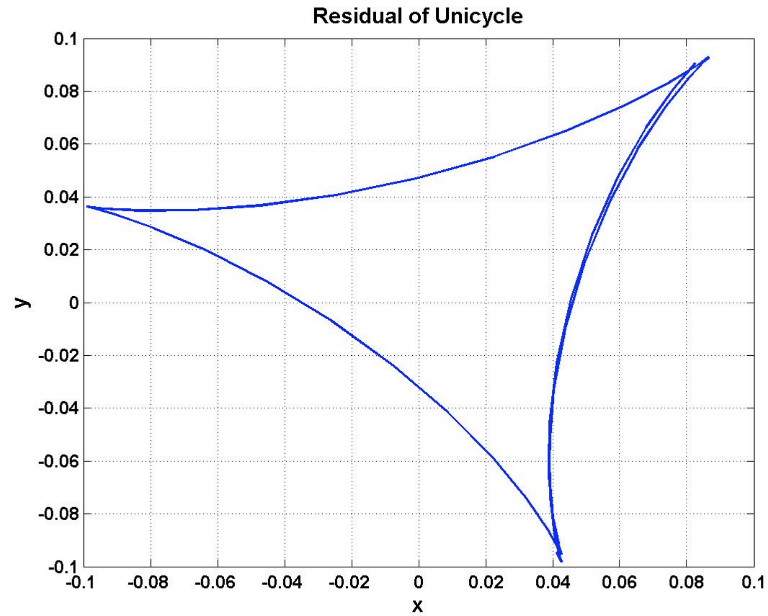


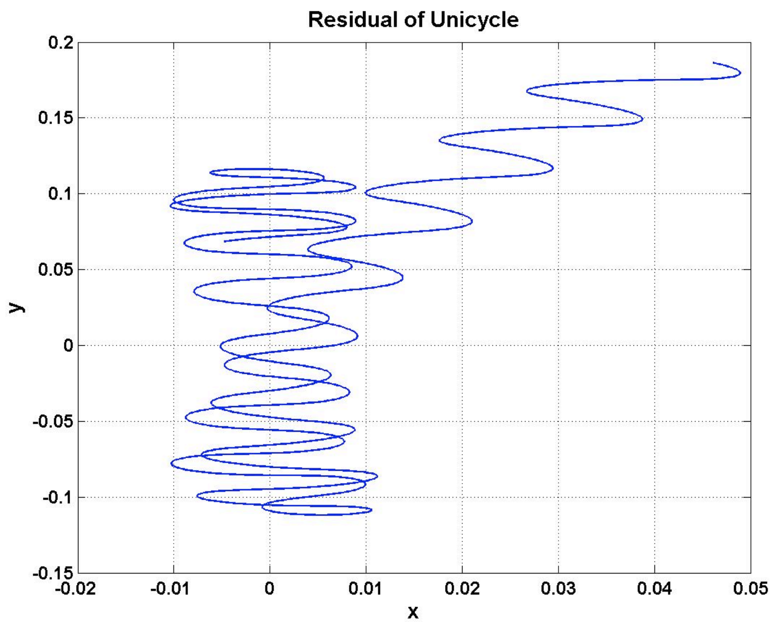
Inspired by…



Summary

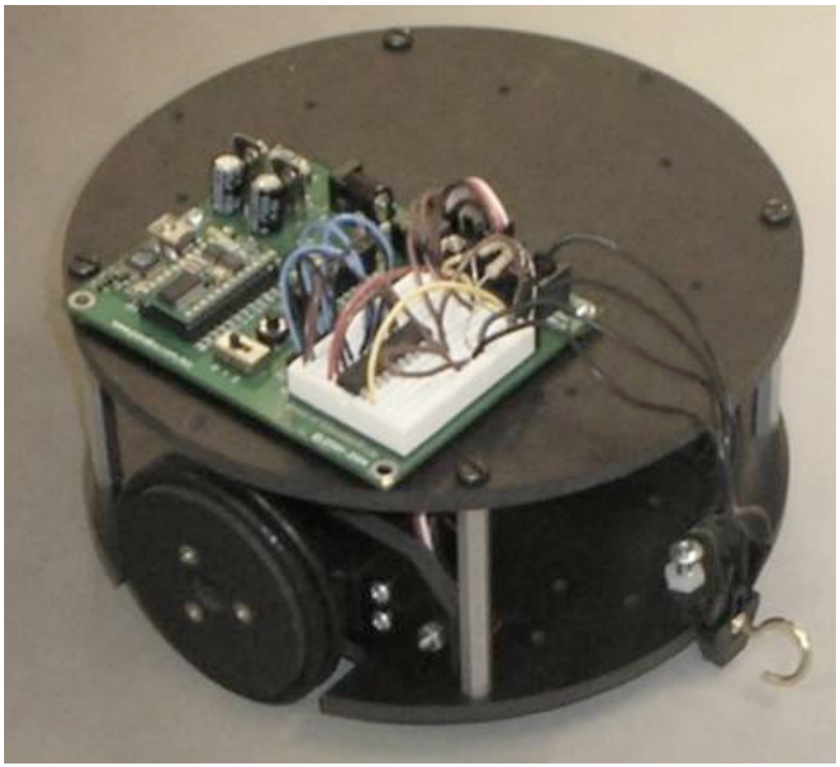

Redsidual of Point Mass
y
x

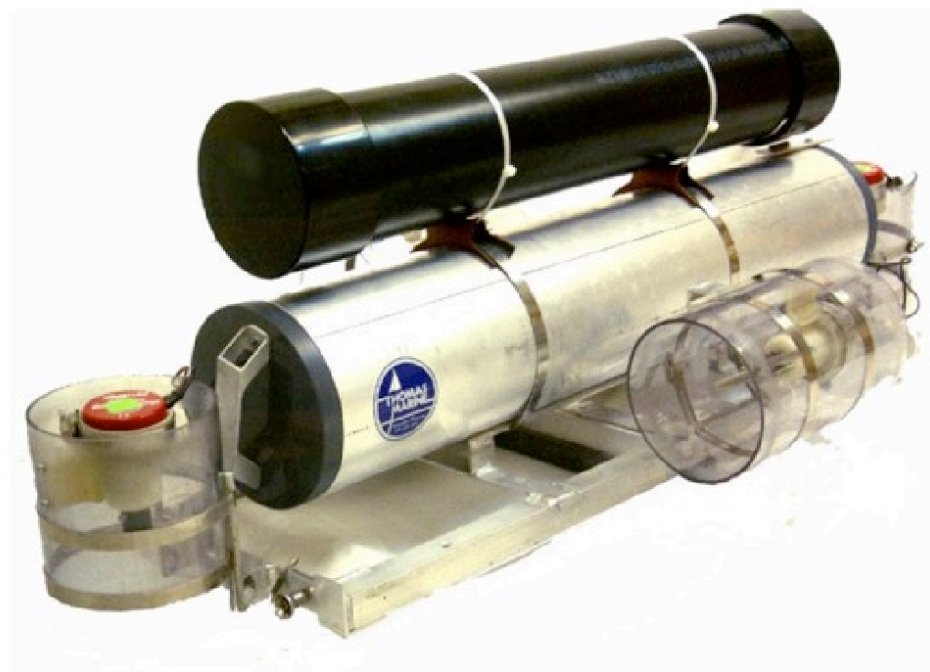

Residual of Unicycle
y
x


Residual of Unicycle
y
x

Track a Diffusive Source

Experimental Results

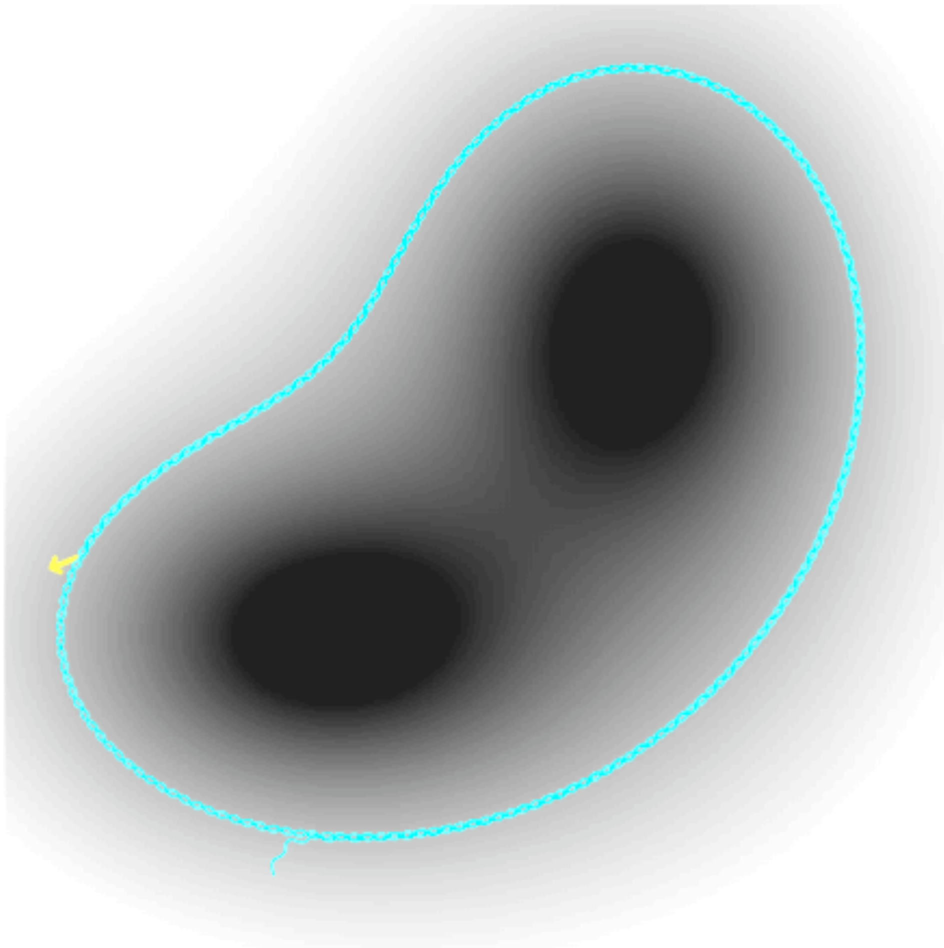




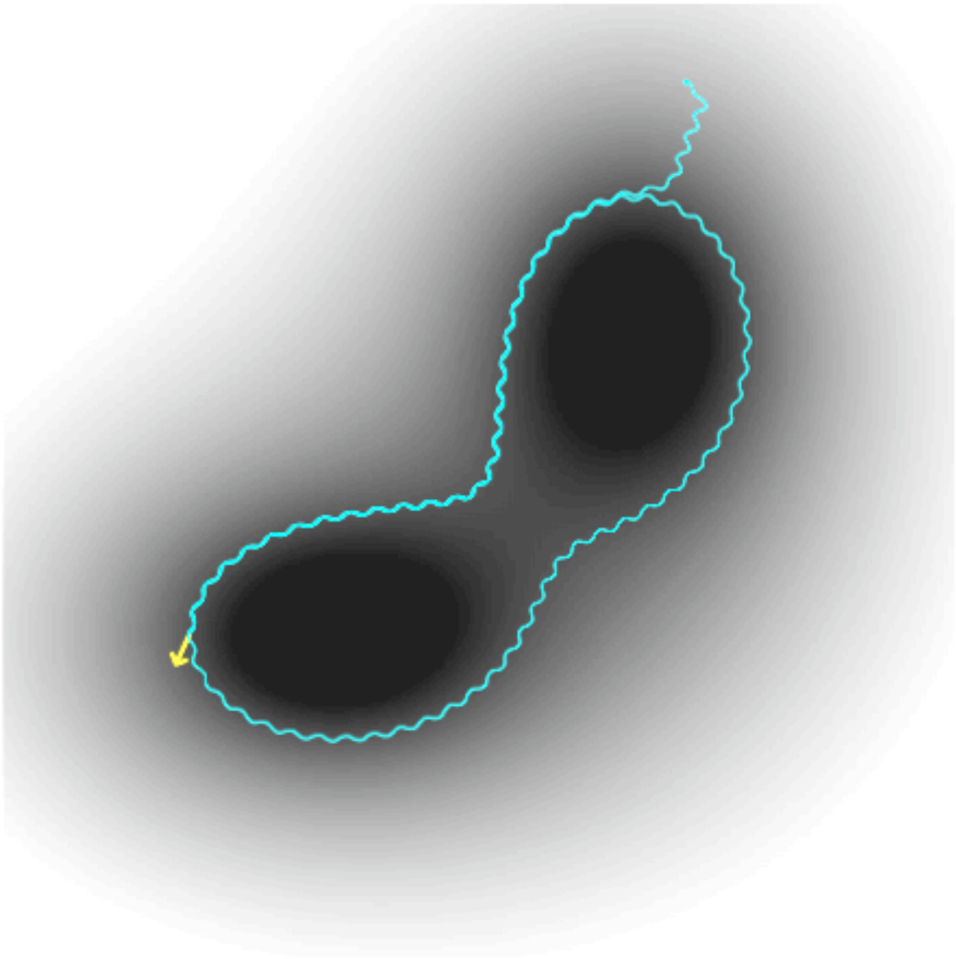
Level Sets

- Methods using **multipe AUVs**:
  - Kalantar & Zimmer
  - N. Leonard, Fiorelli, Ogren
  - Bertozzi
  - Burian, Singh
  - Bennett, J. Leonard

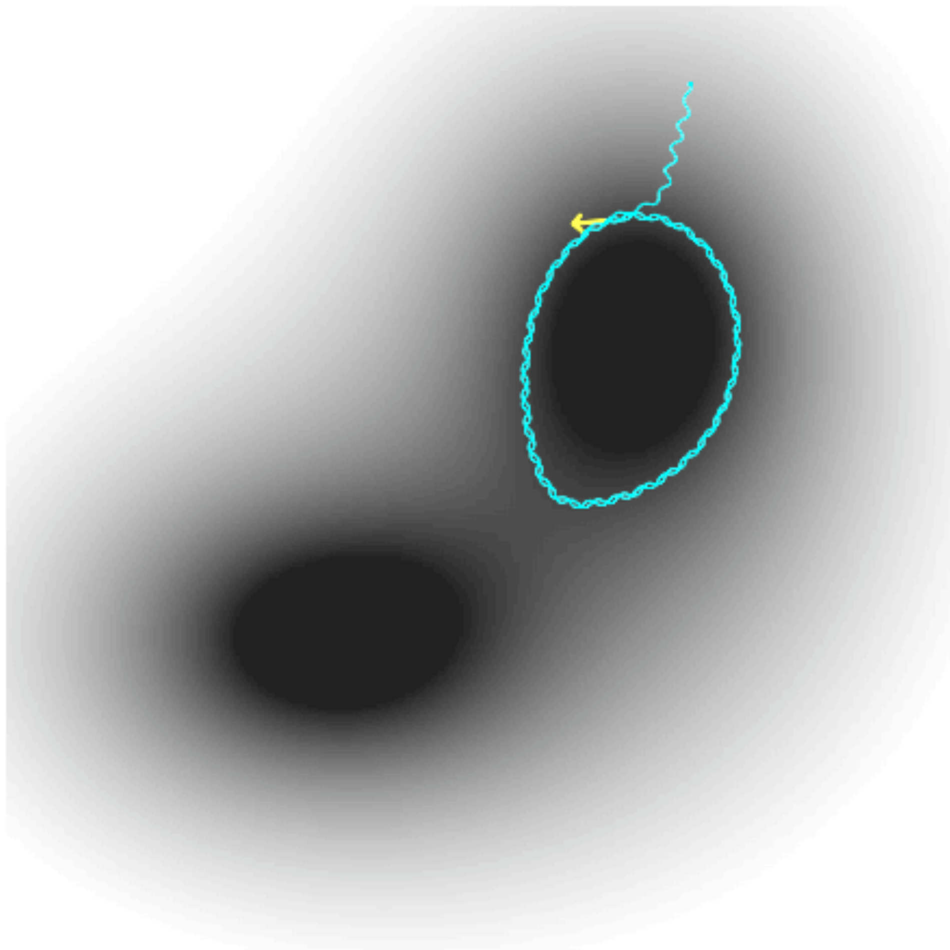
Level Sets



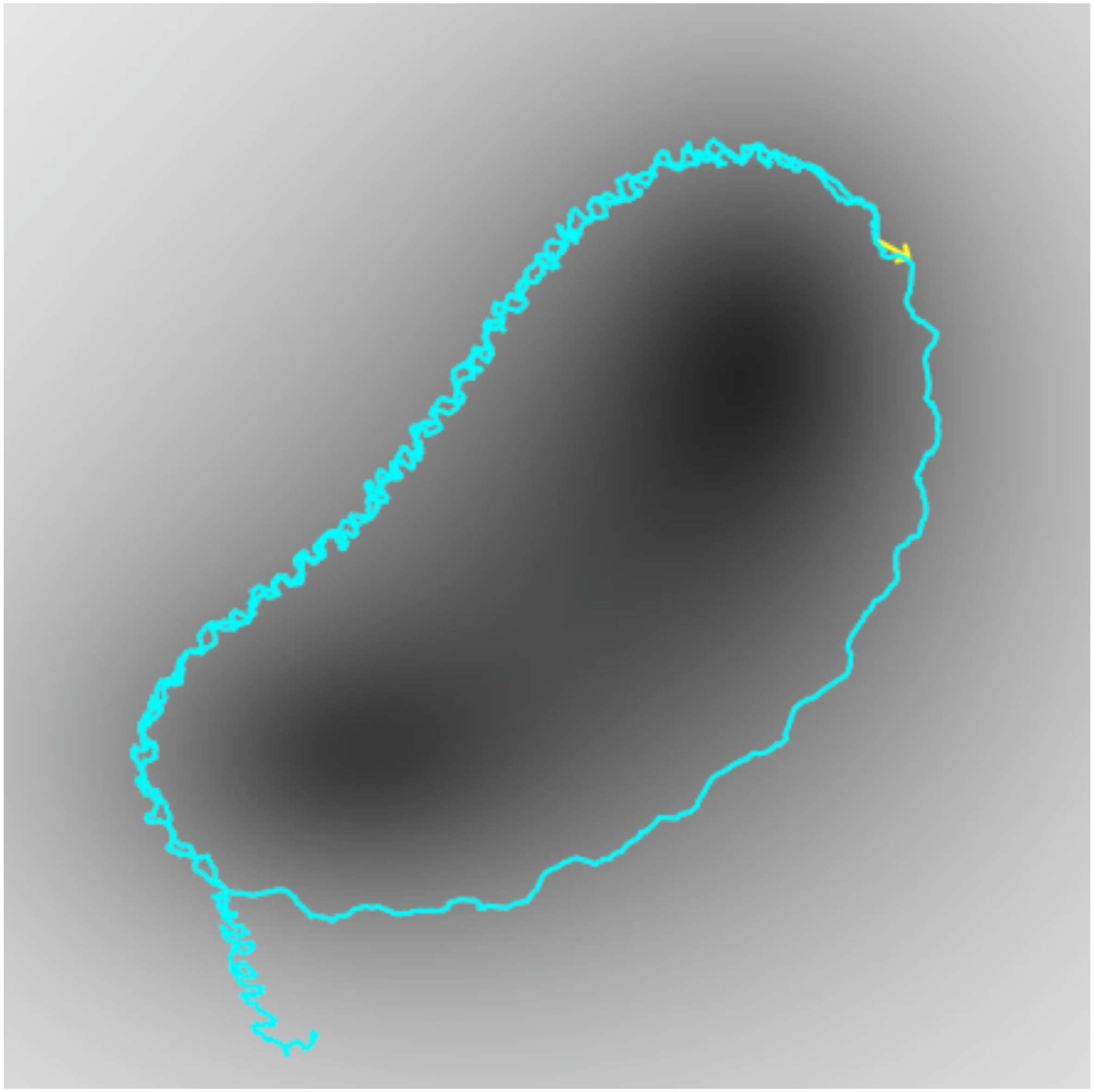
Level Sets



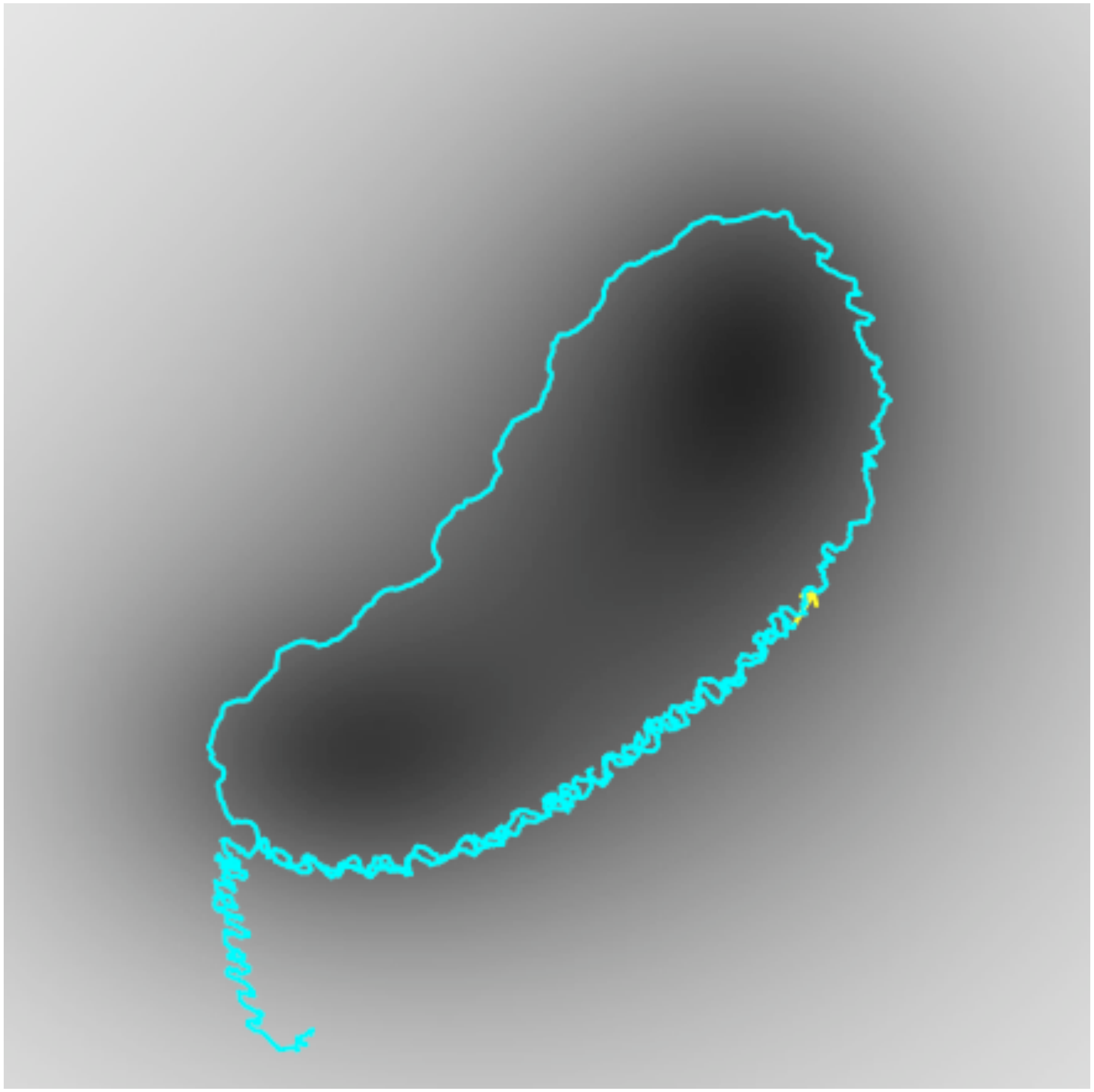
Level Sets



Level Sets

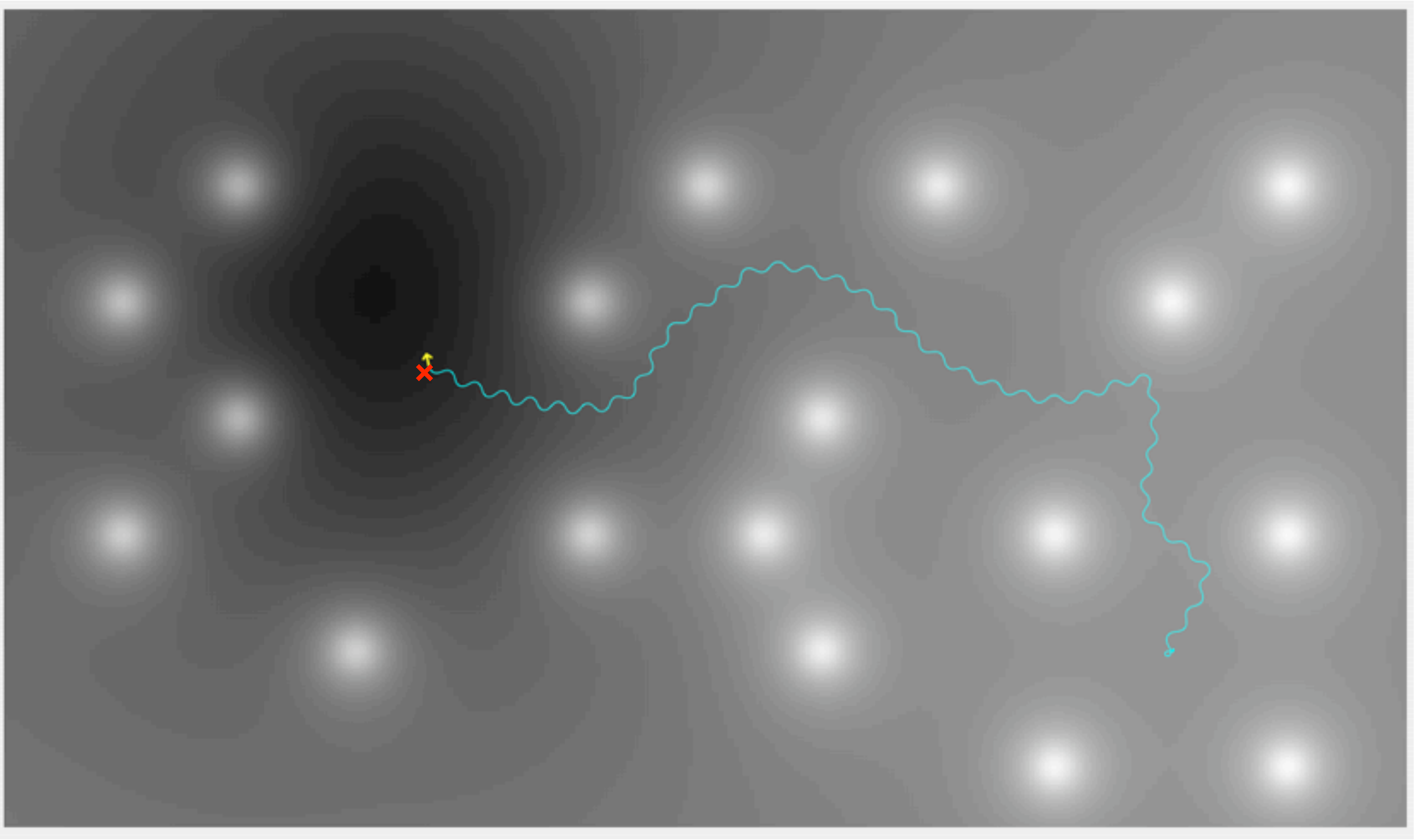


Level Sets



Mobile Robot Experiments

Navigation Through a "Minefield"

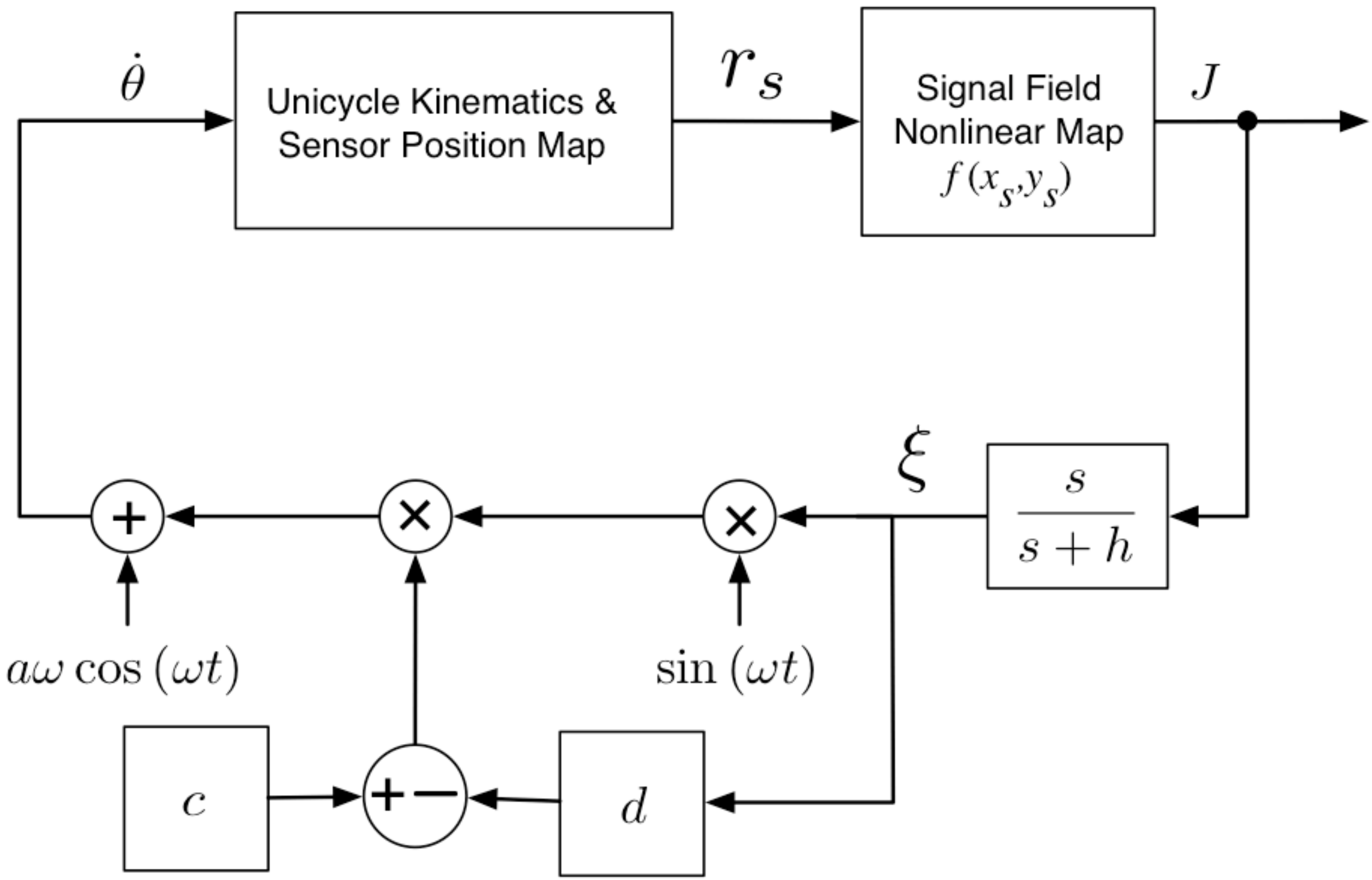


Hardest Problem: Stationary Source

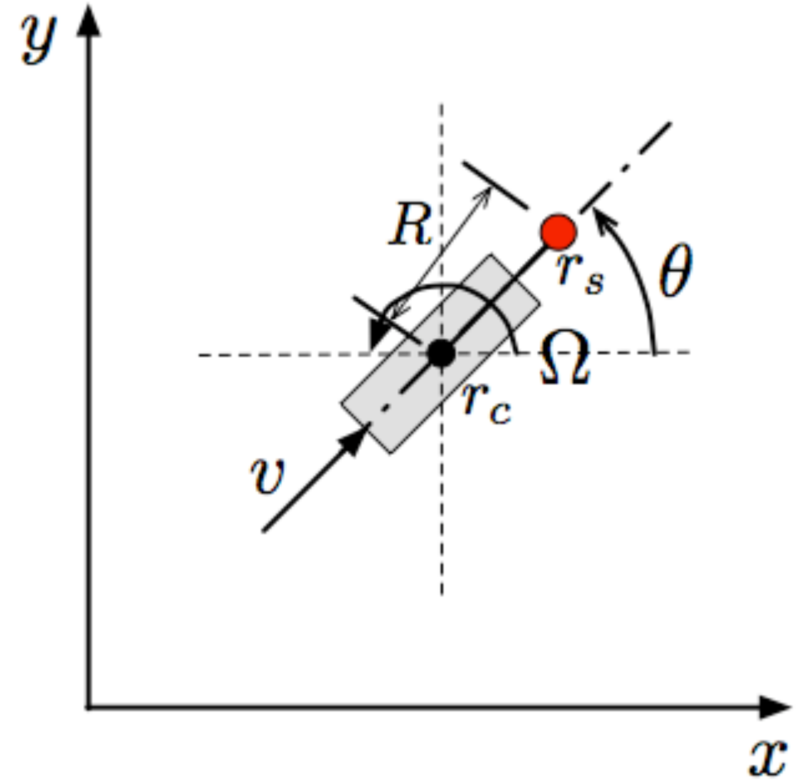
Characterization of *Attractors*

(stability proof)

Stability w/ Stationary Source



Stability w/ Stationary Source



$$\dot{r}_c = V_c e^{j\theta}$$

$$\dot{\theta} = a\omega\cos(\omega t) + \sin(\omega t)\left\{ c\frac{s}{s+h}[J] - d\left(\frac{s}{s+h}[J]\right)^2 \right\}$$

Stability w/ Stationary Source

Theorem. *The vehicle center locally exponentially converges to one of the two (almost periodic) attractors of the form*

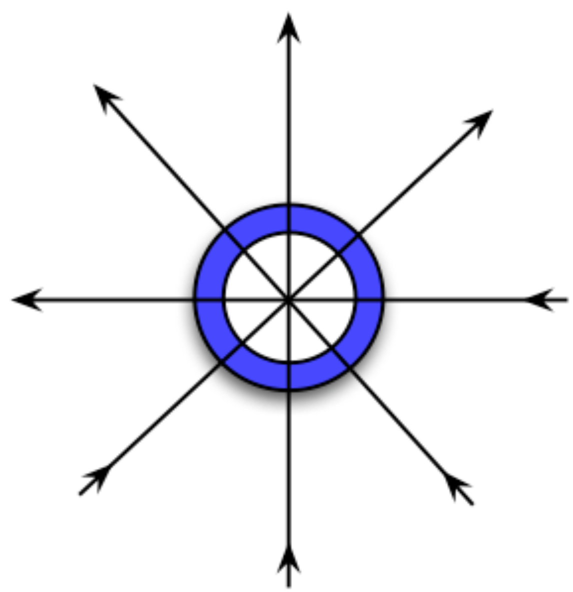
$$x_c^{\mathrm{attr}}(t) = x^* - \left(\rho + \tilde{r}_\pm^{2\pi/\omega}(t)\right)\cos\left(\frac{V_c}{\rho}t + \alpha_\pm^{2\pi/\omega}(t) + \gamma\right)$$

$$y_c^{\mathrm{attr}}(t) = y^* \pm \left(\rho + \tilde{r}_\pm^{2\pi/\omega}(t)\right)\sin\left(\frac{V_c}{\rho}t + \alpha_\pm^{2\pi/\omega}(t) + \gamma\right)$$

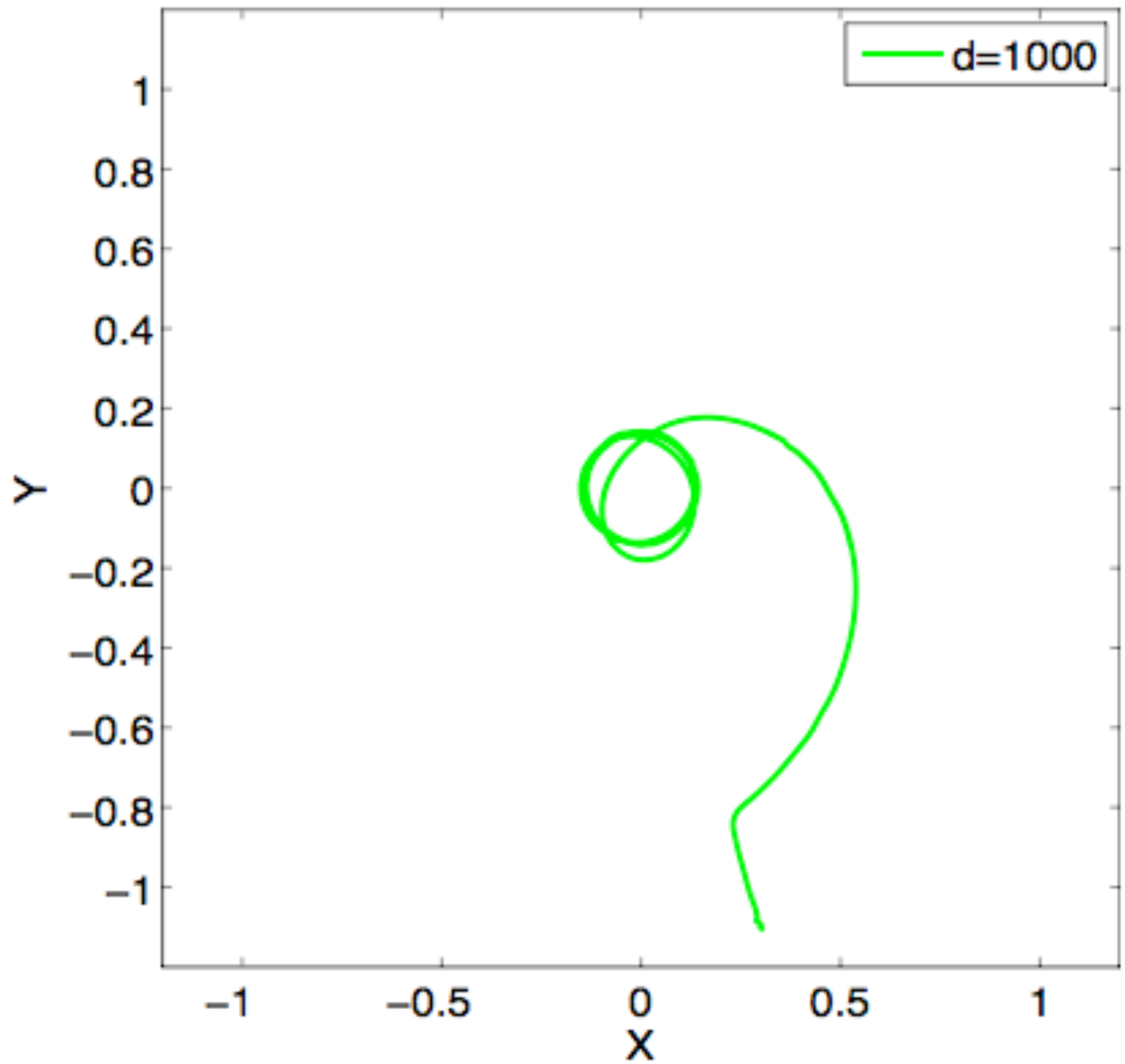
where $\rho = \sqrt{\dfrac{V_c J_0(a)}{2cRq_r J_1(a)}}$

$$O\left(\frac{1}{\omega}\right) + O\left(a^2\right)$$

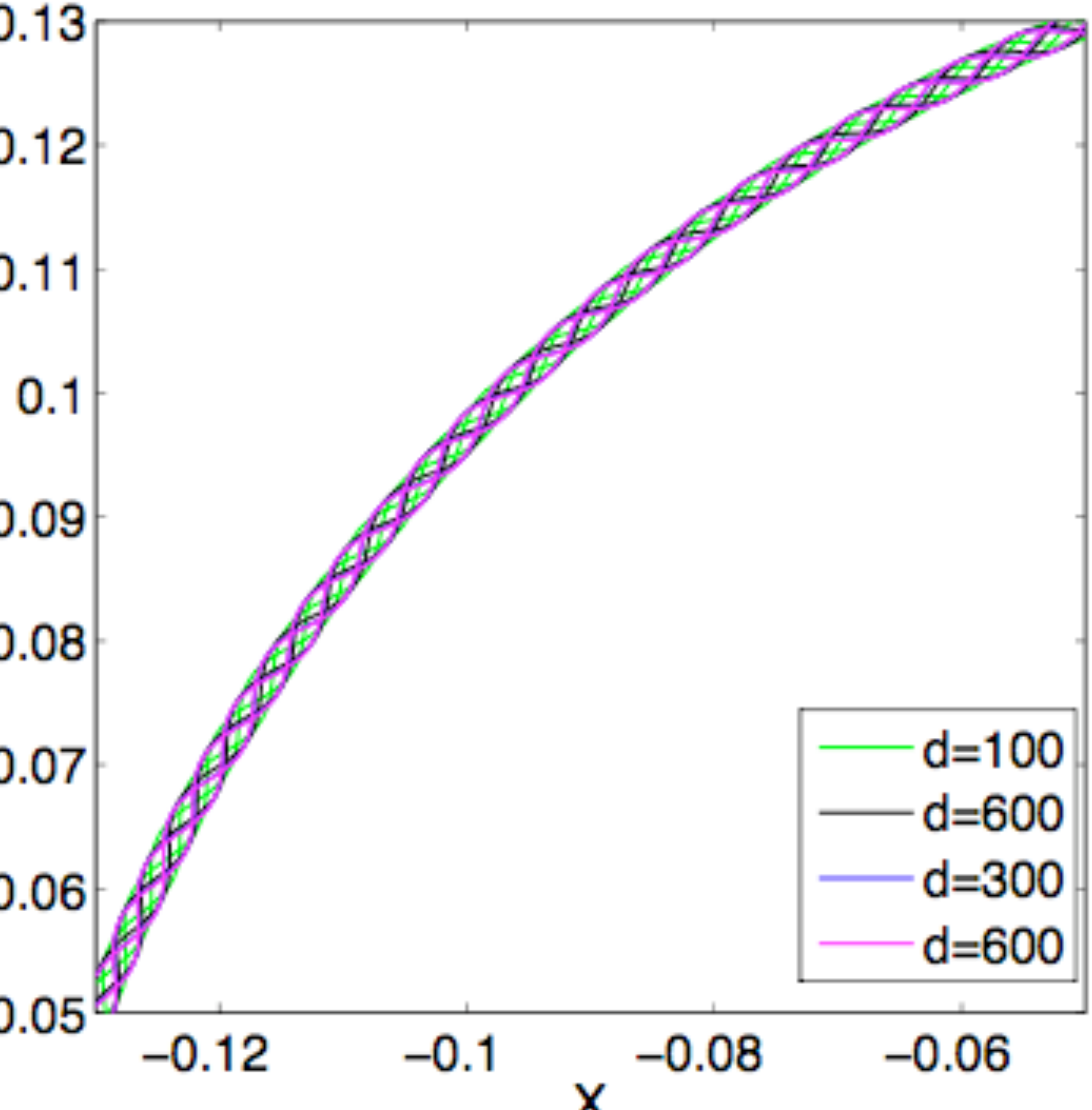
$J_0, J_1 = \text{Bessel functions}$



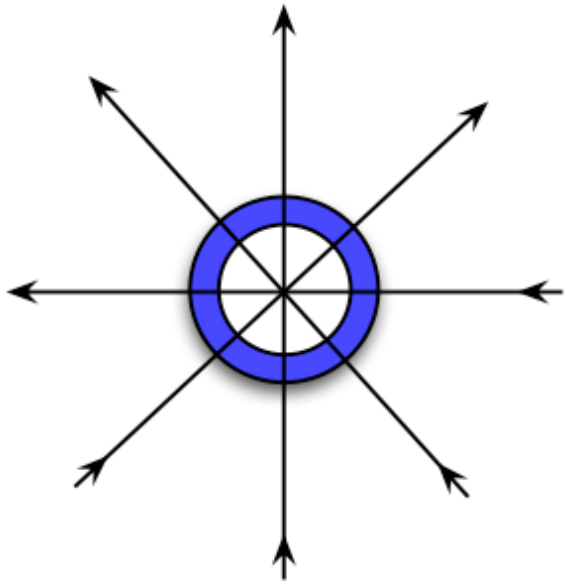
Residual "Hovering" Motion



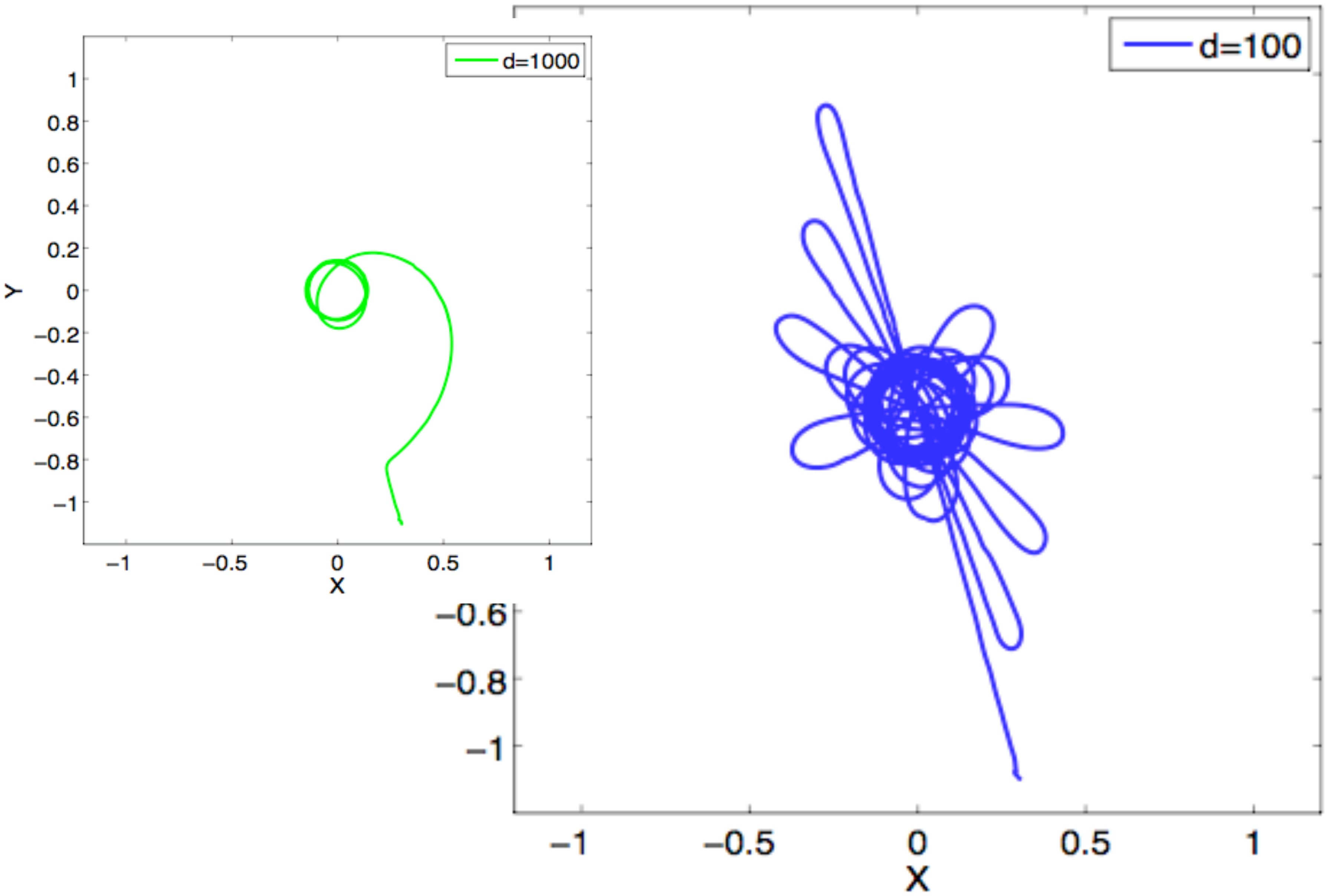
“Hovering” Motion



$$\rho = \sqrt{\frac{V_c J_0(a)}{2cRq_r J_1(a)}}$$

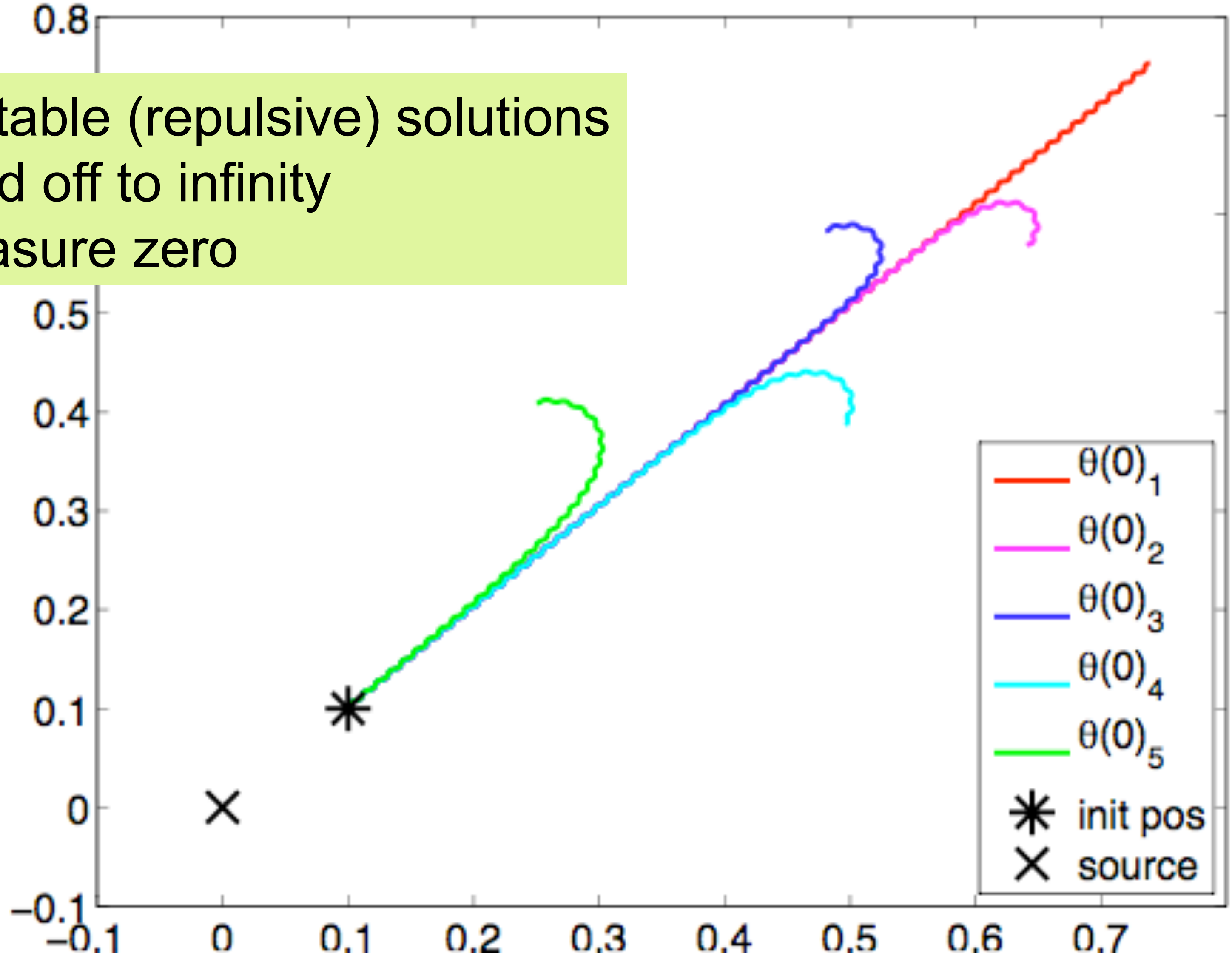


Effect of Small “Nonlinear Damping”

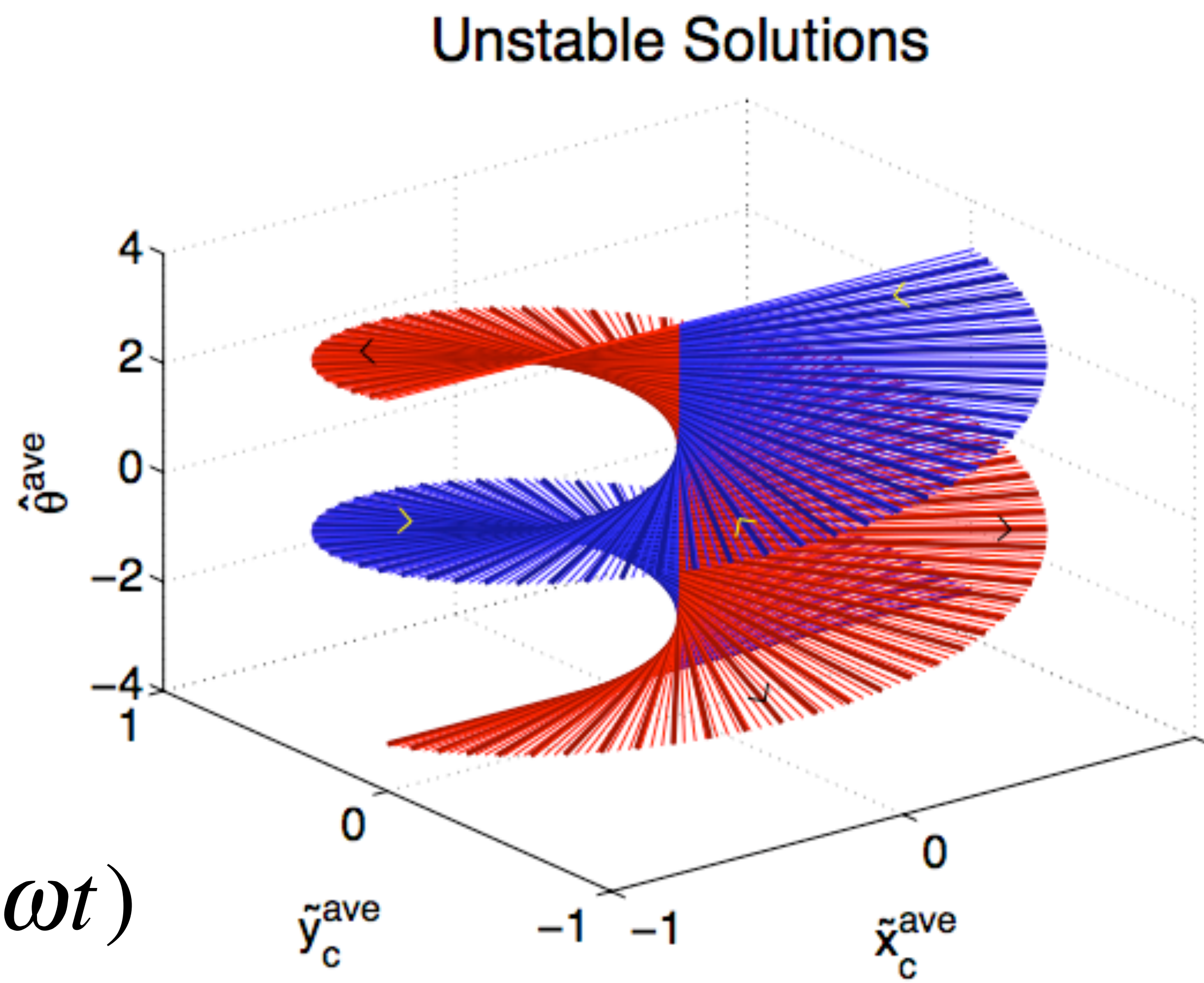


Unstable Trajectories

- unstable (repulsive) solutions
- head off to infinity
- measure zero



The Optimal Heading Manifold



$$\theta^* = \arg\left(r^* - r_c\right)$$

$$\tilde{\theta} = \theta - \theta^* - a\sin(\omega t)$$

$$\tilde{r}_c = \left|r^* - r_c\right|$$

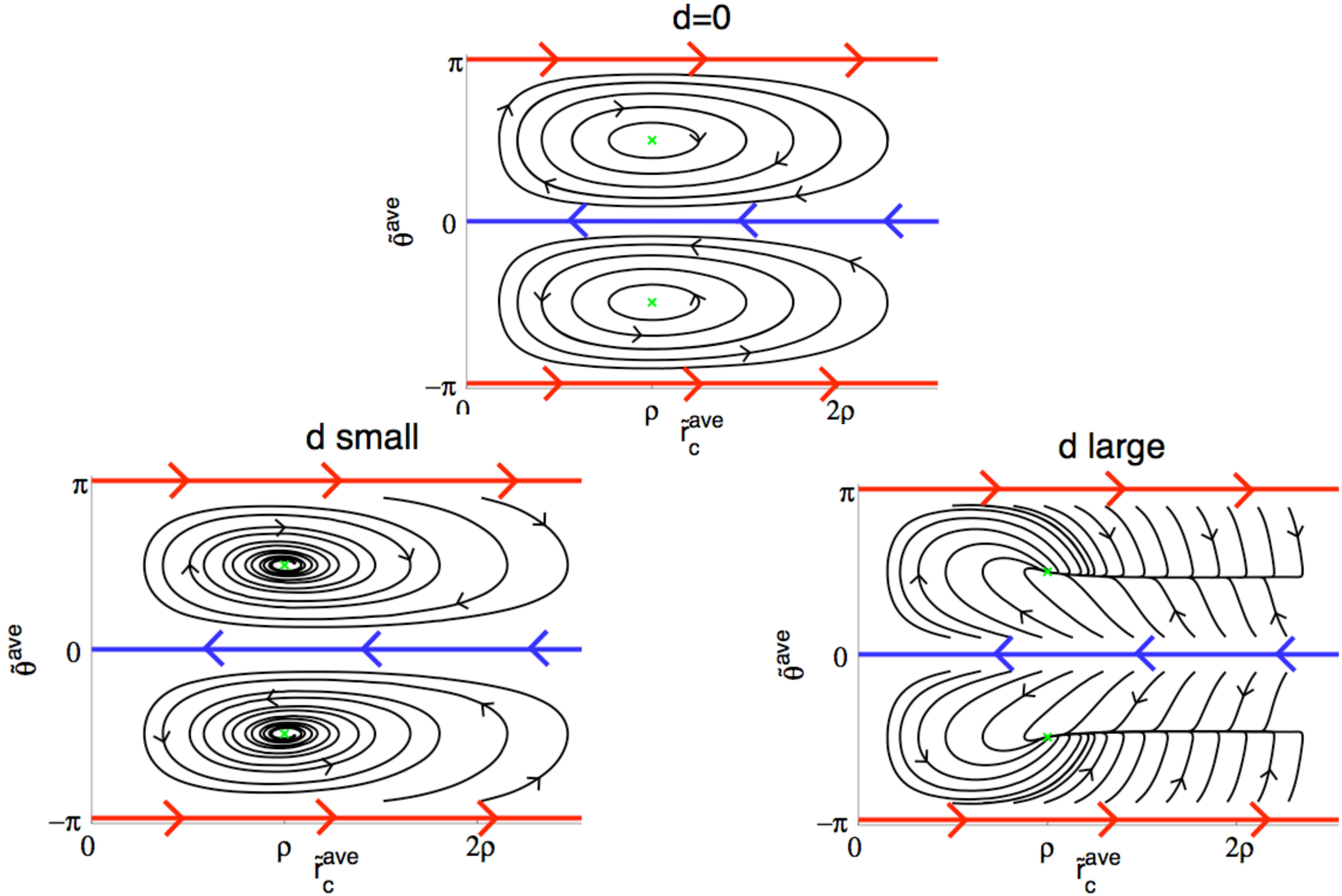
The "Average System"

$$\dot{\tilde{r}}_c^{\text{ave}} = -\frac{V_c J_0(a)}{\omega}\cos\tilde{\theta}^{\text{ave}}$$

$$\dot{\tilde{\theta}}^{\text{ave}} = \frac{1}{\omega}\sin\tilde{\theta}^{\text{ave}}\left\{\frac{V_c J_0(a)}{\tilde{r}_c^{\text{ave}}} - 2q_r R J_1(a)\tilde{r}_c^{\text{ave}}\left(c + 2dq_r\left(R^2 + \tilde{r}_c^{\text{ave}^2}\right)\right)\right\}$$

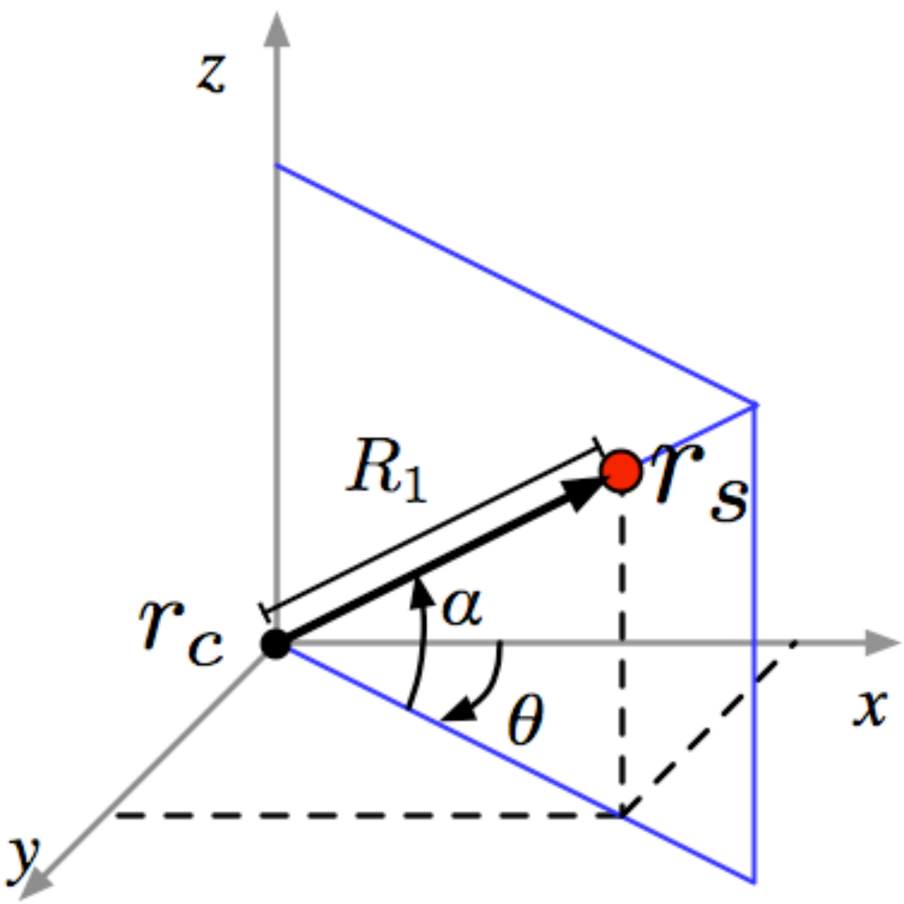
$$+\frac{1}{\omega}2dq_r^2 R^2 \tilde{r}_c^{\text{ave}^2} J_1(2a)\sin\left(2\tilde{\theta}^{\text{ave}}\right)$$

The "Average Dynamics"



Extensions to 3D

3D - UUV or UAV



$$\dot{y}_c = V_c \cos(\alpha) \sin(\theta)$$

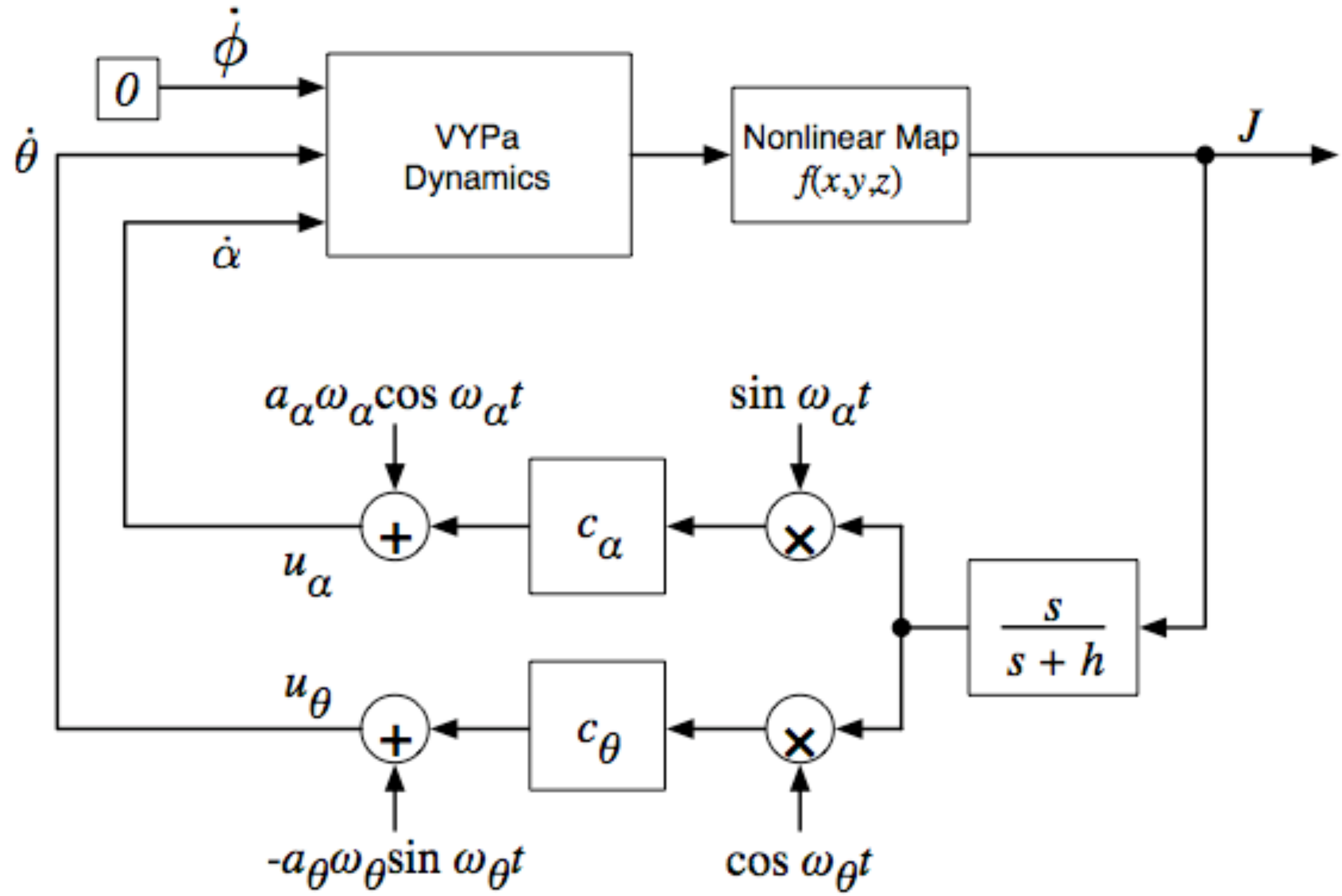
$$\dot{x}_c = V_c \cos(\alpha) \cos(\theta)$$

$$\dot{z}_c = V_c \sin(\alpha)$$

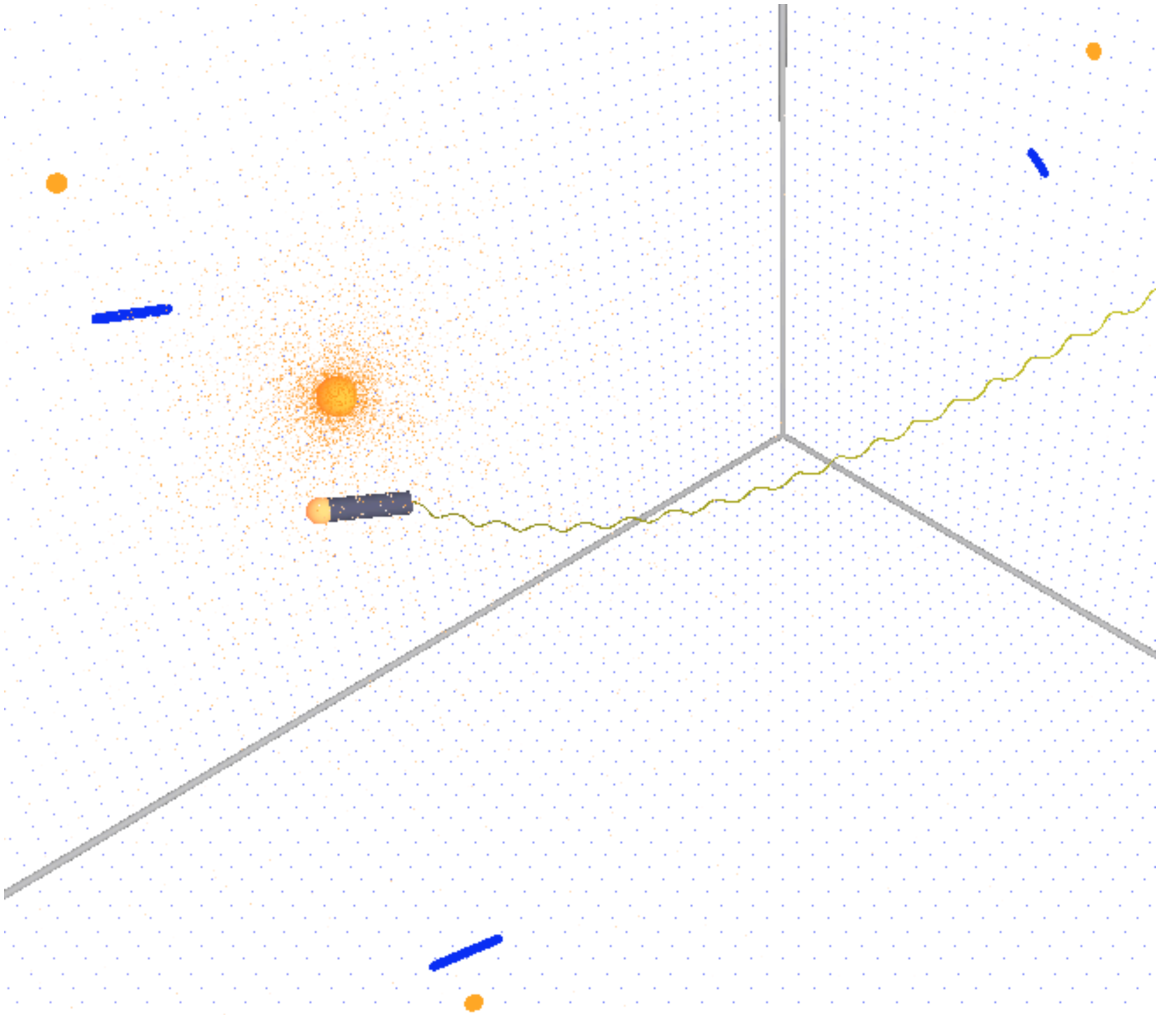
$$\dot{\theta} = \Omega_2$$

$$\dot{\alpha} = \Omega_1$$

Yaw and Pitch Actuated



Simulation Results



Stability w/ Stationary Source

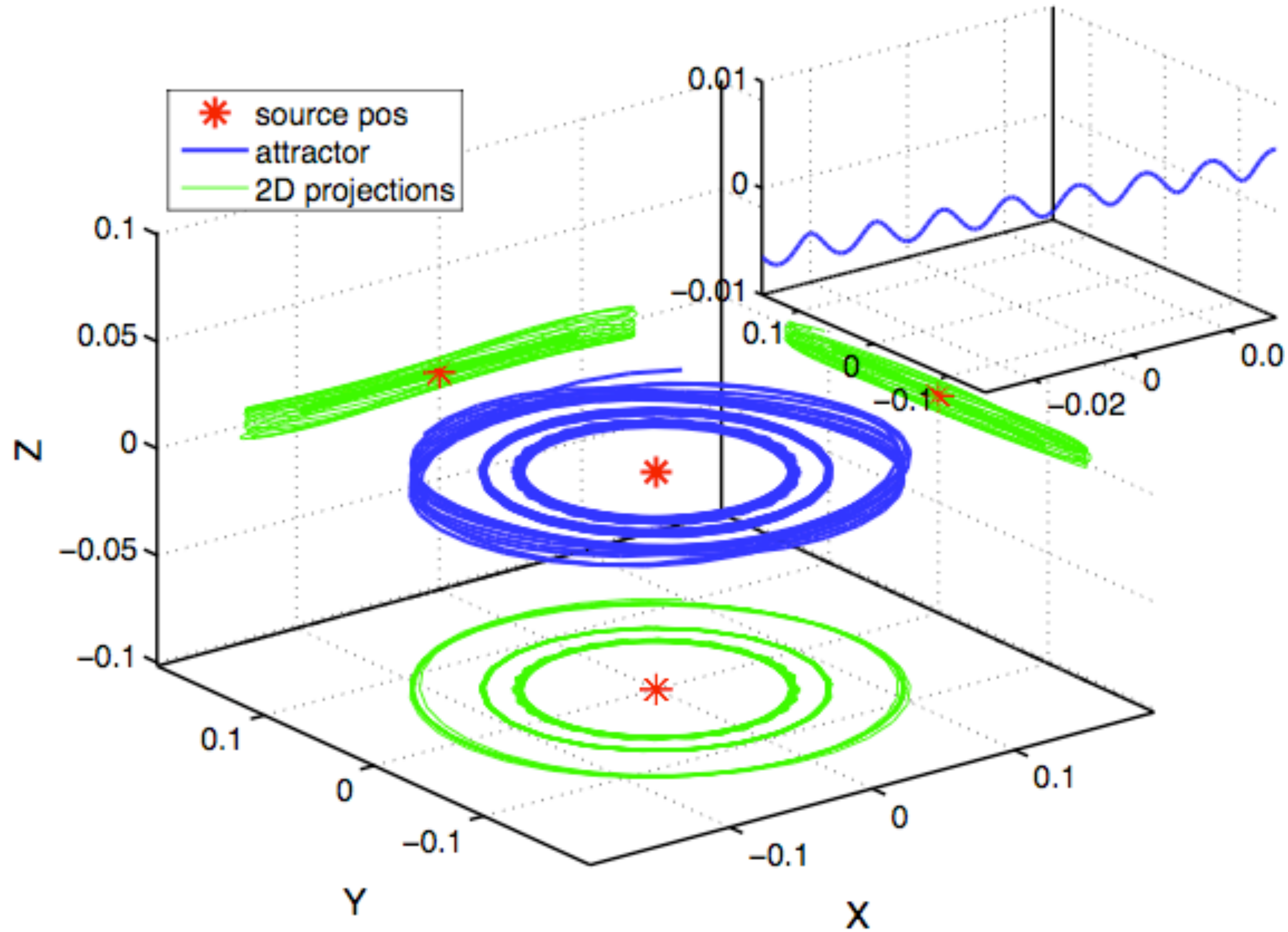
Theorem. *The vehicle center locally exponentially converges to one of the two (almost periodic) attractors of the form*

$$
\begin{aligned}
x_c^{\mathrm{attr}}(t) &= x^* - \left(\rho + \tilde{r}_{\pm}^{2\pi/\omega}(t)\right)\cos\left(\frac{V_c}{\rho}t + \alpha_{\pm}^{2\pi/\omega}(t) + \gamma\right)\cos\left(\beta_{\pm}^{2\pi/\omega}(t)\right) \\
y_c^{\mathrm{attr}}(t) &= y^* \pm \left(\rho + \tilde{r}_{\pm}^{2\pi/\omega}(t)\right)\sin\left(\frac{V_c}{\rho}t + \alpha_{\pm}^{2\pi/\omega}(t) + \gamma\right)\cos\left(\beta_{\pm}^{2\pi/\omega}(t)\right) \\
z_c^{\mathrm{attr}}(t) &= z^* \pm \left(\rho + \tilde{r}_{\pm}^{2\pi/\omega}(t)\right)\sin\left(\beta_{\pm}^{2\pi/\omega}(t)\right)
\end{aligned}
$$

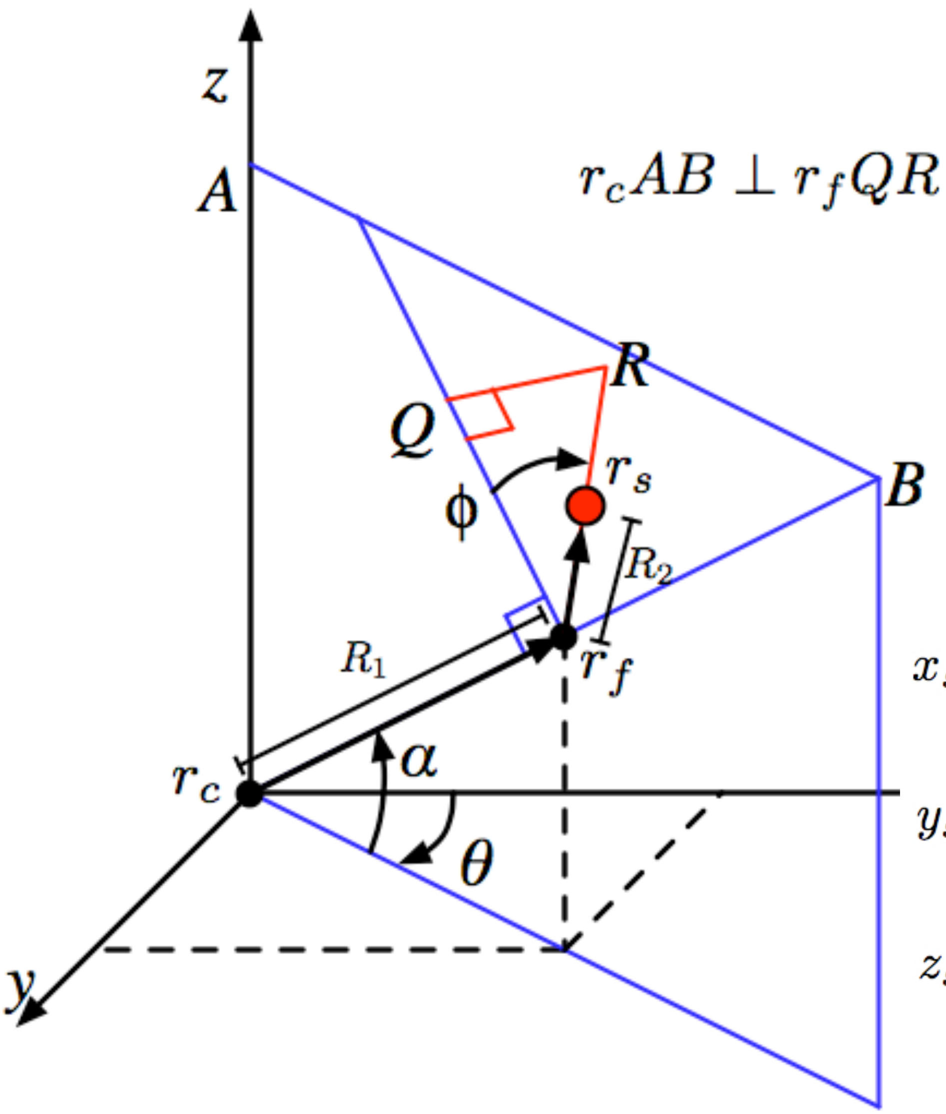
where $\rho = \sqrt{\dfrac{V_c J_0(\sqrt{2}a)}{\sqrt{2}c_\theta q_r R_1 J_1(\sqrt{2}a)}}$

$J_0, J_1 =$ Bessel functions

"Orbits" for different values of $c_{\mathbb{W}}$



Vehicle with Const Speed & Const Pitch Up Velocity, Sensor Off the Vehicle



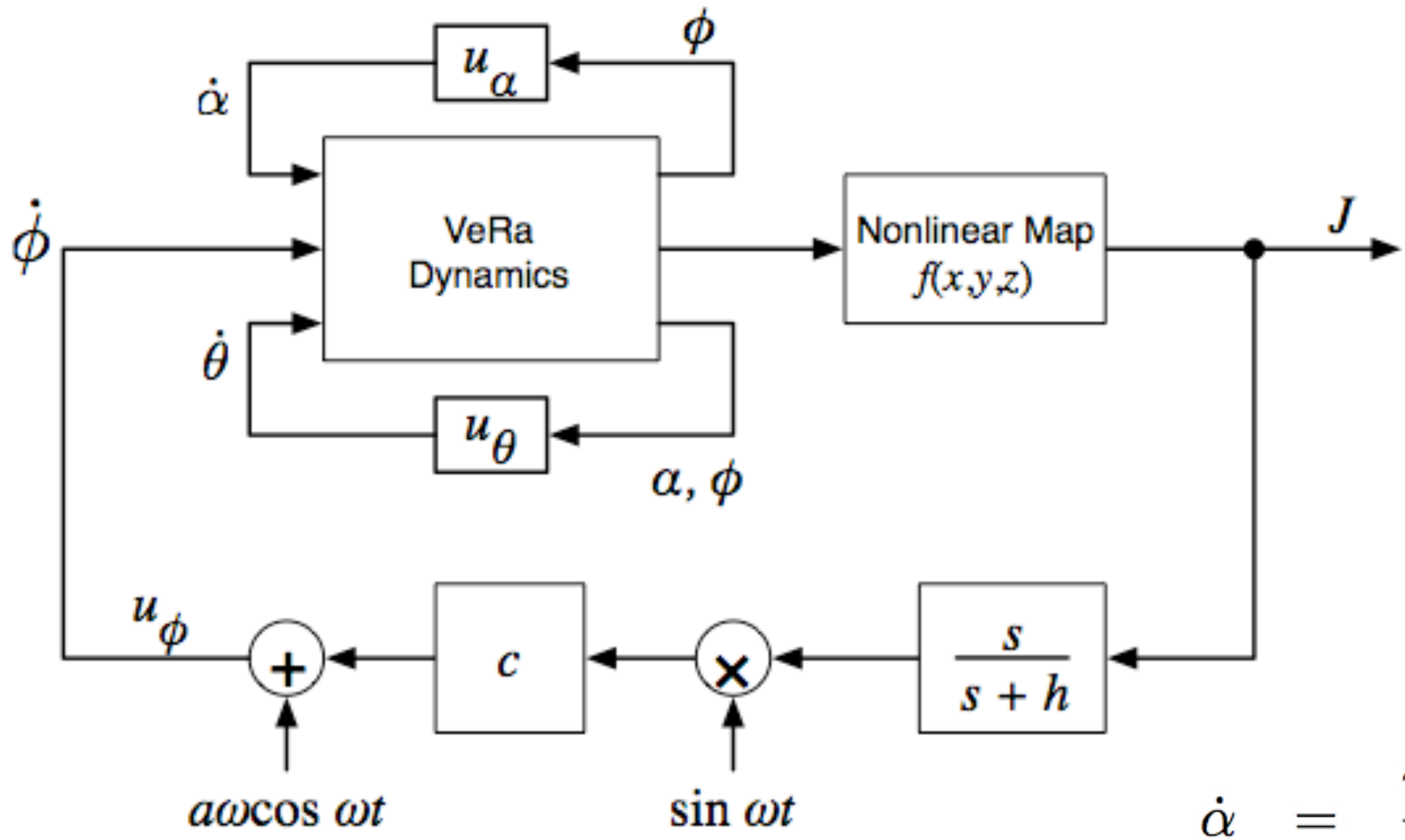
$$\dot{y}_c = V_c \cos(\alpha)\sin(\theta)$$

$$\dot{x}_c = V_c \cos(\alpha)\cos(\theta)$$

$$\dot{z}_c = V_c \sin(\alpha)$$

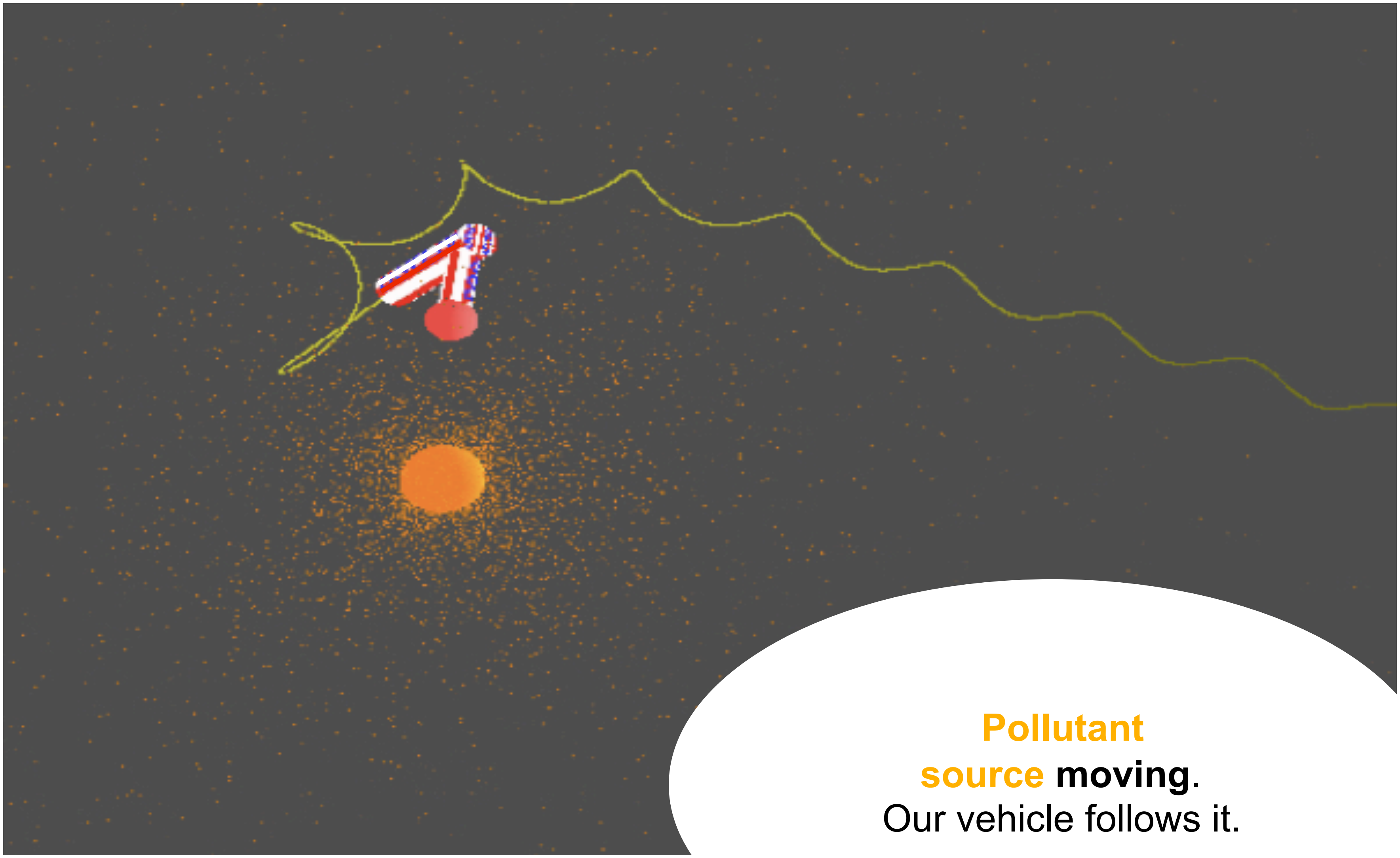
$$\begin{aligned}
x_s &= x_c + R_1 \cos\alpha\cos\theta \\
&\quad + R_2\left(-\cos\phi\sin\alpha\cos\theta + \sin\phi\sin\theta\right) \\
y_s &= y_c + R_1 \cos\alpha\sin\theta \\
&\quad + R_2\left(-\cos\phi\sin\alpha\sin\theta - \sin\phi\cos\theta\right) \\
z_s &= z_c + R_1\sin\alpha + R_2\cos\phi\cos\alpha\,,
\end{aligned}$$

Roll Actuated

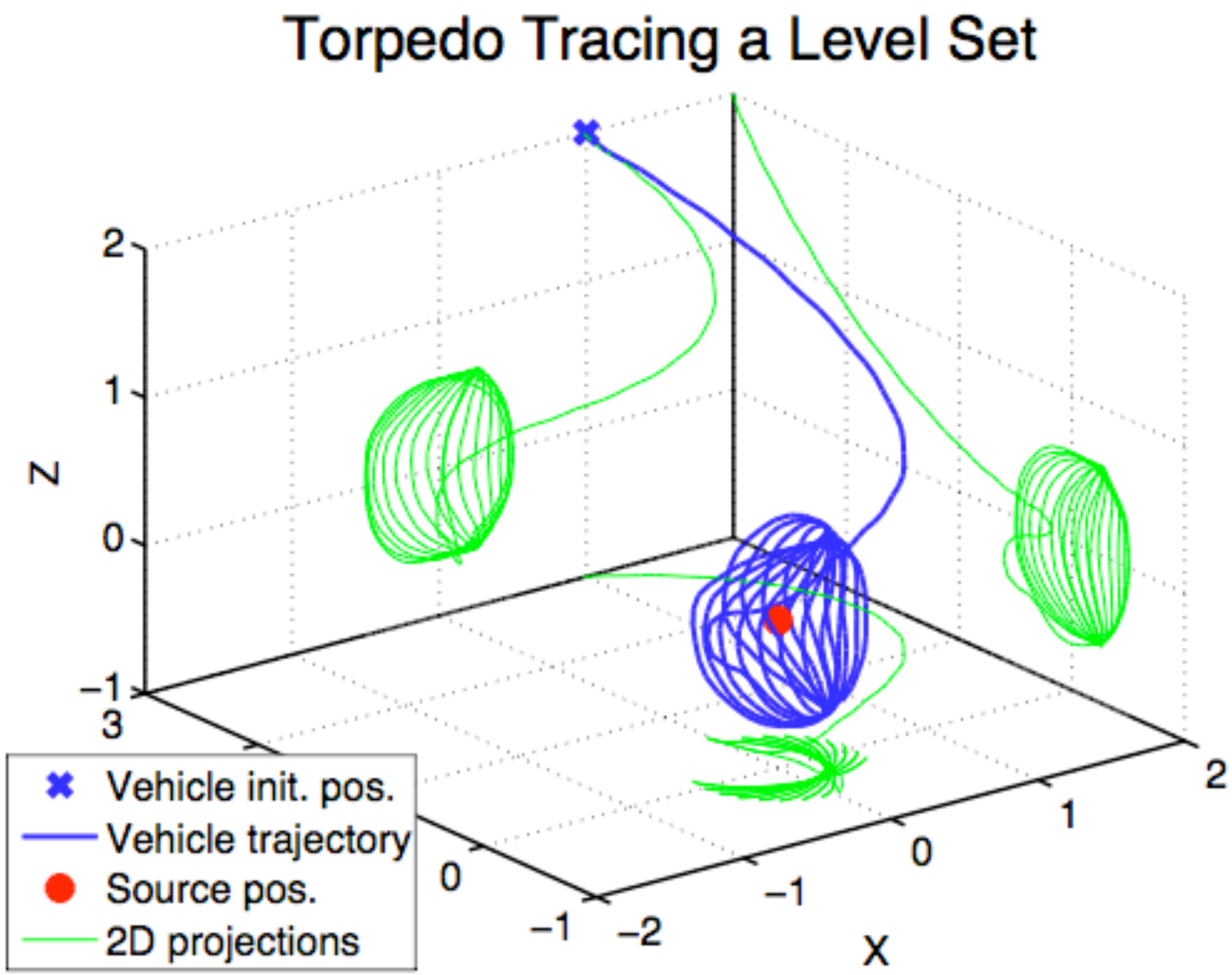


$$
\begin{aligned}
\dot\alpha &= \frac{v_2}{r_1}\cos\phi \\
\dot\theta &= -\frac{v_2}{r_1}\frac{\sin\phi}{\cos\alpha}
\end{aligned}
$$

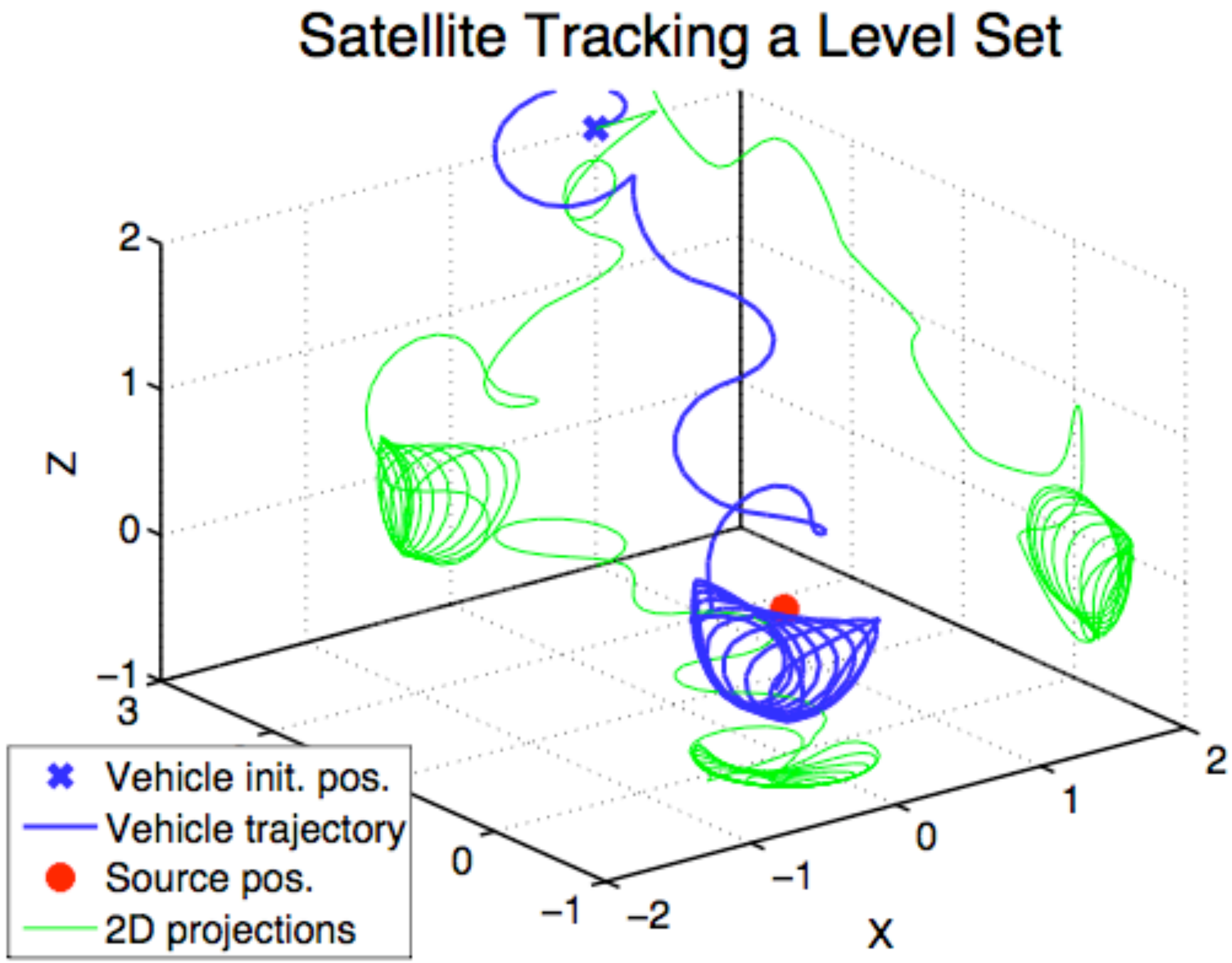
Simulation Results



3D Boundary Tracing: Yaw+Pitch Actuation

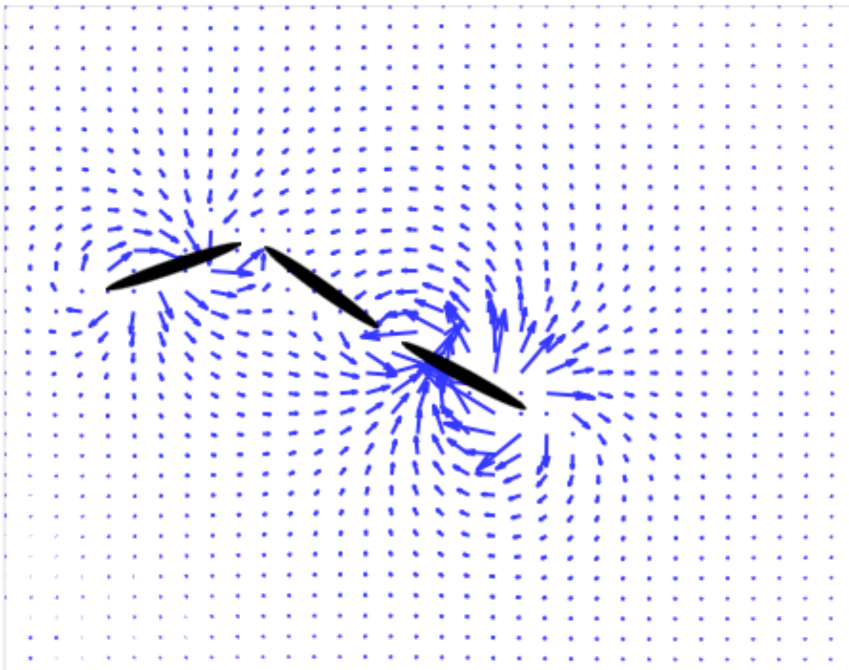


3D Boundary Tracing: Roll Actuation

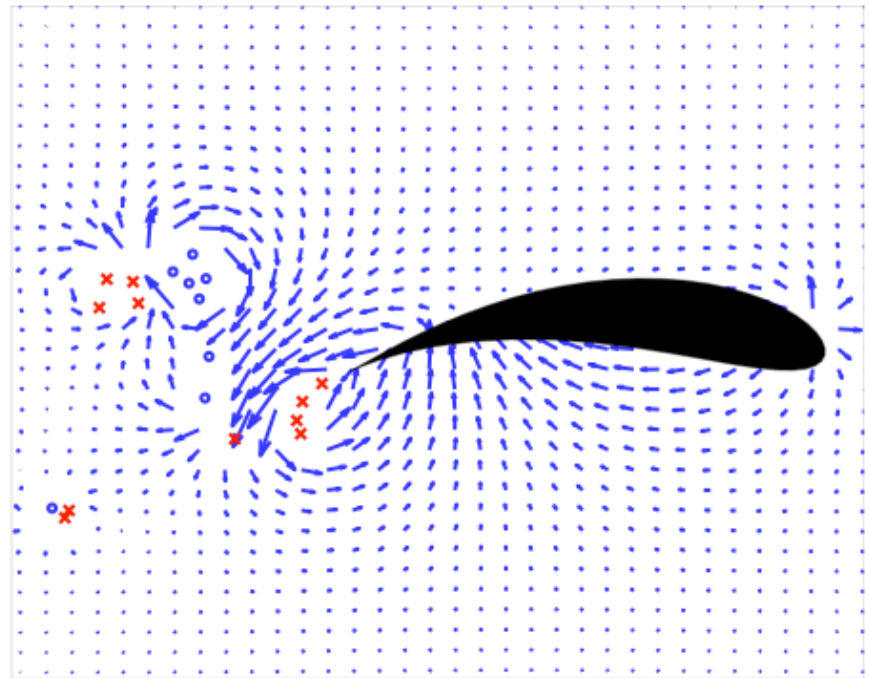


Source Seeking with Fish Locomotion Models

Fish Locomotion

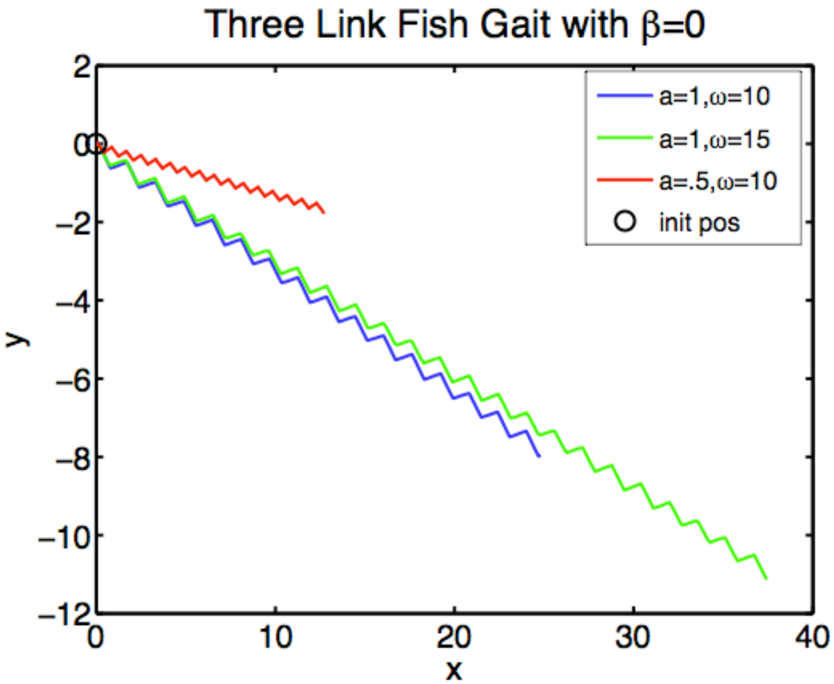


Eva Kanso - Jerry Marsden model

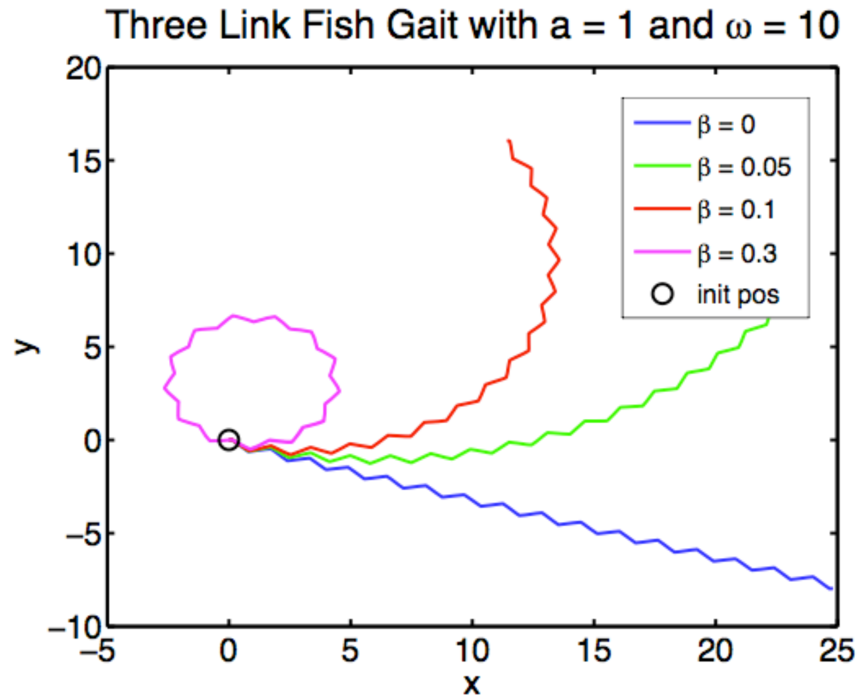


Scott Kelly model

Three Links - Two Inputs



move forward



turn in a circle

$$\begin{aligned}\theta_1 &= -a\cos(\omega t) \\ \theta_2 &= a\sin(\omega t)\end{aligned}$$

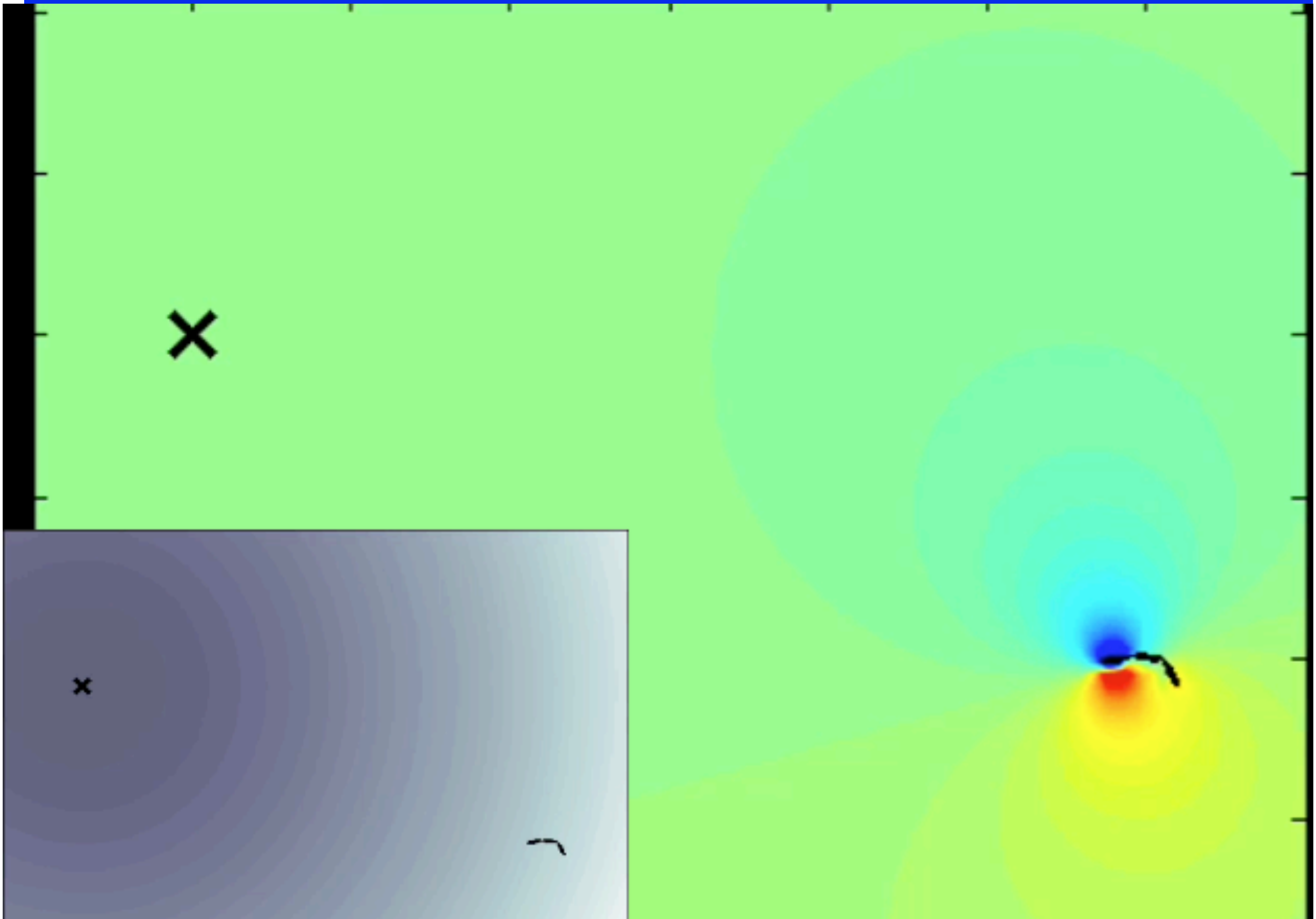
$$\begin{aligned}\theta_1 &= -a\cos(\omega t) + \beta \\ \theta_2 &= a\sin(\omega t) - \beta\end{aligned}$$

Tuning of the Fish "**Body Bending**"

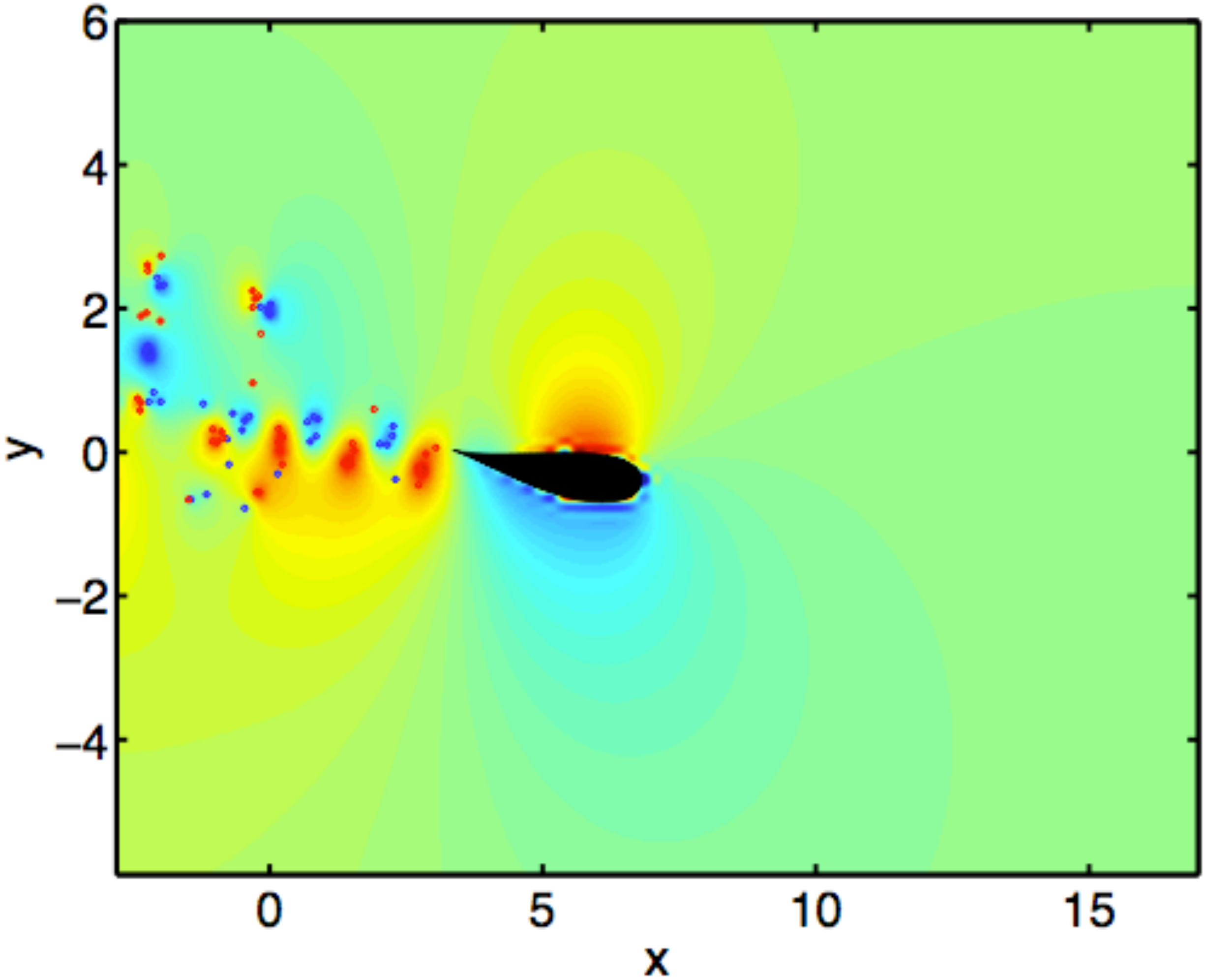
Control:

$$
\begin{aligned}
\dot{\beta} &= -c\xi\cos(\omega t) \\
\xi &= \frac{s}{s+h}[J]
\end{aligned}
$$

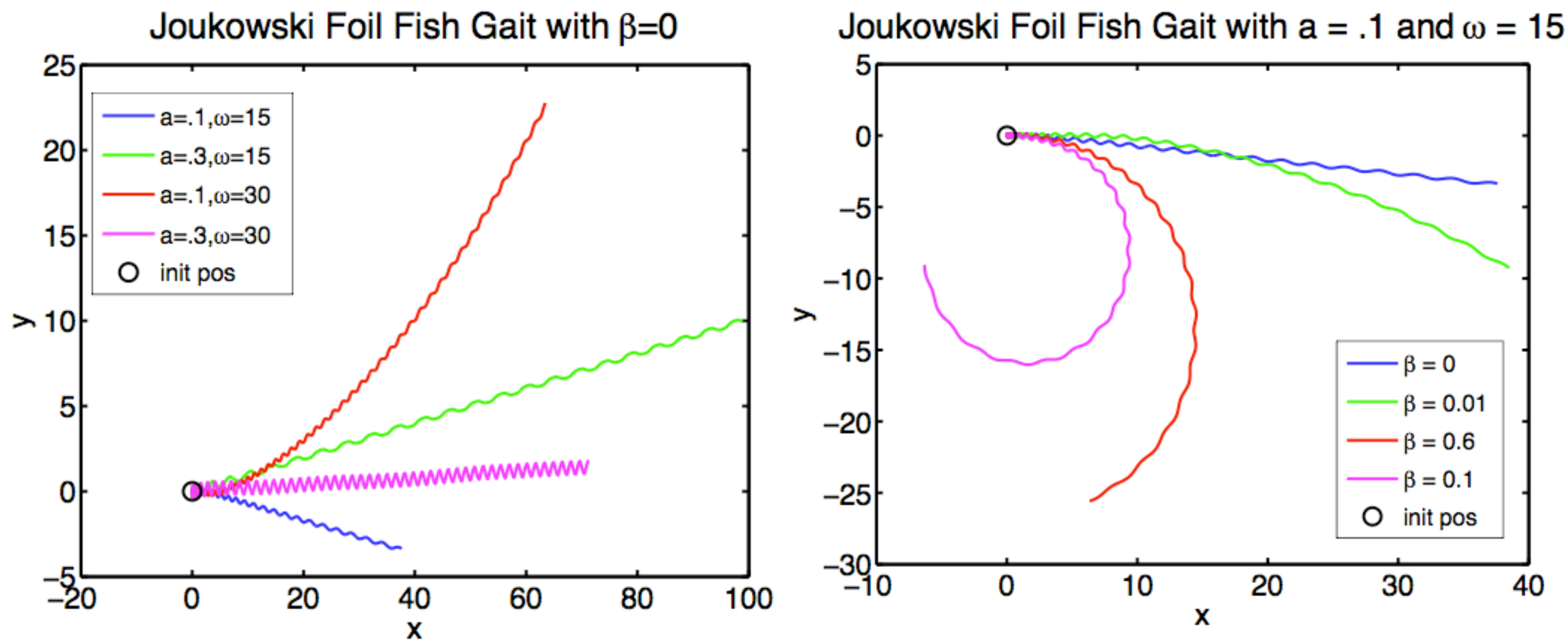
Simulations: Three Link Fish



Joukowsky Foil Fish w/ Vortex Model



One input, with vortex interaction



move forward **turn in a circle**

$$\zeta_y = a\sin(\omega t)$$

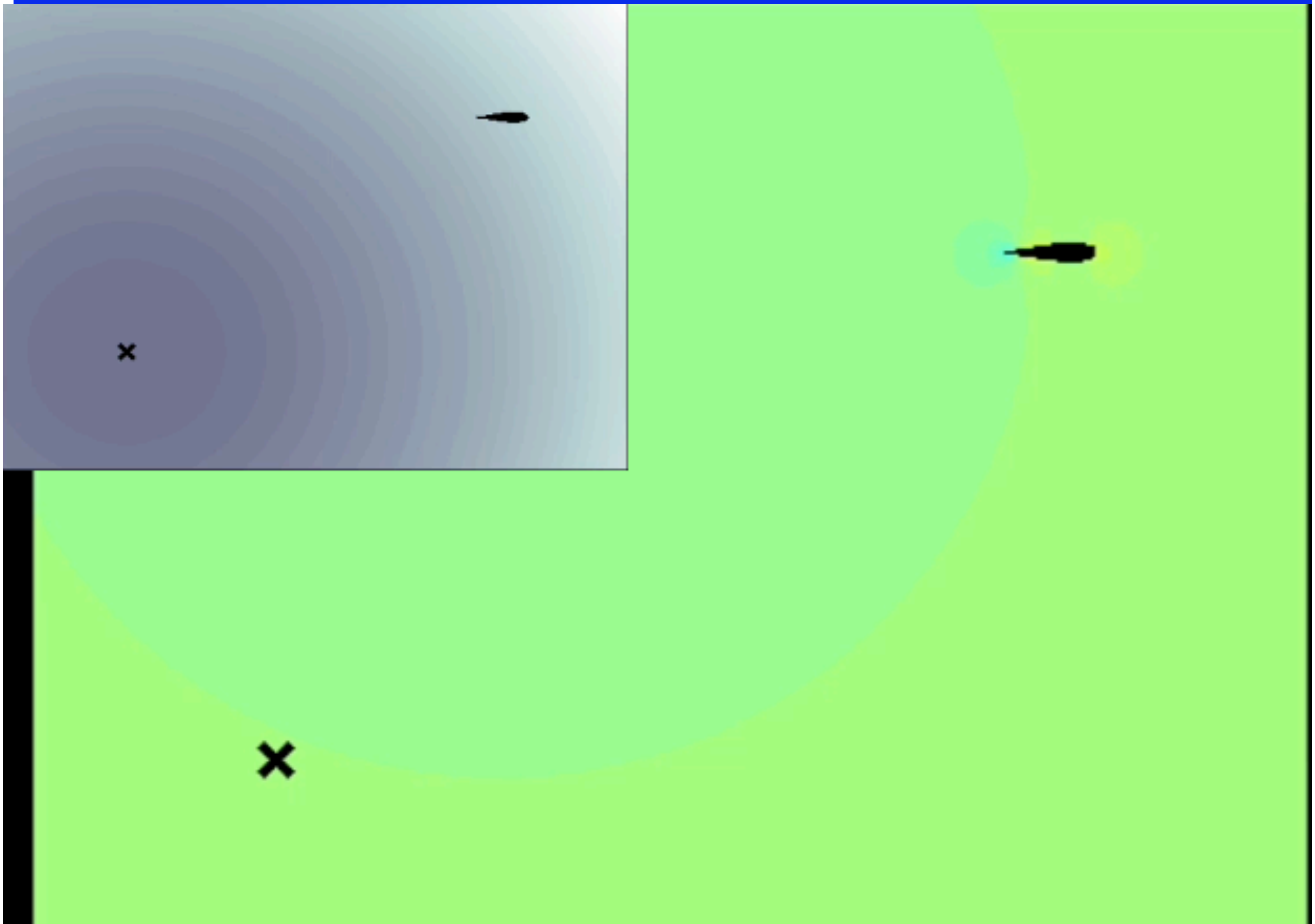
$$\zeta_y = a\sin(\omega t) + \beta$$

Tuning of the Fish "**Body Bending**"

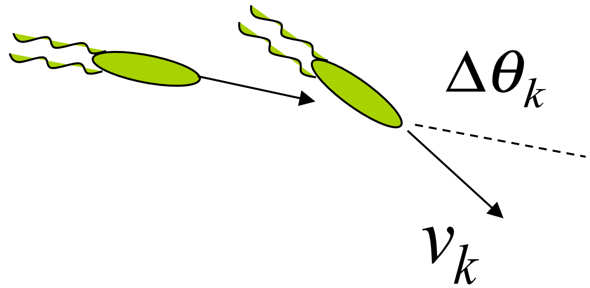
Control:

$$\begin{aligned} \dot{\beta} &= c\xi \sin(\omega t) \\ \xi &= \left(\frac{s}{s+h}\right)^2 [J] \end{aligned}$$

Simulations: Joukowski Foil Fish



Bacterial Locomotion/Chemotaxis



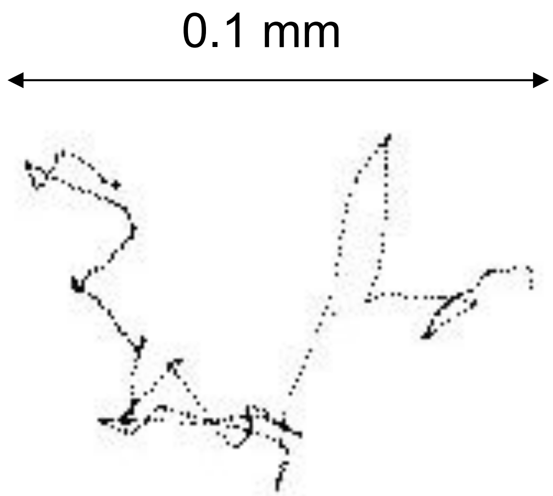
E. Coli motility has two phases, **run** and **tumble**

- During run phase all flagella spin counter clockwise and bacterium moves forward
- During tumble phase some flagella spin clockwise and bacterium changes orientation

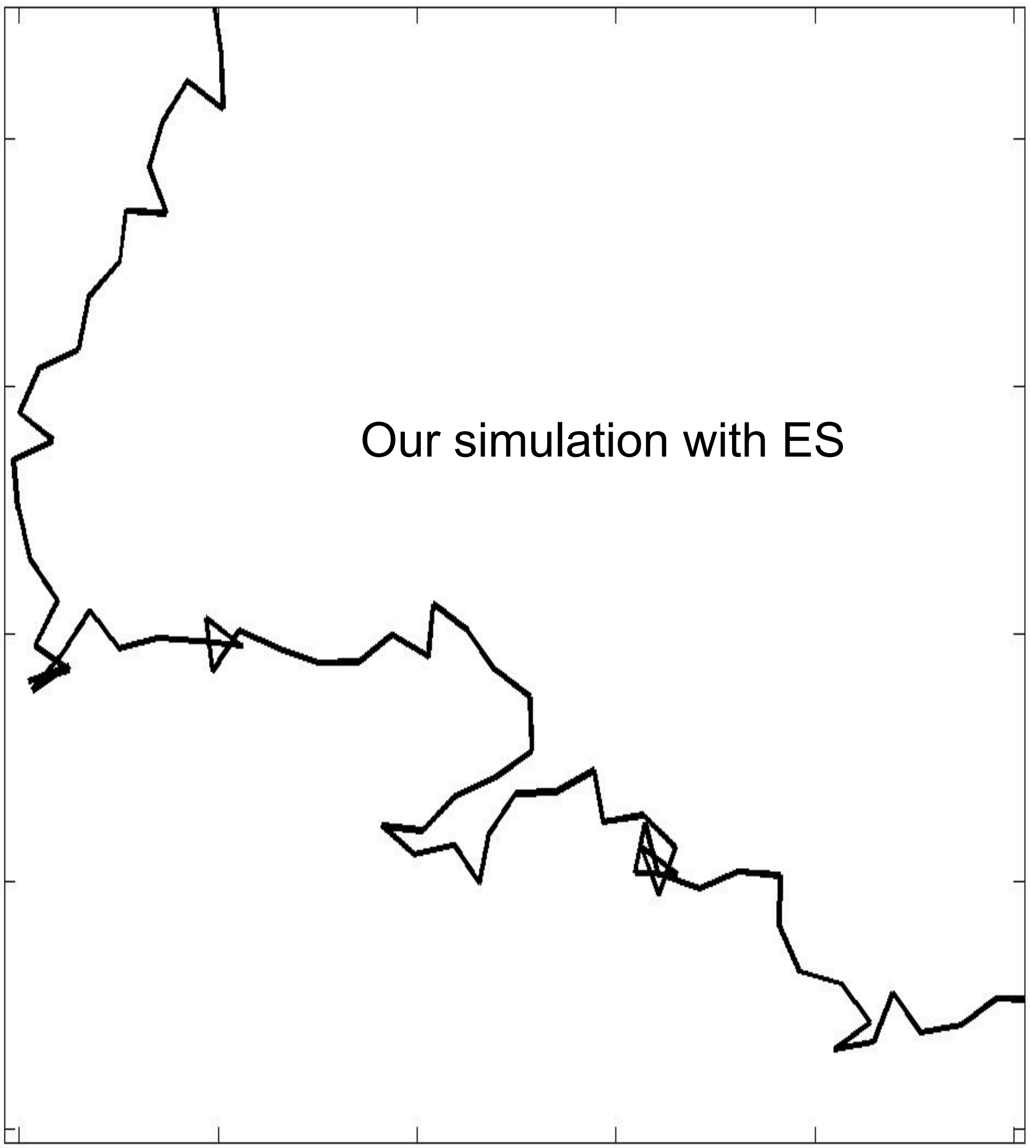
Emulate as *Stochastic Source Seeking*

$$\theta_{k+1} = \theta_k + w_k + \gamma w_{k-1} \frac{z-1}{z+h}\left[J\left(x\left(\theta_k\right), y\left(\theta_k\right)\right)\right]$$

E. Coli Perform Ext. Seeking



30 sec single bacterium experiment (Berg, Harvard, 2000)



Our simulation with ES

Summary

Summary

- **Extremum seeking**
  - a form of *optimization* - continuous in the input
  - a form of *adaptive* control - but not model based

- **Source seeking for nonholonomic vehices - in lieu of simultaneously solving**
  - motion planning/trajectory generation
  - state and parameter estimation
  - trajectory tracking
  - (subject to an optimality cost)

Thank You