Extremum Seeking Feedback Tools for Real-Time Optimization

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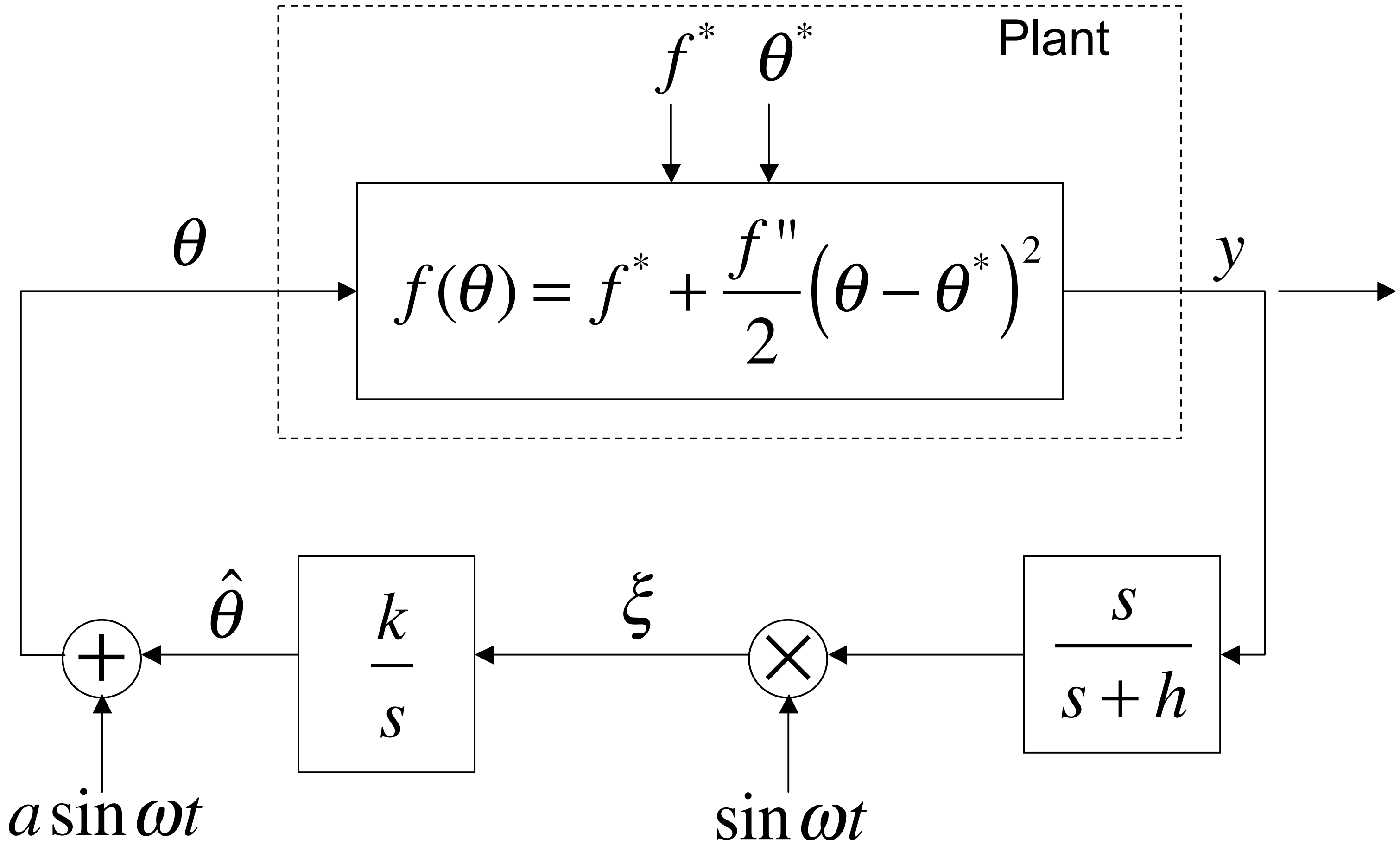
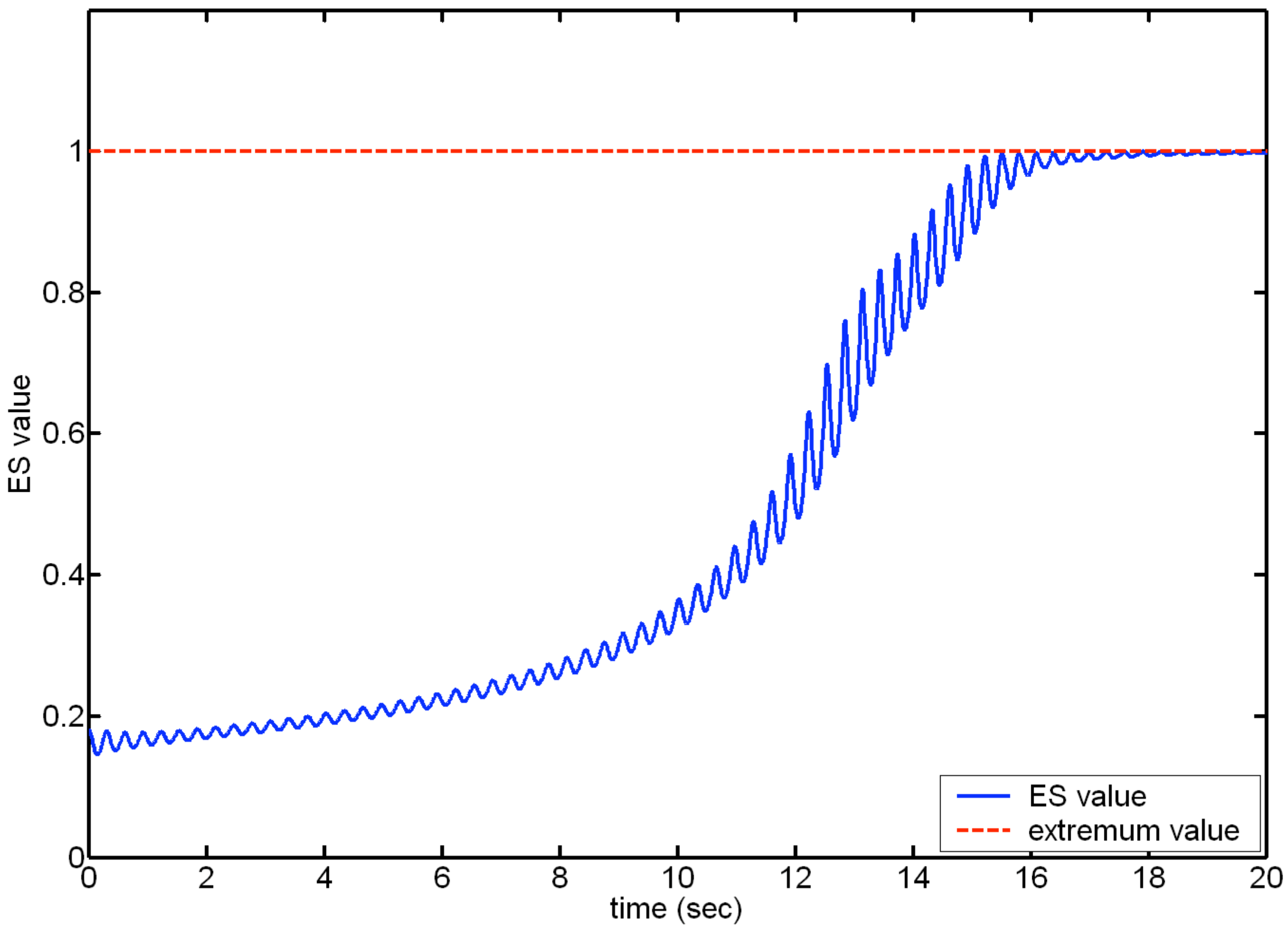
University of California, San Diego

Large Scale Robust Optimization, Sandia, August 2005

Example of Single-Parameter Maximum Seeking



Example of Single-Parameter Maximum Seeking



Theory

- History
- Single parameter ES, how it works, and stability analysis by averaging
- ES with plant dynamics and compensators for performance improvement
- Internal model principle for tracking parameter changes
- Multi-parameter ES
- Slope seeking
- ES in discrete time
- Limit cycle minimization via ES

Applications

- Anti-skid braking
- Compressor instabilities in jet engines
- Combustion instabilities
- Flow separation control in diffusers
- Thermoacoustic coolers
- Automotive engine mapping
- Beam matching in particle accelerators
- Formation flight
- Bioreactors
- PID tuning
- Autonomous vehicles without position sensing

History

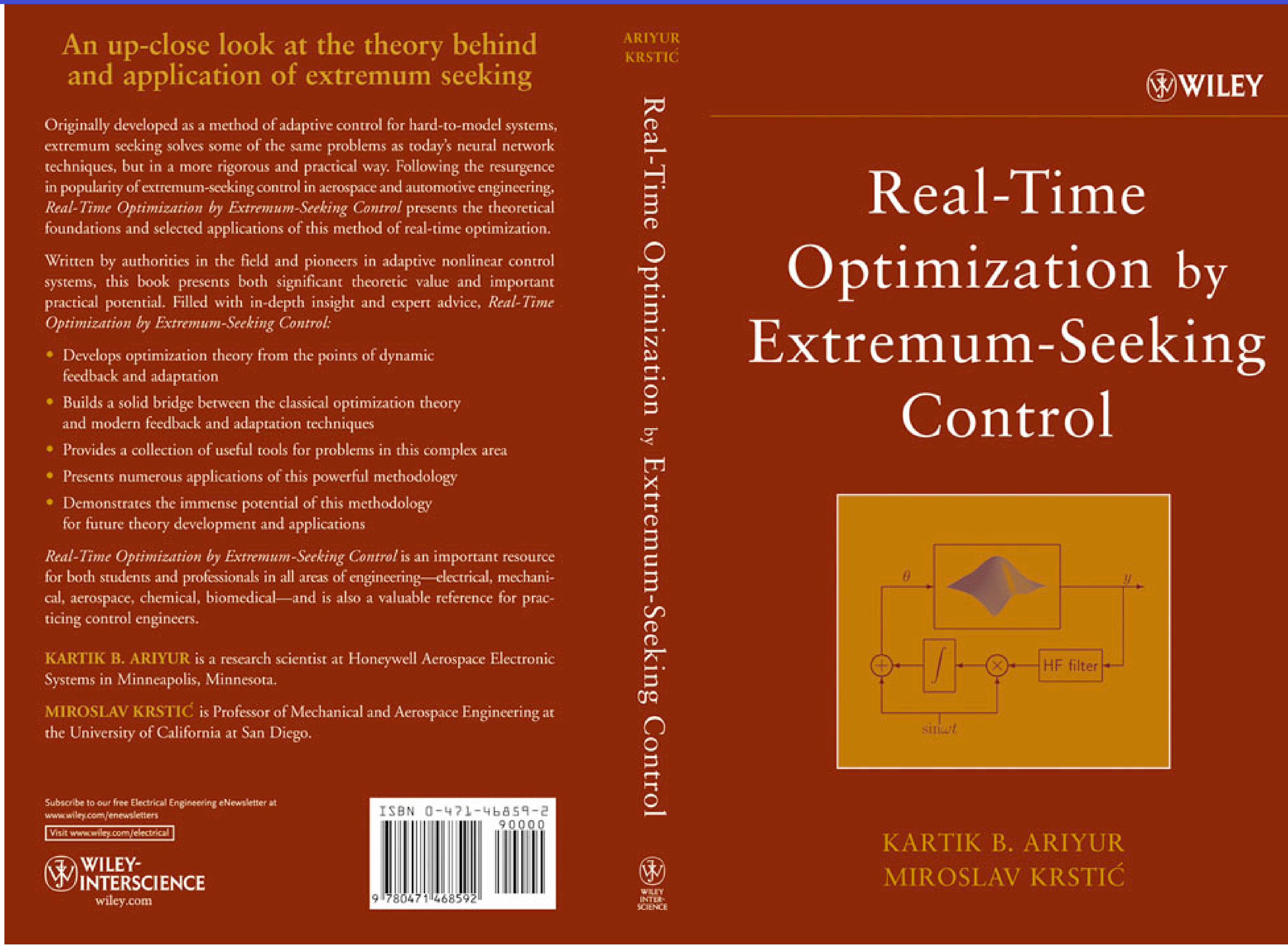
- **Leblanc (1922)** - electric railways
- **Early Russian literature (1940's)** - many papers
- **Drapper and Li (1951)** - application to IC engine spark timing tuning
- **Tsien (1954)** - a chapter in his book on Engineering Cybernetics
- **Feldbaum (1959)** - book *Computers in Automatic Control Systems*
- **Blackman (1962 chap. in book by Westcott)** - nice intuitive presentation of ES
- **Wilde (1964)** - a book
- **Chinaev (1969)** - a handbook on self-tuning systems
- Papers by[Morosanov], [Ostrovskii], [Pervozvanskii], [Kazakevich], [Frey, Deem, and Altpeter], [Jacobs and Shering], [Korovin and Utkin] - late 50s - early 70's
- **Meerkov (1967, 1968)** - papers with averaging analysis
- **Sternby (1980)** - survey
- **Astrom and Wittenmark (1995 book)** - rates ES as one of the most promising areas for adaptive control

Recent Developments

- **Krstic and Wang (2000, *Automatica*)** - stability proof for single-parameter general dynamic nonlinear plants
- **Choi, Ariyur, Wang, Krstic** - discrete-time, limit cycle minimization, IMC for parameter tracking, etc.
- **Rotea; Walsh; Ariyur** - multi-parameter ES
- **Ariyur** - slope seeking
- **Tan, Nesic, Mareels (2005)** - semi-global stability of ES
- Other approaches: **Guay, Dochain, Titica**, and coworkers; **Zak, Ozguner**, and coworkers; **Banavar, Chichka, Speyer**; **Popovic, Teel**; etc.

- **Applications** outside of my group:
  - Electromechanical valve actuator (Peterson and Stephanopoulou)
  - Artificial heart (Antaki and Paden)
  - Exercise machine (Zhang and Dawson)
  - Chemical reactors and petroleum processing (several research groups)
  - Shape optimization for magnetic head in hard disk drives (UCSD)

ES Book


An up-close look at the theory behind and application of extremum seeking
Originally developed as a method of adaptive control for hard-to-model systems, extremum seeking solves some of the same problems as today's neural network techniques, but in a more rigorous and practical way. Following the resurgence in popularity of extremum-seeking control in aerospace and automotive engineering, *Real-Time Optimization by Extremum-Seeking Control* presents the theoretical foundations and selected applications of this method of real-time optimization.
Written by authorities in the field and pioneers in adaptive nonlinear control systems, this book presents both significant theoretic value and important practical potential. Filled with in-depth insight and expert advice, *Real-Time Optimization by Extremum-Seeking Control:*
- Develops optimization theory from the points of dynamic feedback and adaptation
- Builds a solid bridge between the classical optimization theory and modern feedback and adaptation techniques
- Provides a collection of useful tools for problems in this complex area
- Presents numerous applications of this powerful methodology
- Demonstrates the immense potential of this methodology for future theory development and applications
Real-Time Optimization by Extremum-Seeking Control is an important resource for both students and professionals in all areas of engineering—electrical, mechanical, aerospace, chemical, biomedical—and is also a valuable reference for practicing control engineers.
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MIROSLAV KRSTIĆ is Professor of Mechanical and Aerospace Engineering at the University of California at San Diego.
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HF filter
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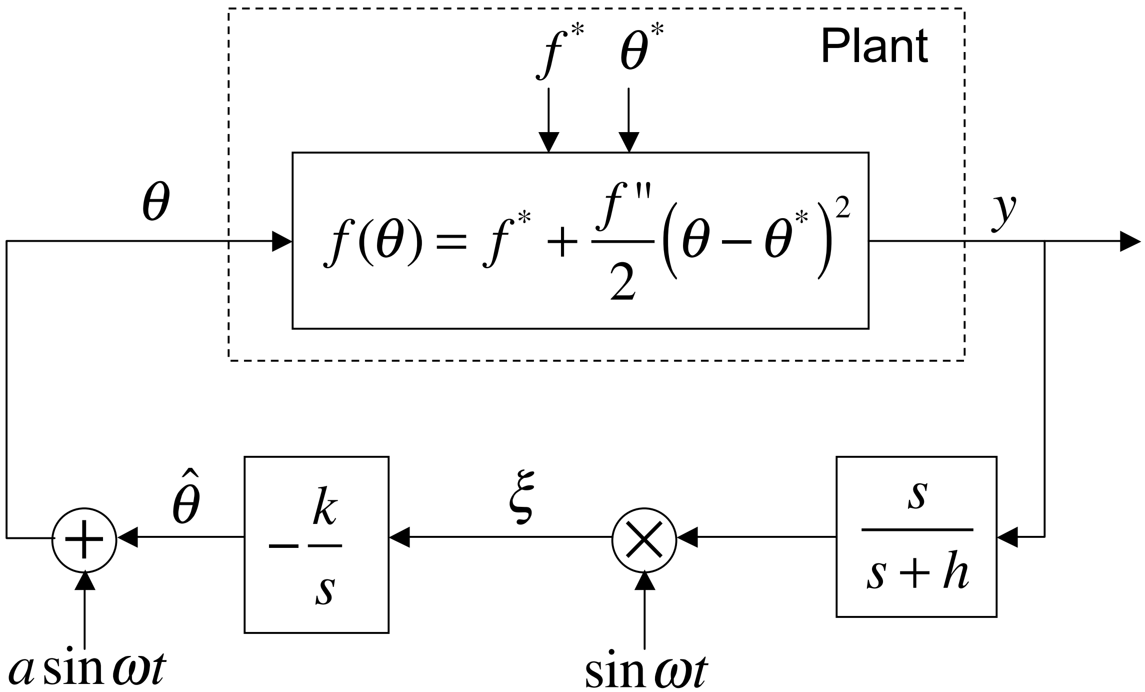
KARTIK B. ARIYUR
MIROSLAV KRSTIĆ

Basic Extremum Seeking - Static Map



$y =$ output to be minimized

$f^* =$ minimum of the map

$f'' =$ second derivative (positive - $f(\theta)$ has a min.)

$\theta^* =$ unknown parameter

$\hat{\theta} =$ estimate of θ^*

$k =$ adaptation gain (positive) of the integrator $\frac{1}{s}$

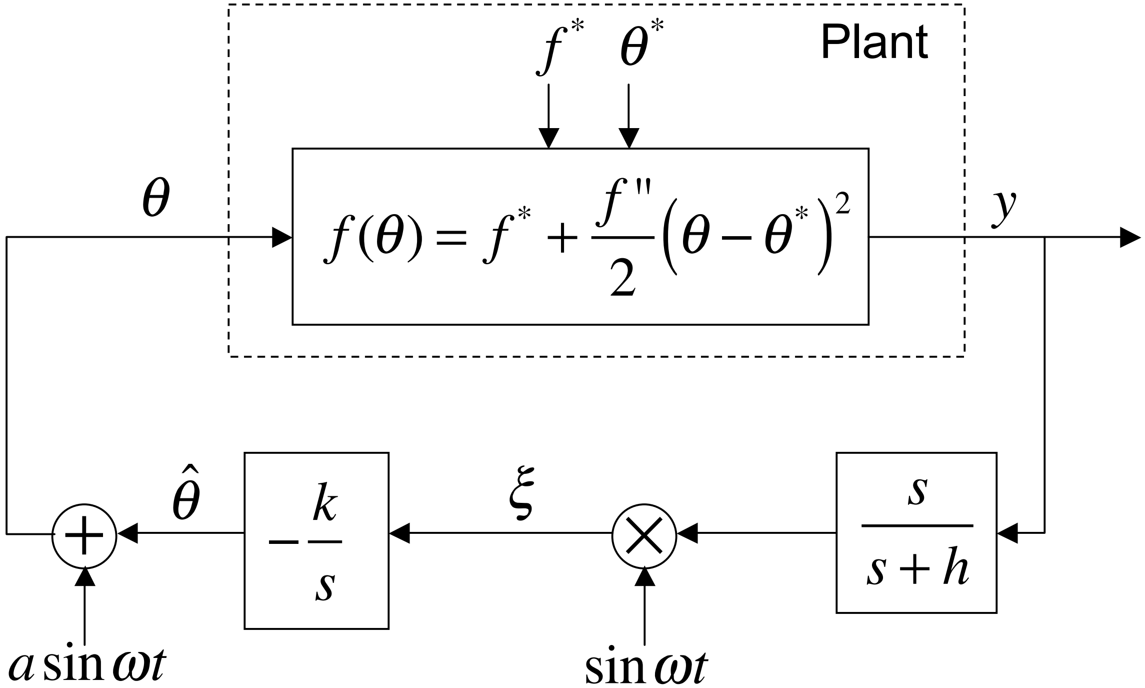
$a =$ amplitude of the probing signal

$\omega =$ frequency of the probing signal

$h =$ cut-off frequency of the "washout filter" $\frac{s}{s+h}$

$+/\times =$ modulation/demodulation

How Does It Work?



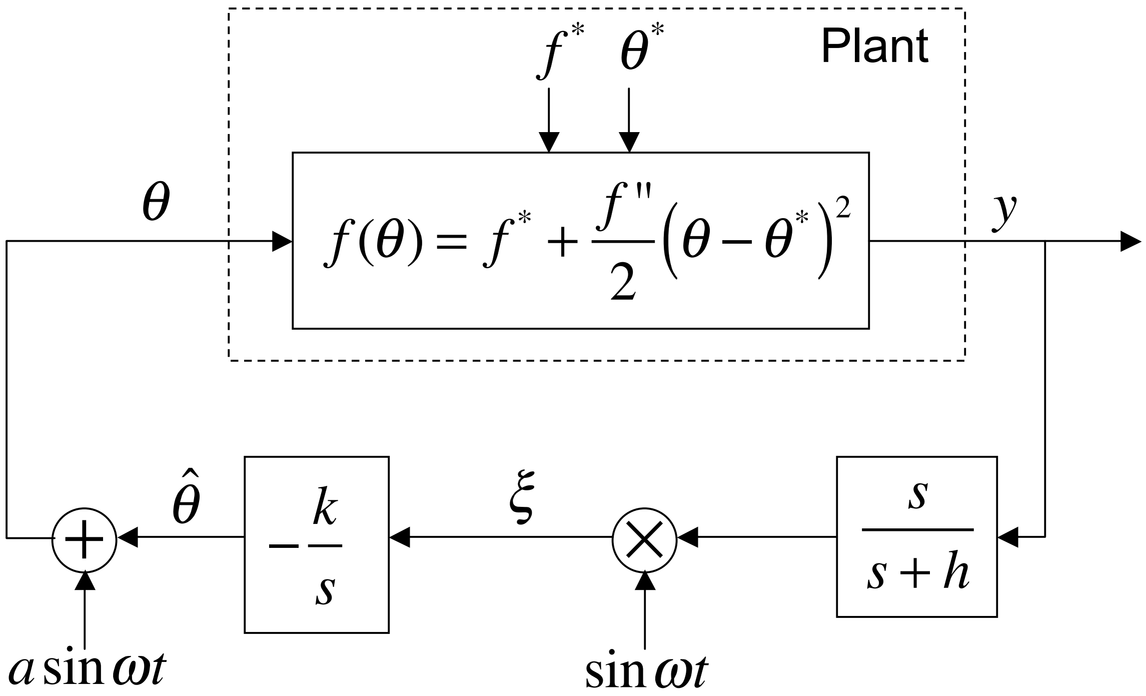
Estimation error: $\tilde{\theta} = \theta^* - \hat{\theta}$

$$y = f^* + \frac{a^2 f''}{4} + \frac{f''}{2}\tilde{\theta}^2 - af''\tilde{\theta}\sin\omega t + \frac{a^2 f''}{4}\cos 2\omega t$$

Loc. Analysis - neglect quadratic terms:

$$y \approx f^* + \frac{a^2 f''}{4} \qquad - af''\tilde{\theta}\sin\omega t + \frac{a^2 f''}{4}\cos 2\omega t$$

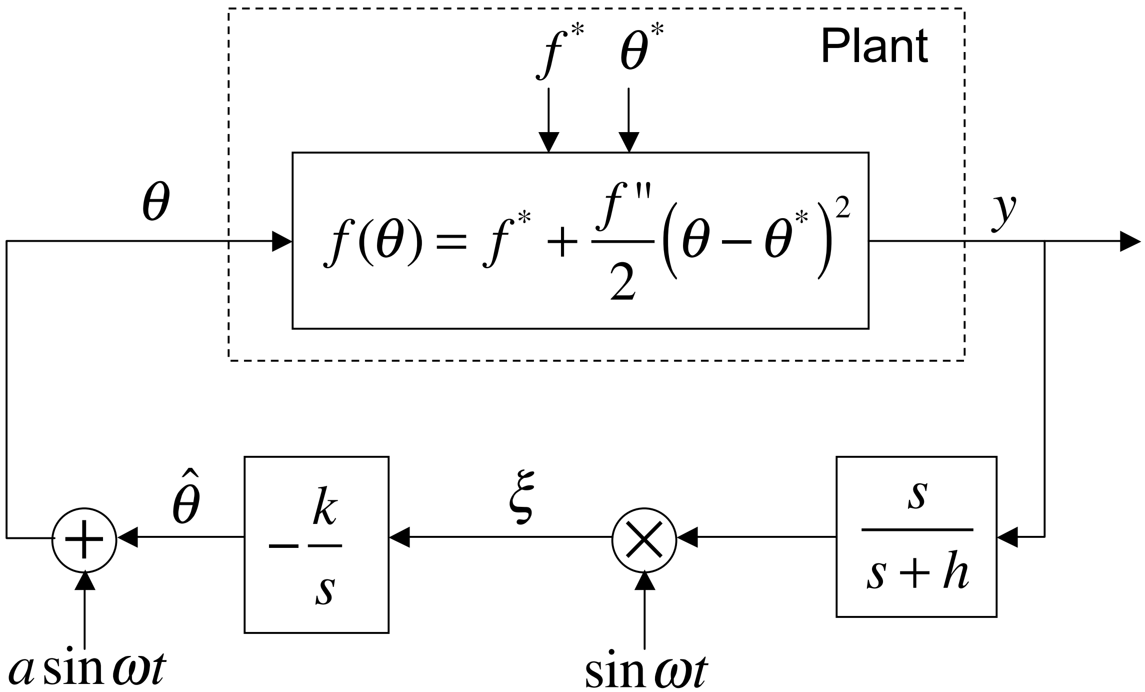
How Does It Work?



$$y \approx f^* + \frac{a^2 f''}{4} - af''\tilde{\theta} \sin \omega t + \frac{a^2 f''}{4} \cos 2\omega t$$

$$\frac{s}{s+h}[y] \approx \qquad - af''\tilde{\theta} \sin \omega t + \frac{a^2 f''}{4} \cos 2\omega t$$

How Does It Work?

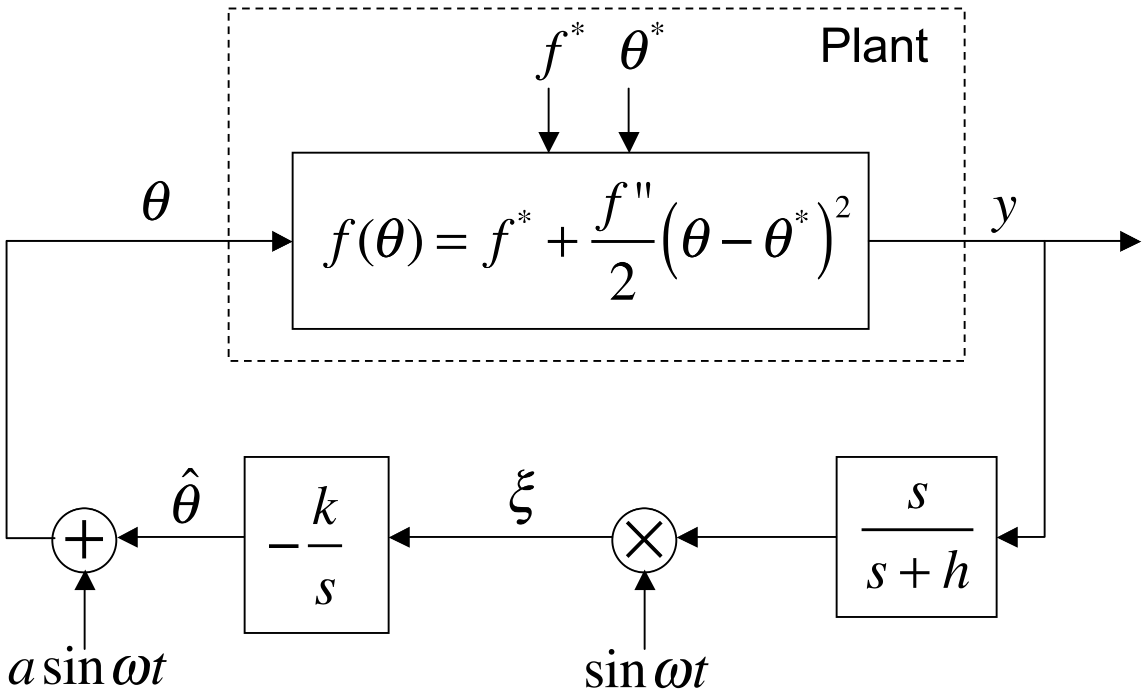


Demodulation:

$$\xi = \sin\omega t \frac{s}{s+h}[y] \approx -af''\tilde{\theta}\sin^2\omega t + \frac{a^2 f''}{4}\cos 2\omega t \sin\omega t$$

$$\xi \approx -\frac{a^2 f''}{4}\tilde{\theta} + \frac{a^2 f''}{4}\tilde{\theta}\cos 2\omega t + \frac{a^2 f''}{8}\left(\sin\omega t - \sin 3\omega t\right)$$

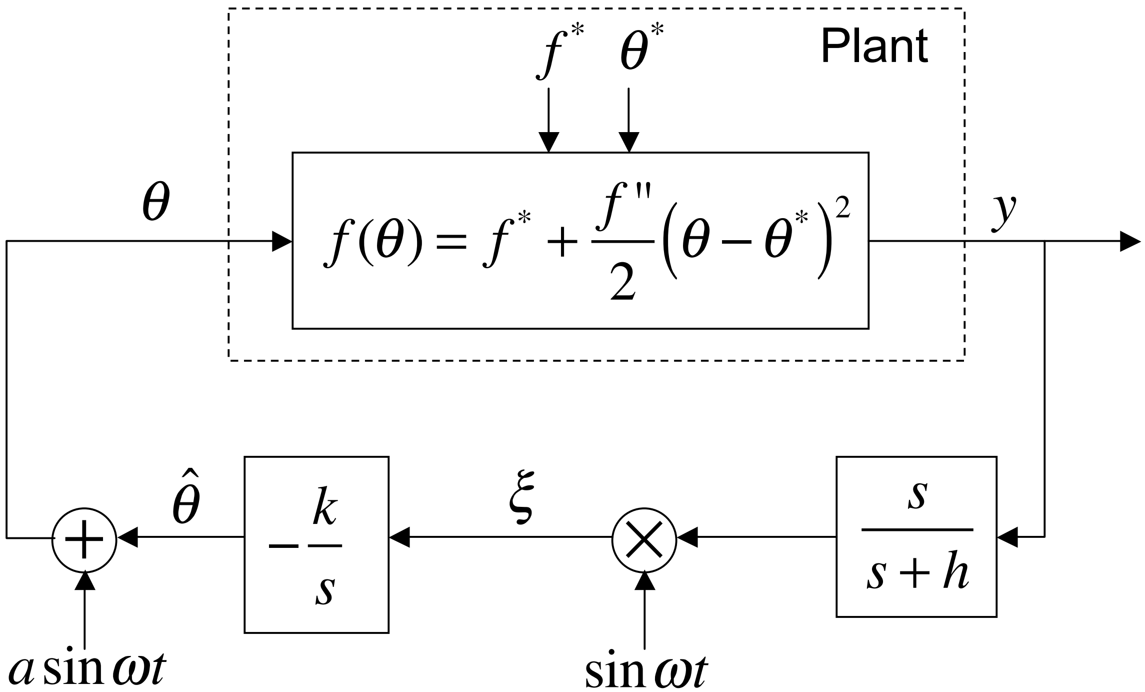
How Does It Work?



Since $\tilde{\theta} = \theta^* - \hat{\theta}$

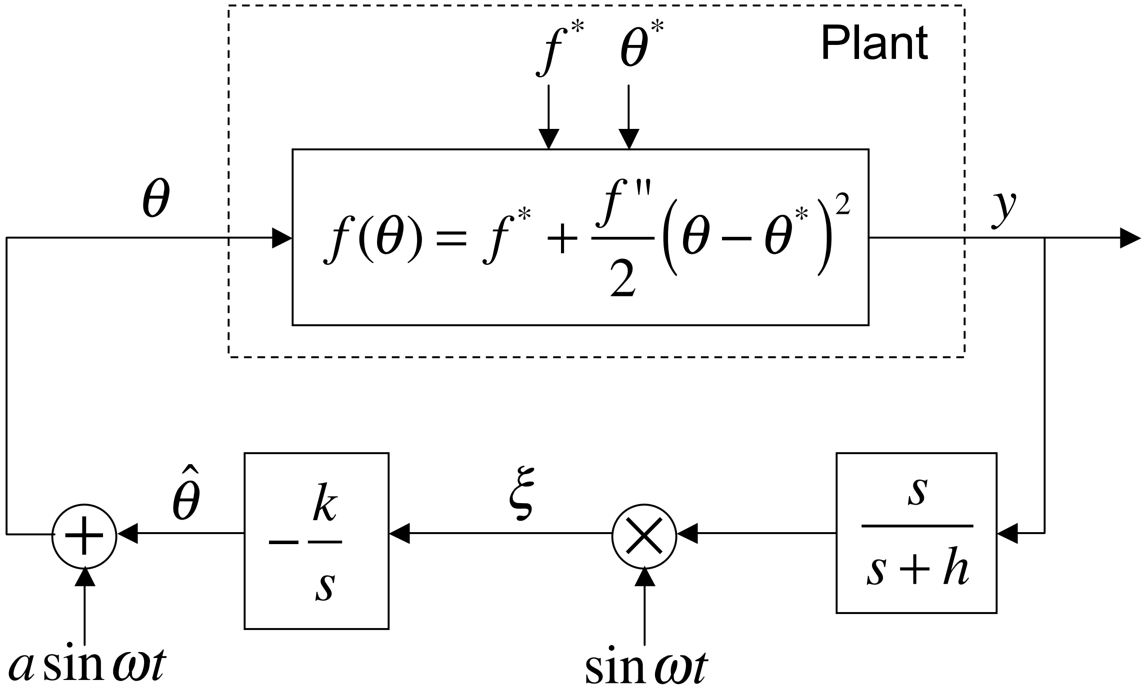
then $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$

How Does It Work?



$$\tilde{\theta} \approx \frac{k}{s}\left[-\frac{a^2 f''}{4}\tilde{\theta} \underbrace{+\frac{a^2 f''}{4}\tilde{\theta}\cos 2\omega t + \frac{a^2 f''}{8}\left(\sin \omega t - \sin 3\omega t\right)}_{\text{high frequency terms - attenuated by integrator}}\right]$$

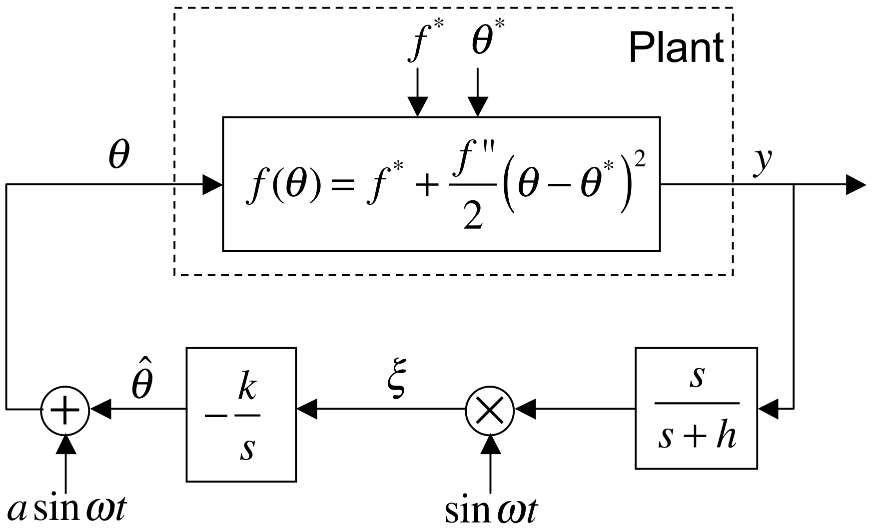
How Does It Work?



$$\boxed{\dot{\tilde{\theta}} \approx -\frac{ka^2 f''}{4}\tilde{\theta}}$$

Stable because $k, a, f'' > 0$

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

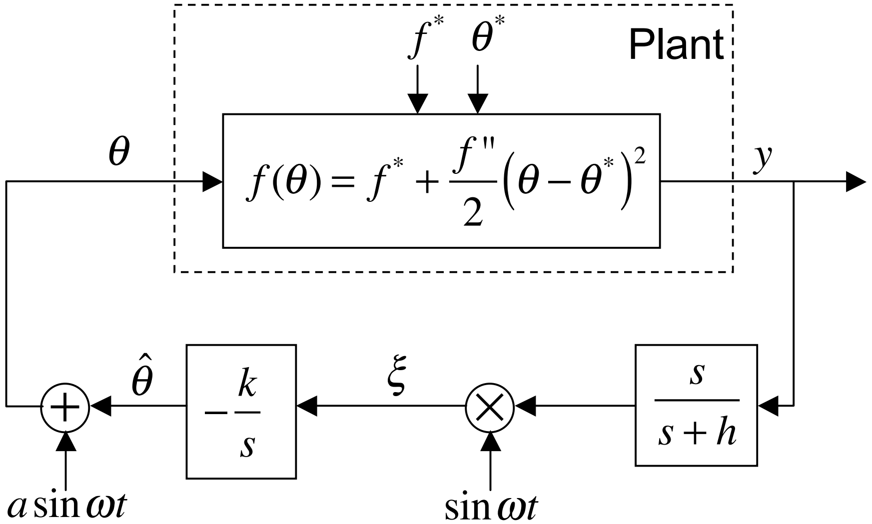
$$\tau = \omega t$$

Full nonlinear time-varying model:

$$\frac{d}{d\tau}\tilde{\theta} = \frac{k}{\omega}\left(\frac{f''}{2}\left(\tilde{\theta} - a\sin\tau\right)^2 - e\right)\sin\tau$$

$$\frac{d}{d\tau}e = \frac{h}{\omega}\left(-e - \frac{f''}{2}\left(\tilde{\theta} - a\sin\tau\right)^2\right)$$

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

$$\tau = \omega t$$

Average system:

$$\frac{d}{d\tau}\tilde{\theta}_{\mathrm{av}} = -\frac{kaf''}{2\omega}\tilde{\theta}_{\mathrm{av}}$$

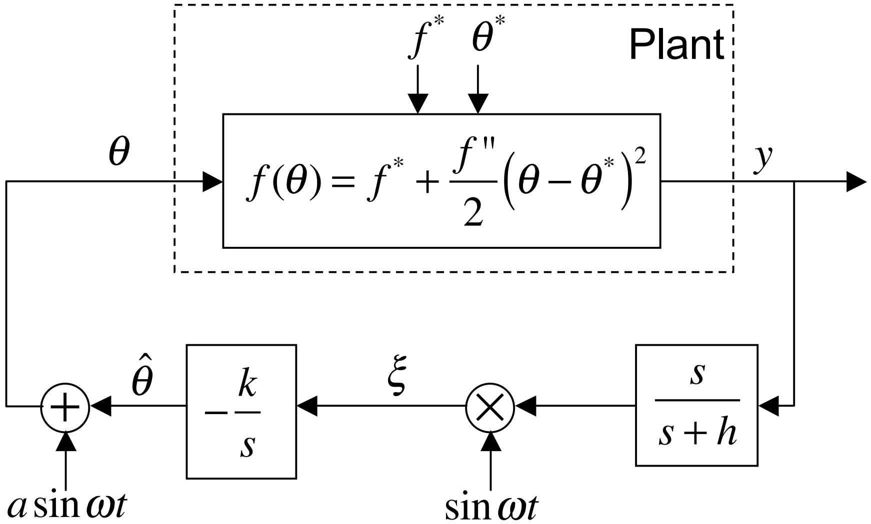
$$\frac{d}{d\tau}e_{\mathrm{av}} = \frac{h}{\omega}\left(-e_{\mathrm{av}} - \frac{f''}{2}\left(\tilde{\theta}_{\mathrm{av}}^2 + \frac{a^2}{2}\right)\right)$$

Average equilibrium:

$$\tilde{\theta}_{\mathrm{av}} = 0$$

$$e_{\mathrm{av}} = -\frac{a^2 f''}{4}$$

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

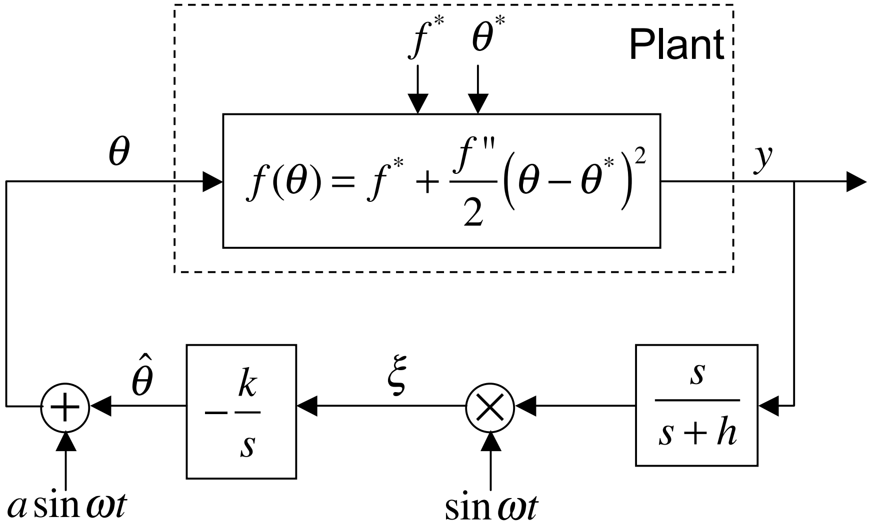
$$e = f^* - \frac{h}{s+h}[y]$$

$$\tau = \omega t$$

Jacobian of the average system:

$$J_{\text{av}} = \begin{bmatrix} -\frac{kaf''}{2\omega} & 0 \\ 0 & -\frac{h}{\omega} \end{bmatrix}$$

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

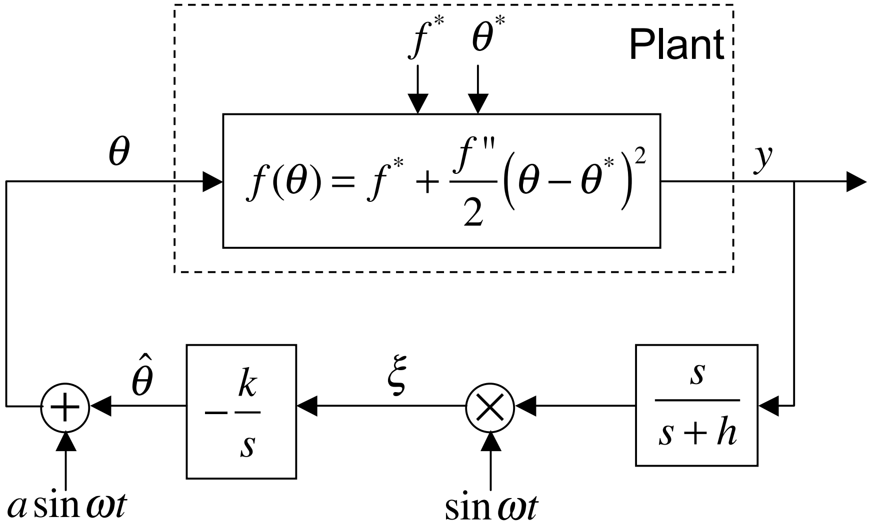
$$\tau = \omega t$$

Theorem. *For sufficiently large* ω *there exists a unique exponentially stable periodic solution of period* 2π/ω *and it satisfies*

$$\left|\tilde{\theta}_{2\pi/\omega}(t)\right| + \left|e_{2\pi/\omega}(t) - \frac{a^2 f''}{4}\right| \leq O\left(\frac{1}{\omega}\right), \qquad \forall t \geq 0$$

Speed of convergence proportional to $1/\omega$, a^2, k, f''

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

$$\tau = \omega t$$

Output performance:

$$y - f^* \to f'' O\left(\frac{1}{\omega^2} + a^2\right)$$

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Extremum-Seeking Control of Combustion Instabilities

Andrzej Banaszuk

United Technologies Research Center, E. Hartford, CT, U.S.A.

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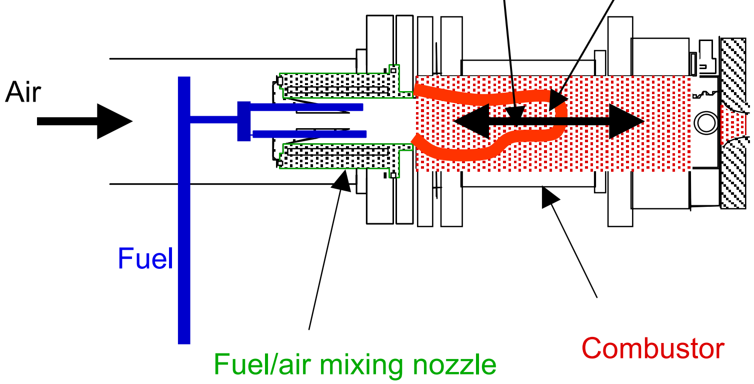
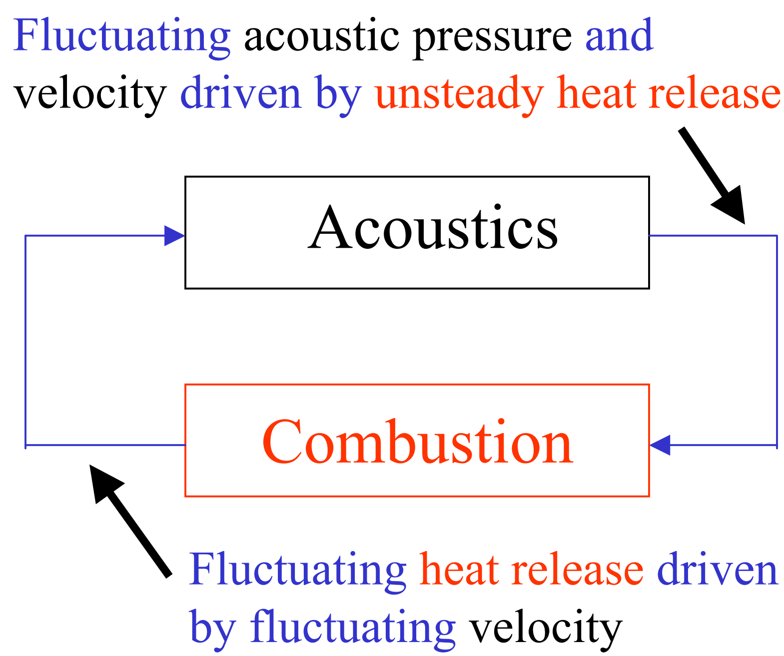
Workshop on Real-Time Optimization by Extremum-Seeking Control, ACC 05

Partially sponsored by AFOSR

Thermo-acoustic instability

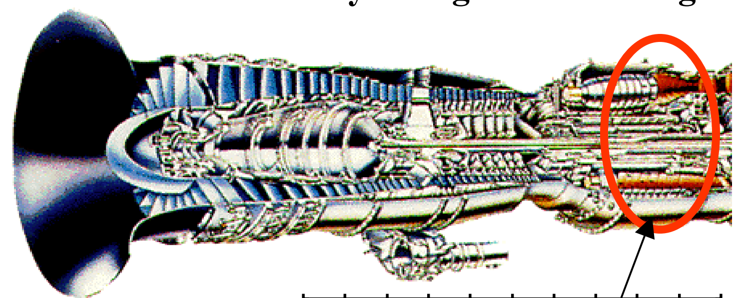
Coupling of acoustics with heat release results in pressure oscillations

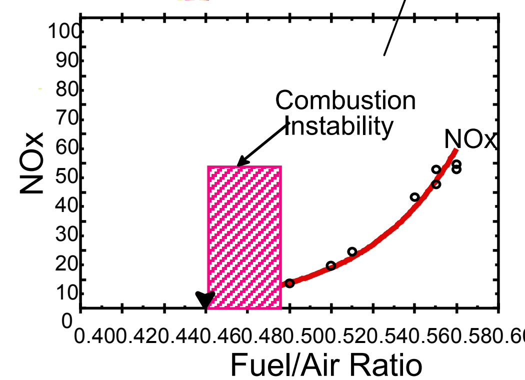




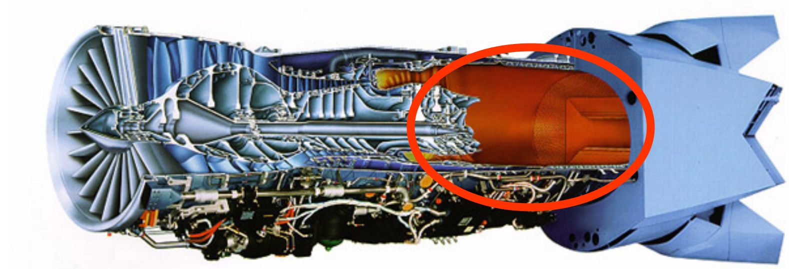
Thermo-acoustic instabilities affect cost and performance of gas turbines and rocket engines

25MW Pratt & Whitney FT8 gas turbine engine

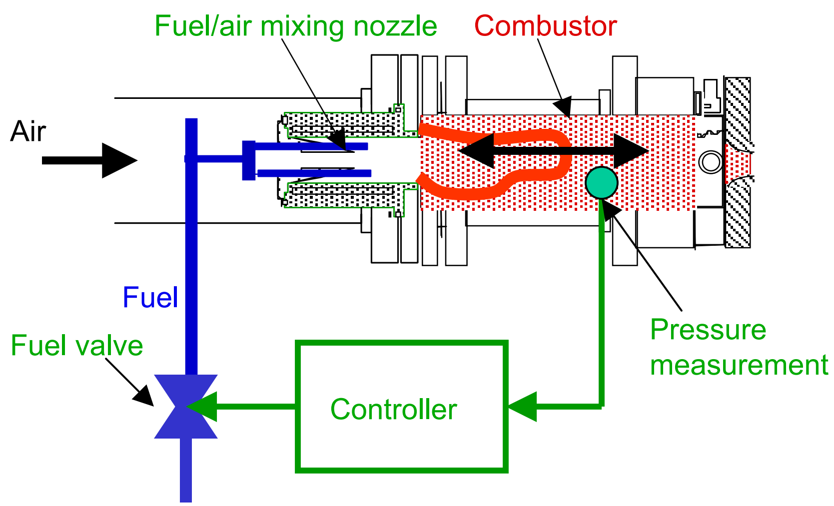


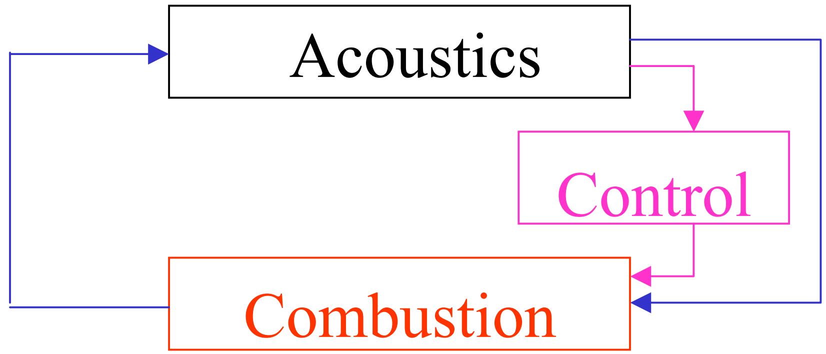


- •Power generation: pressure oscillations prevent low emission operation
- •Military aeroengines: afterburner screech and rumble limit performance, passive fixes increase weight, maintenance costs
- • Rockets: passive fixes increase weight, limit payload

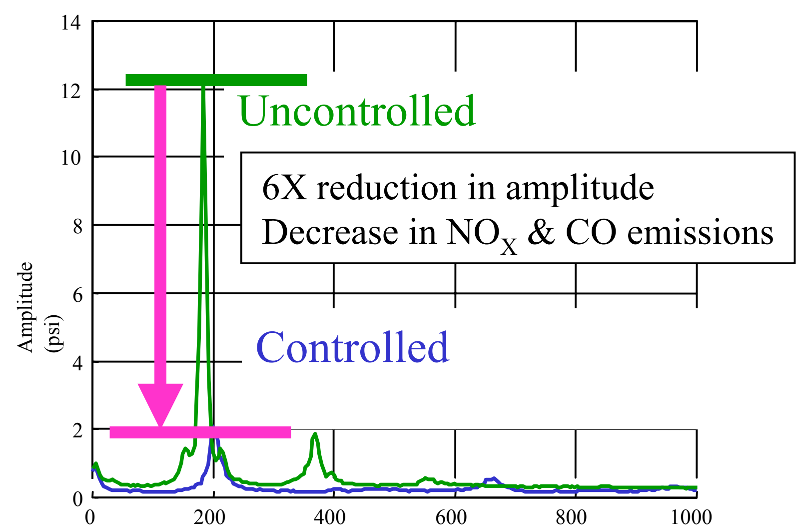


Active fuel control can suppress oscillations





Experiment in UTRC 4MW Single Nozzle Rig



- Numerous demonstrations at universities and industrial rigs
- Rolls-Royce demonstrated control in afterburner
- Siemens implemented control in 260MW gas turbine engine
- Phase-shifting control effective, but no models to guide selection of parameters

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Summary of results: two extremum seeking schemes were successfully demonstrated on UTRC 4MW single nozzle rig in August 1998.

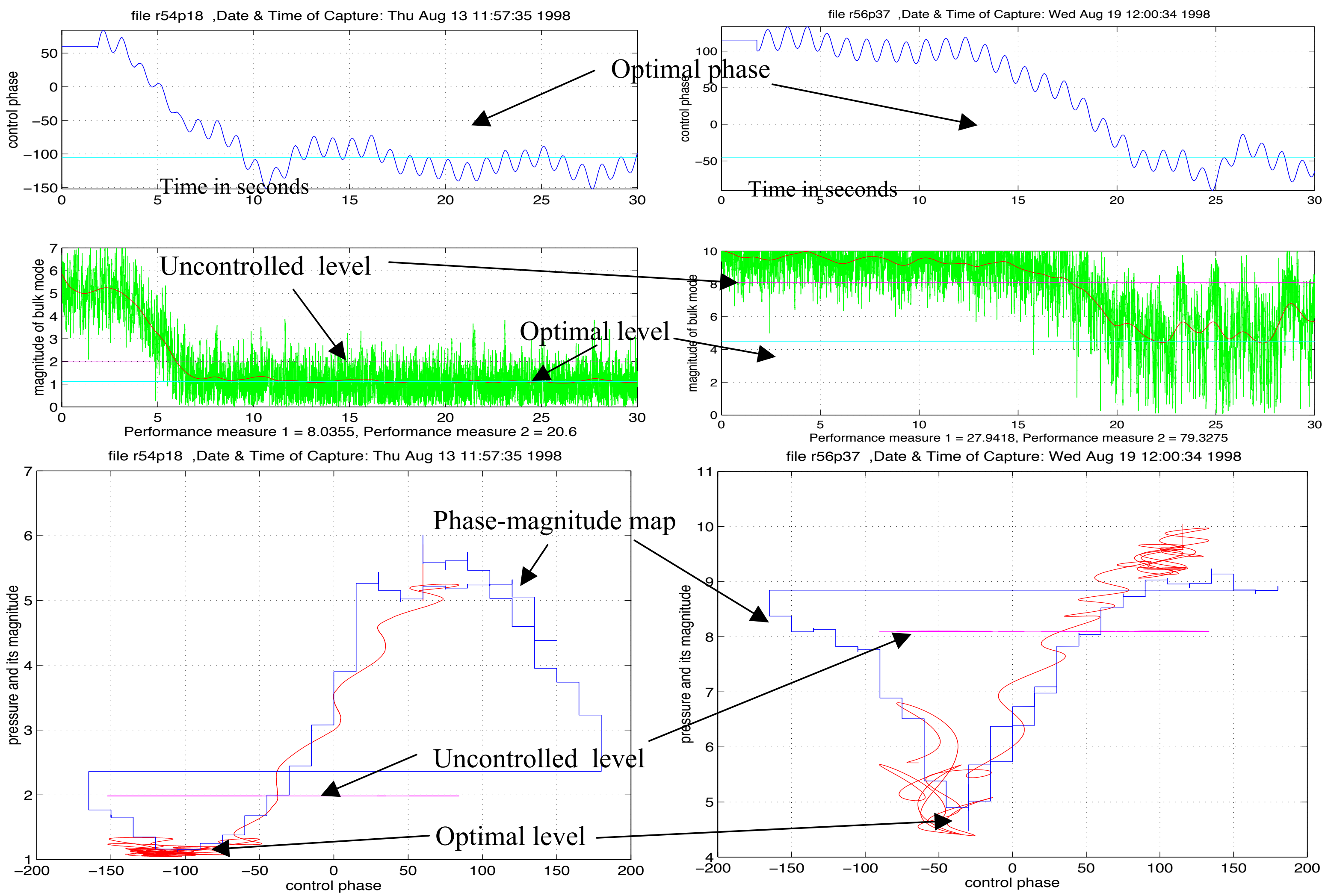
Excellent performance at high power

Improved performance at low power

Control phase in degrees

Pressure magnitude in psi

Pressure magnitude in psi



Control phase in degrees

Control phase in degrees

Formation Flight Optimization

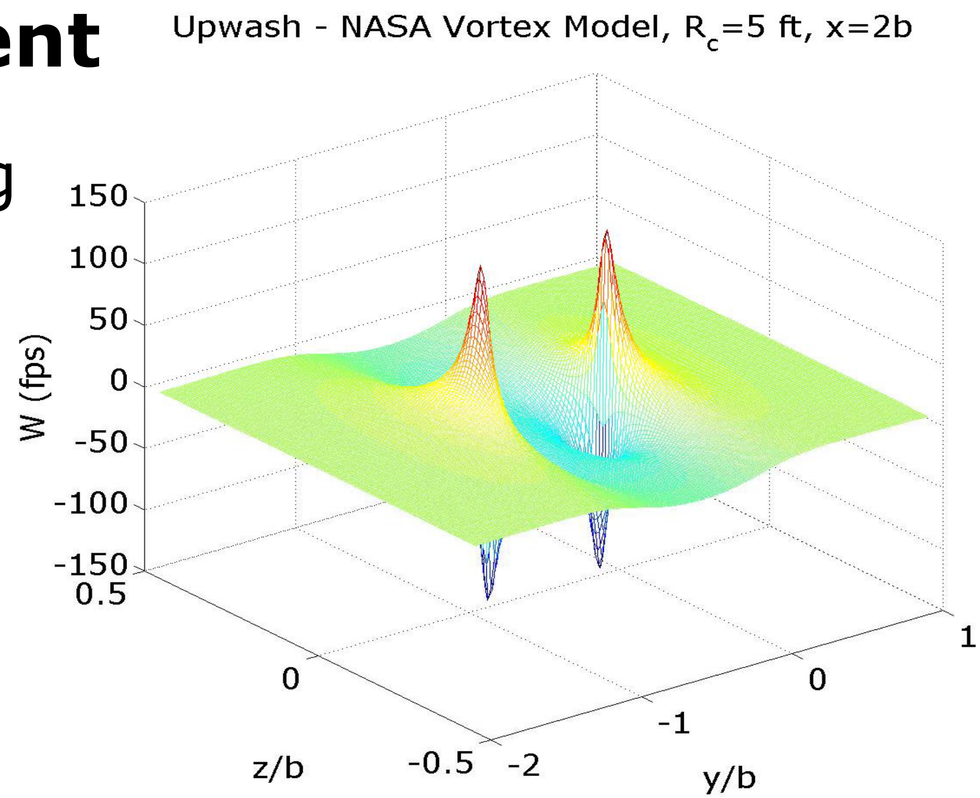
(Acknowledgement: Paolo Binetti)

Benefits of Formation Flight

- Reduction in power demand
- Alleviation of airspace congestion

Obstacles to Attainment

- Sensitivity to positioning
- Difficulty in precise measurements
- Absence of precise modeling



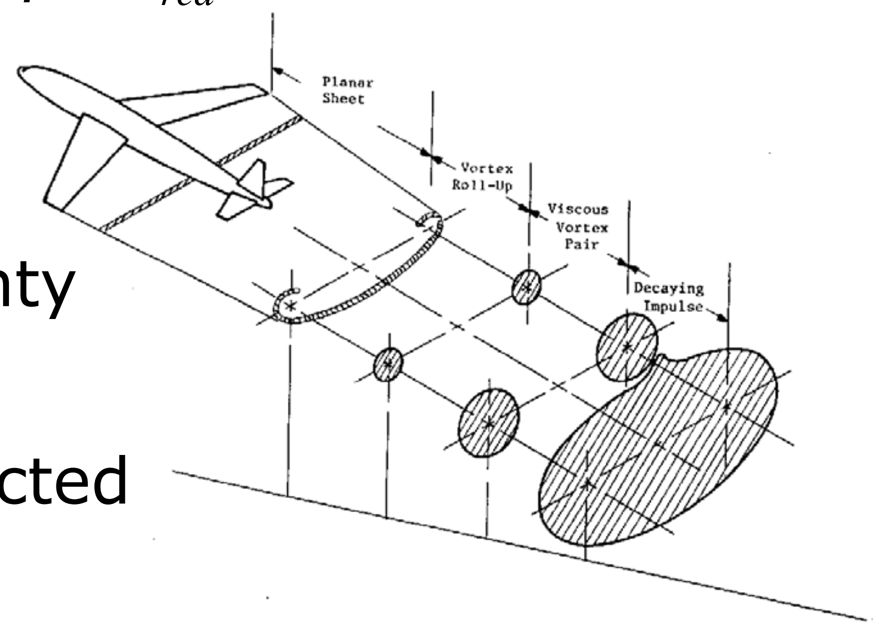
Physics of Aircraft Wakes and Wake Modeling

- Wake structure—modeled as two NASA counter-rotating vortices

$$V_\theta(r) = \frac{\Gamma}{2\pi r}\frac{r^2}{\left(r^2 + r_c^2\right)},\ \Gamma = \frac{W}{\rho V_\infty b_{red}},\ b_{red} = \frac{\pi}{4}b$$

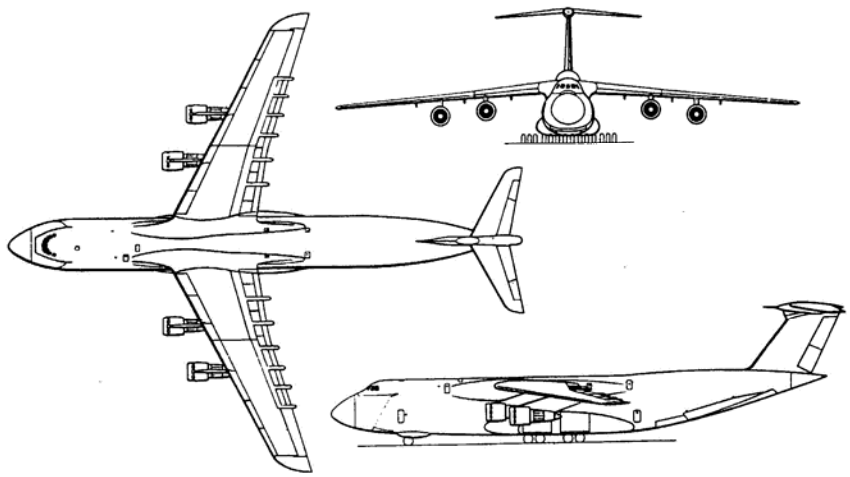
- Wake transport—
  a modeling uncertainty

- Wake decay—neglected



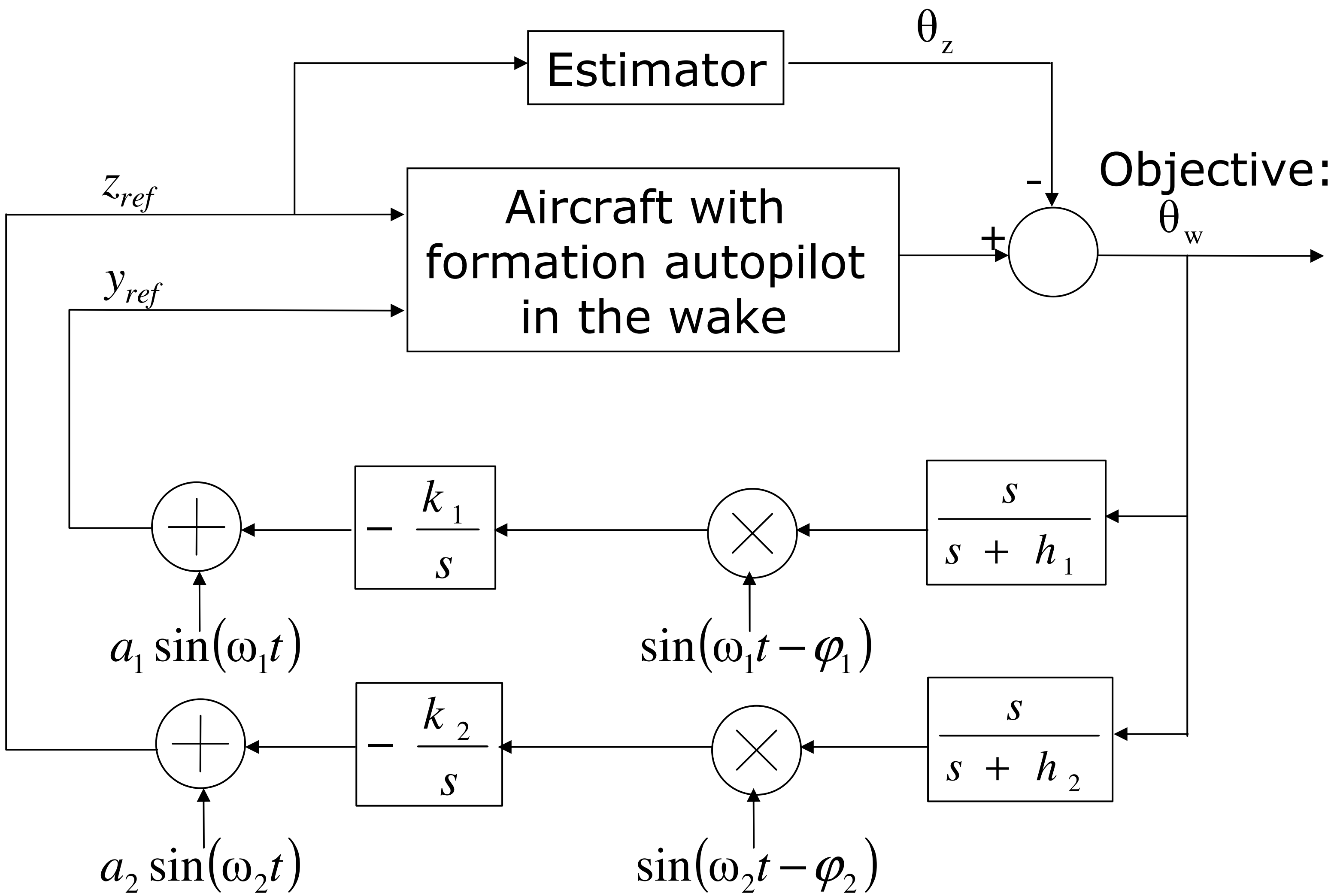
Selection of the Problem

Why the C-5?

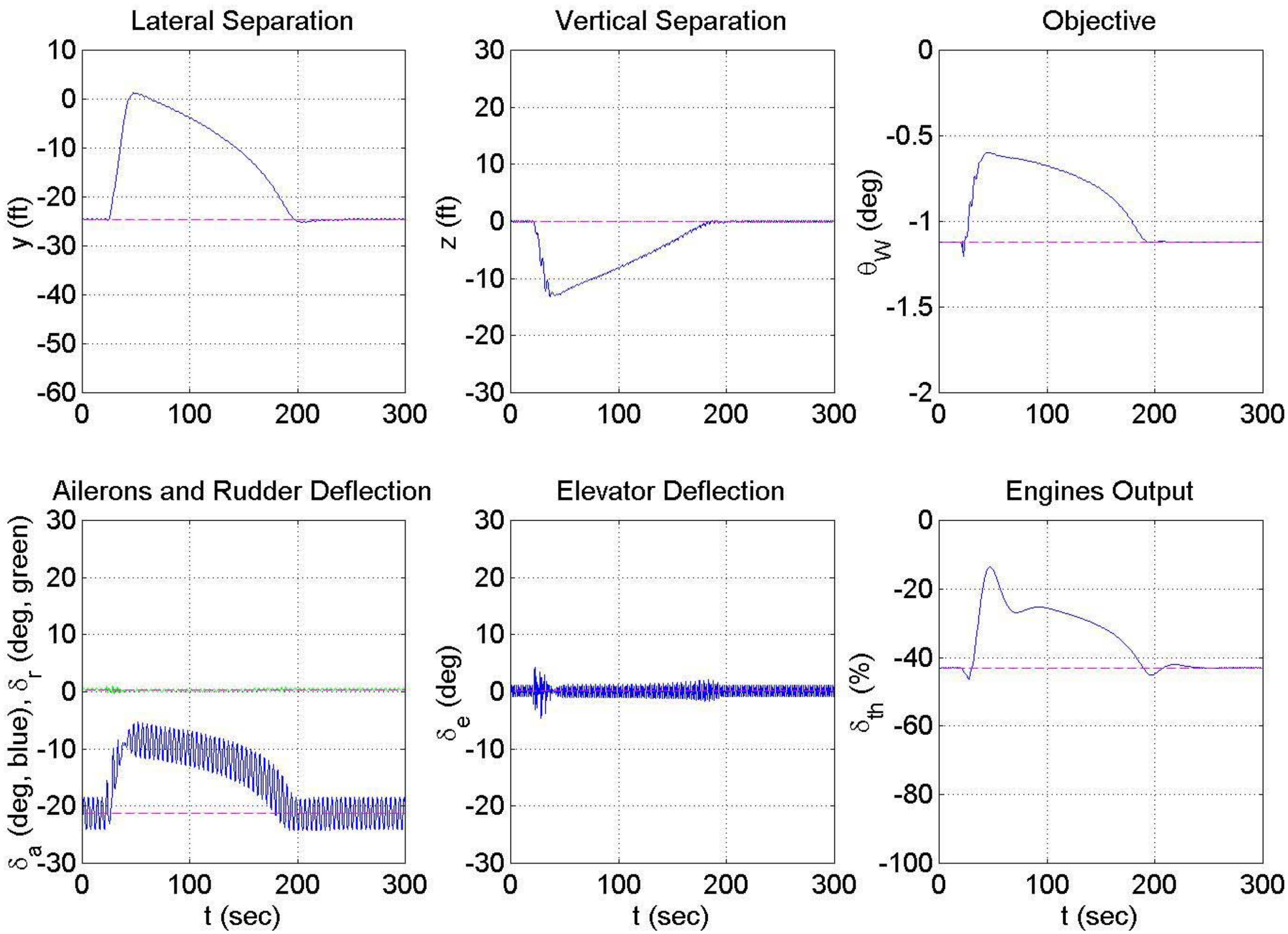


- Large savings in fuel consumption
- Representative of large transports
- They will be in service for the next 40 years
- Availability of wake data

Extremum Seeking for Formation Flight



Simulation for brief CAT Encounter …



Extremum Seeking Control of Thermoacoustic Coolers

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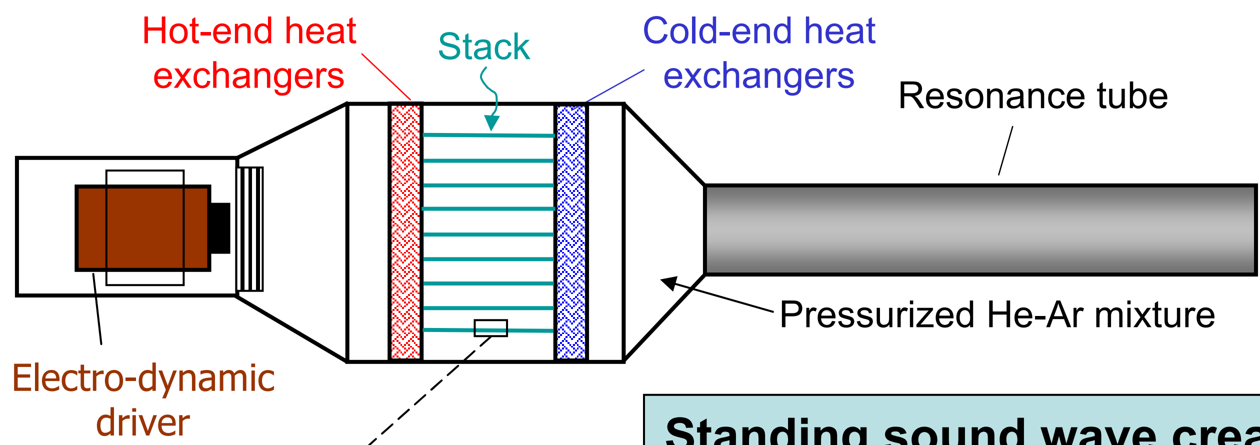
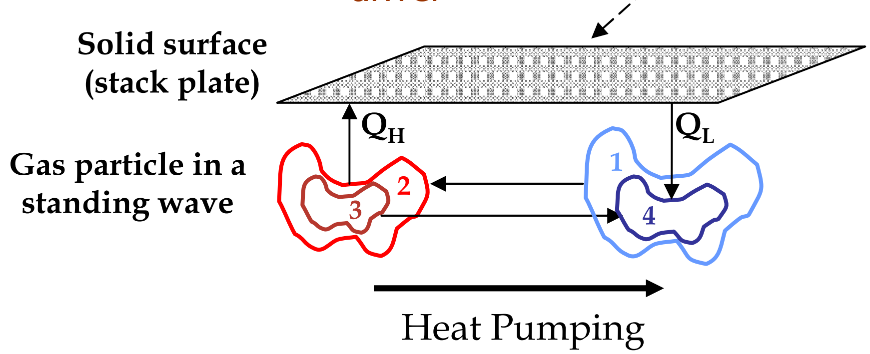
June 7, 2005
Purdue University
rotea@purdue.edu

REF. Extremum Seeking Control of Tunable Thermo-acoustic Coolers, Y. Li, M. Rotea, G. Chiu, L. Mongeau, I. Paek, IEEE Transactions on Control Systems Technology, Vol. 13, No. 4, July 2005

Thermoacoustic Cooling

Electric energy → **Acoustic energy** → **Heat pumping**





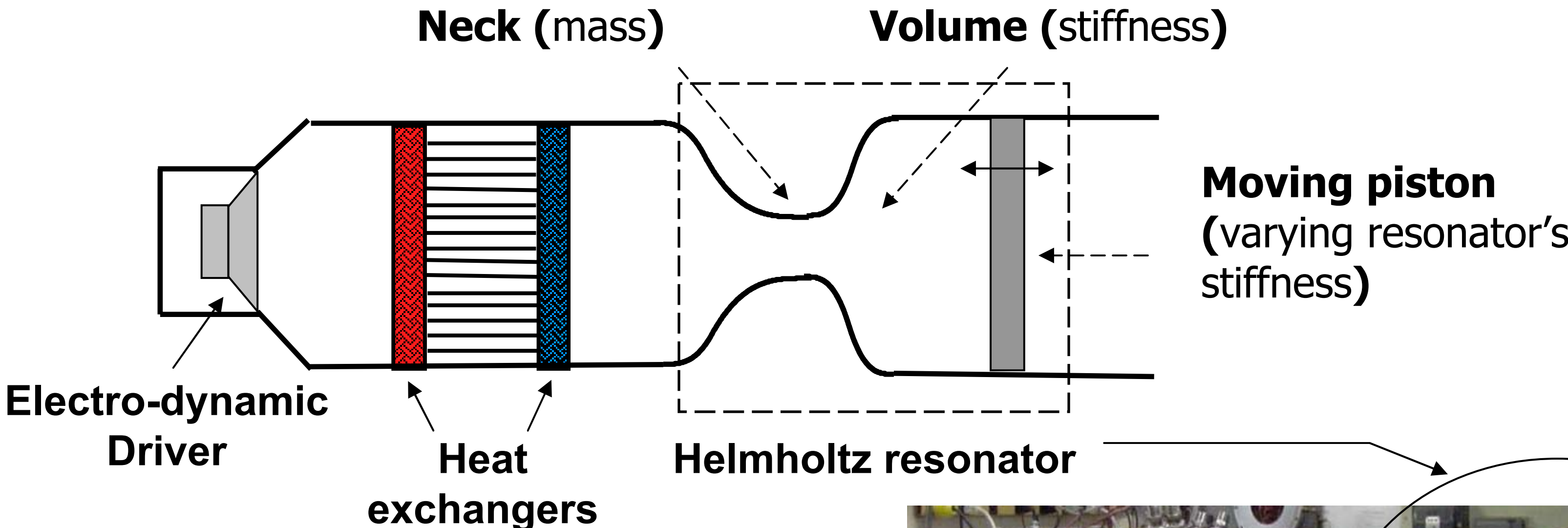
Standing sound wave creates the refrigeration cycle

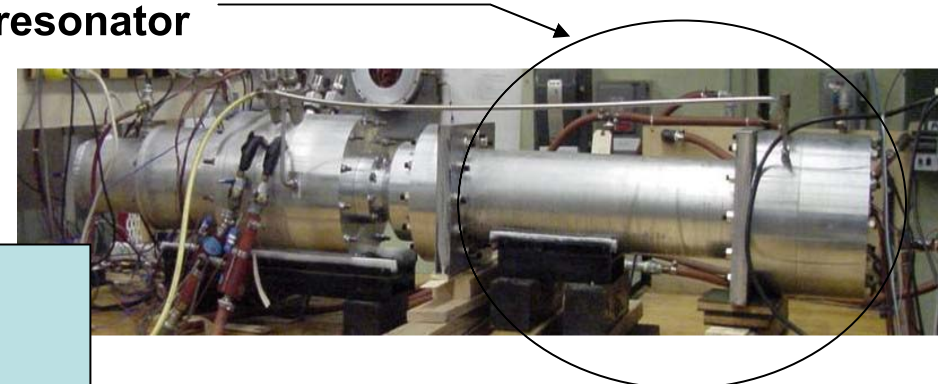
1-2: adiabatic compression and displacement
2-3: isobaric heat transfer (gas to solid)
3-4: adiabatic expansion and displacement
4-1: isobaric heat transfer (solid to gas)

Thermoacoustic Cooling

- Benefits
  - Environmentally benign (inert gases)
  - Simple, easy to maintain configuration
- Limitations
  - Very hard to model → hard to tune key parameters (driver, stack location, duct geometry) for best performance
- Existing prototypes
  - Acoustic Stirling Heat Engine (Los Alamos National Lab)
  - Triton 10 kW refrigerator (Penn State)
  - Space Thermo-acoustic Refrigerator (NASA)
  - Purdue 300 W standing wave unit
  - Miniature thermo-acoustic cooler (Rockwell Science Center)

Tunable Thermoacoustic Cooler



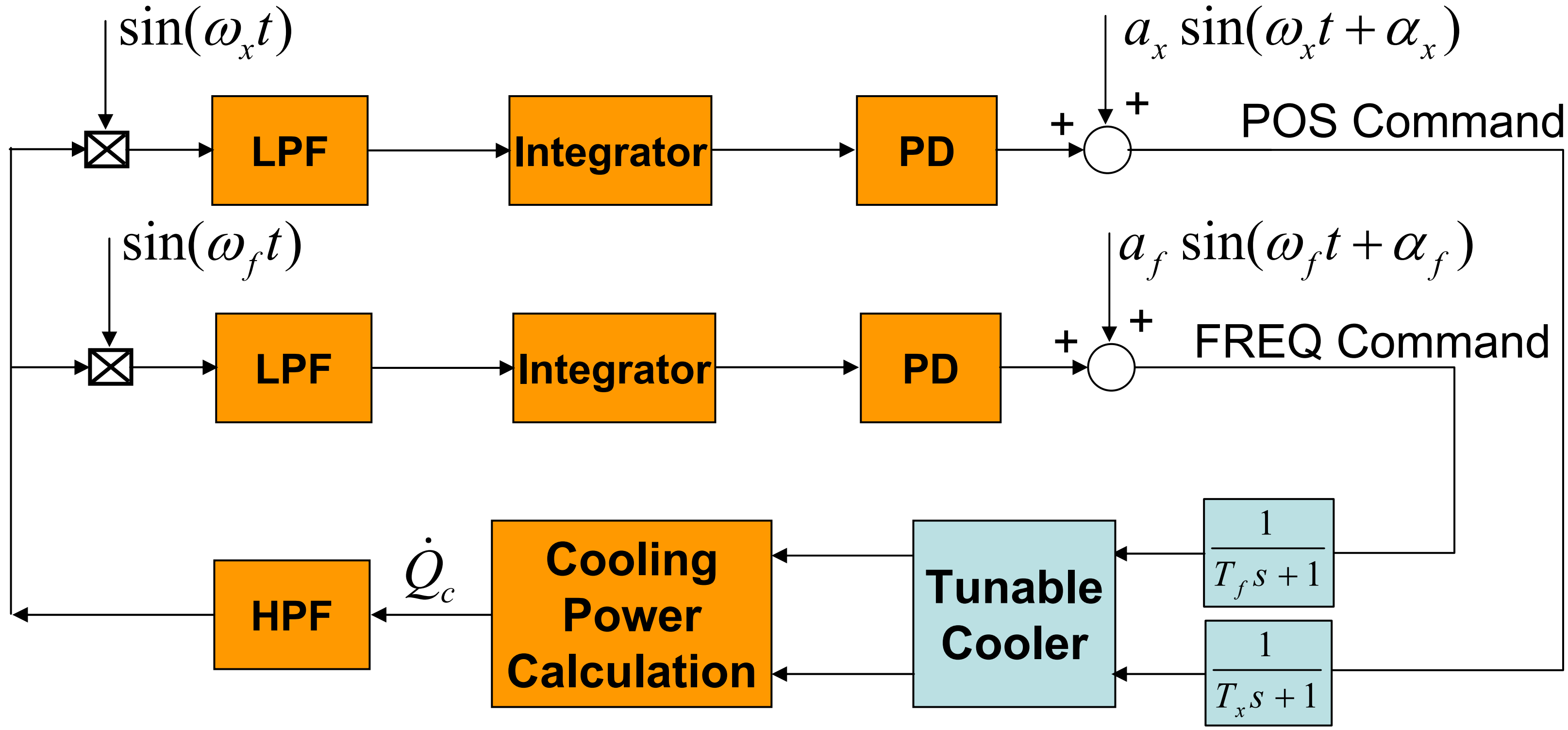


Tuning Variables

Performance and efficiency sensitive to variations in

- Piston position (acoustic impedance)
- Driver frequency

Extremum Seeking Controller

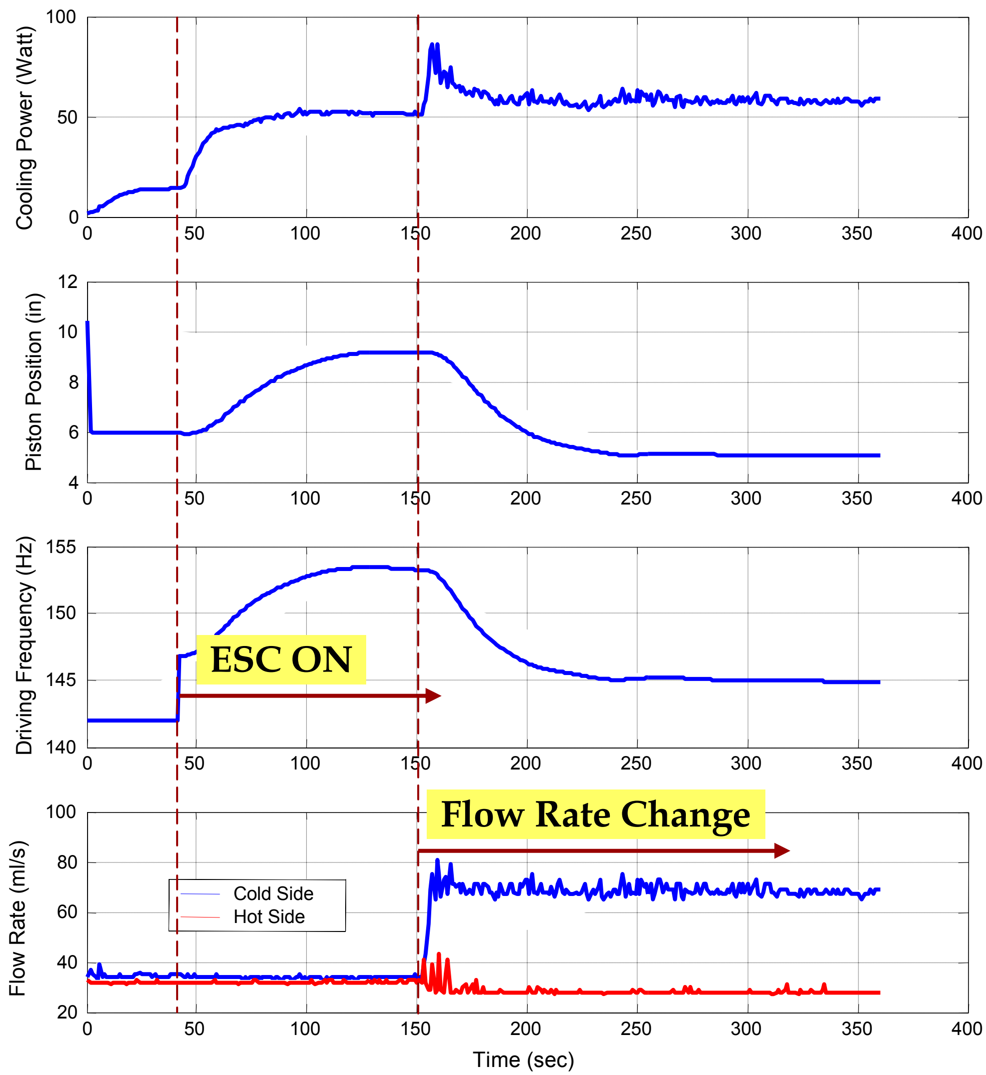


(Tuning mechanism)

<u>ESC Parameters</u>

– High pass filter (HPF)
– Low pass filters (LPF)
– Dither frequencies, amplitudes and phases (ω_x, a_x, α_x; ω_x, a_x, α_x)
– PD gains (4 gains)

Experiment – Varying Operating Condition



ESC tracks optimum after cold-side flow rate is increased

Extremum-Seeking Control of Flow Separation in a Planar Diffuser

Andrzej Banaszuk

United Technologies Research Center, E. Hartford, CT, U.S.A.

Acknowledgements:

Satish Narayanan, *UTRC: experiment*

Youping Zhang, *Numerical Technologies: algorithm*

Workshop on Real-Time Optimization by Extremum-Seeking Control, ACC 05

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Partially sponsored by AFOSR

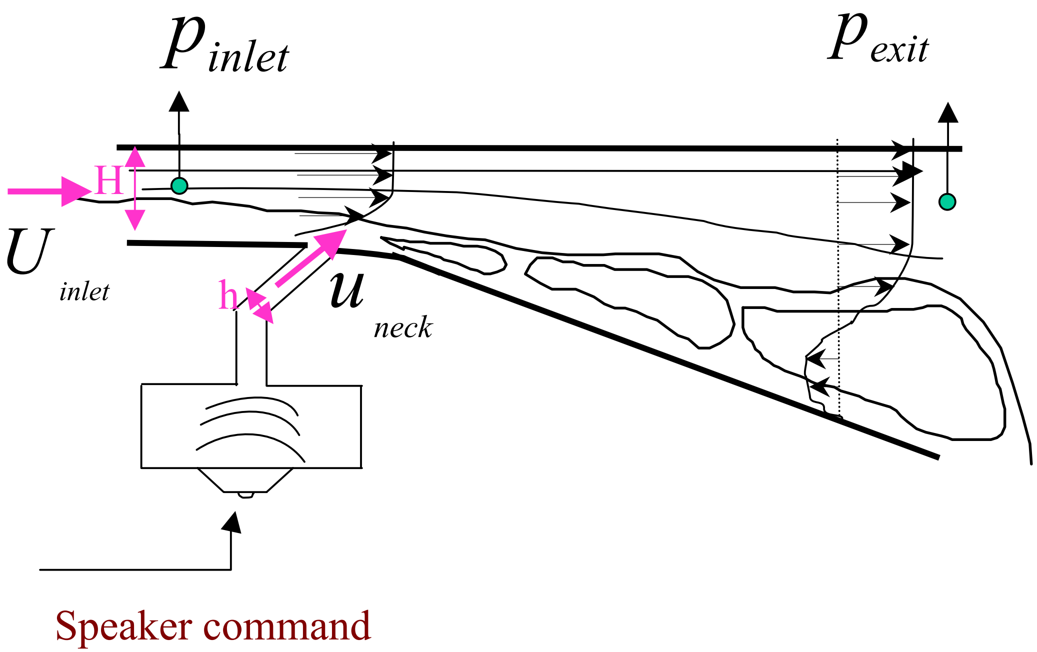
Objective of pressure recovery control

Performance

$$C_p(t) = \frac{p_{exit} - p_{inlet}}{\frac{1}{2}\rho U^2_{inlet}}$$

Control effort

$$C_\mu(t) = \frac{u^2_{neck}\, h}{U^2_{inlet}\, H}$$



$$\frac{\overline{C_p}}{\overline{C_\mu}} = \frac{\text{bang}}{\text{buck}}$$

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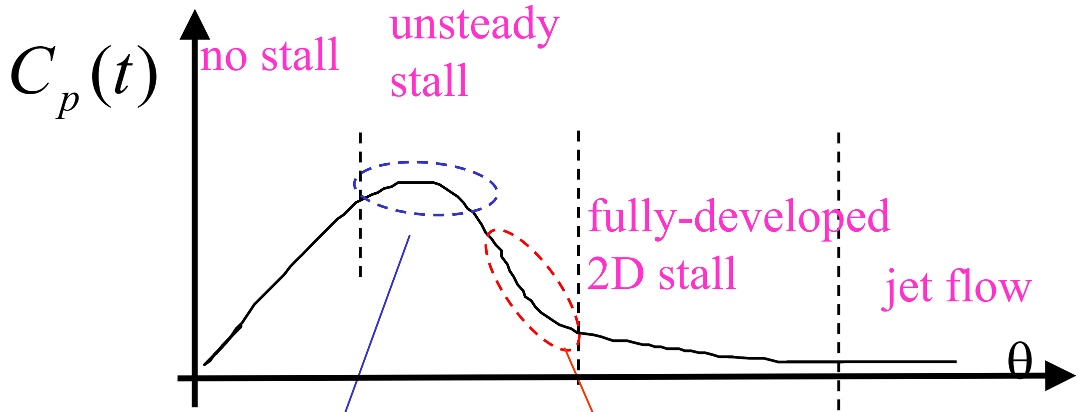
Experimental Setup

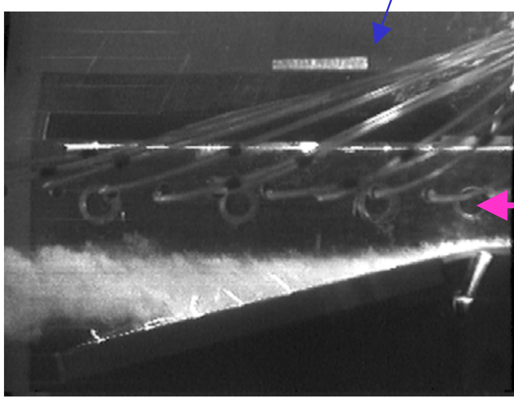
Pressure recovery as function of diffuser angle (no control)



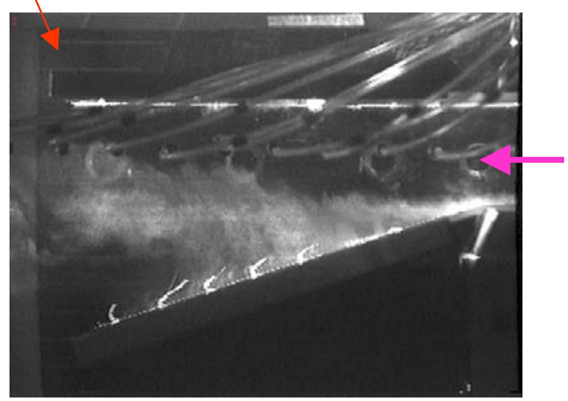
UTRC diffuser rig

- fully turbulent BL
- $40{,}000 < Re_H < 140{,}000$
- $Re_{\theta e} > 300$; $M < 0.1$
- Actuation: $C_\mu \sim 0.001$

$\theta \approx 14^o$



$\theta \approx 18^o$



- Optimum uncontrolled performance
- Insignificant improvement with control

- Poor uncontrolled performance
- Significant improvement with control

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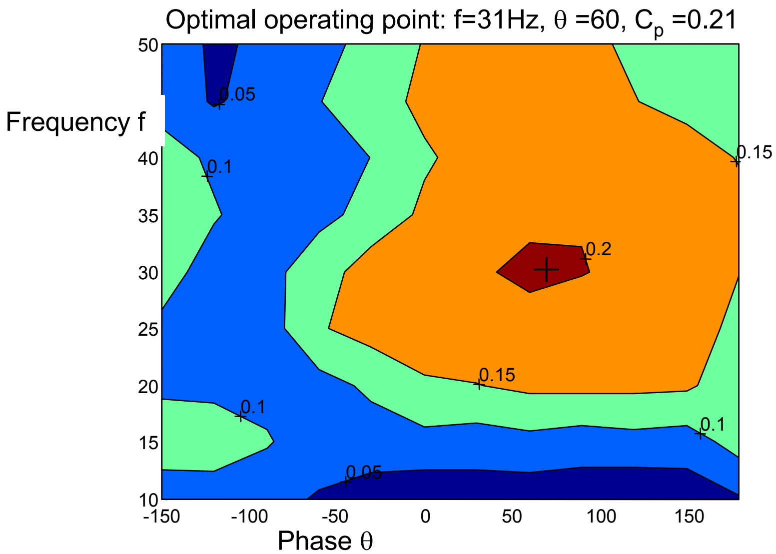
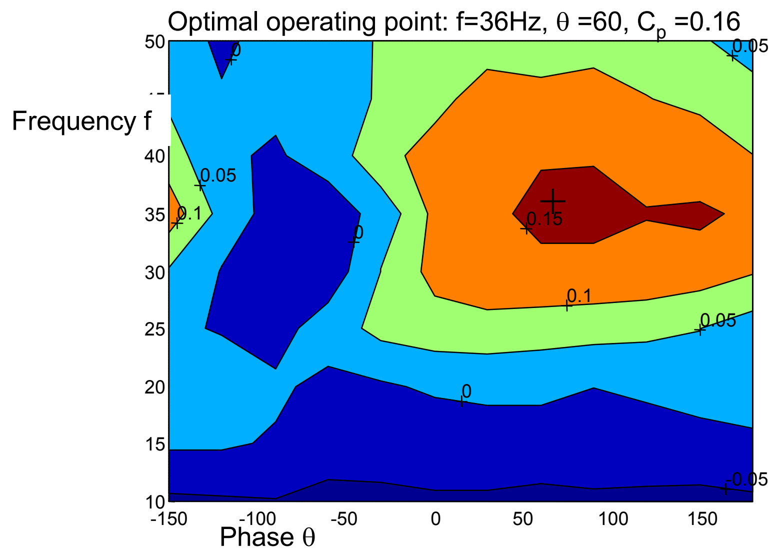
Need: control algorithm to optimize performance

Two frequency control law: $U(t)=A_1*(\sin(2*\pi*f*t) + \sin(2*\pi*2f*t-\theta))$

U = 20m/s



U = 30m/s



Objective:

- Optimize performance without exhaustive search

Challenges:

- Noisy measurement
- Flow transients
- Keeping up with operating condition change

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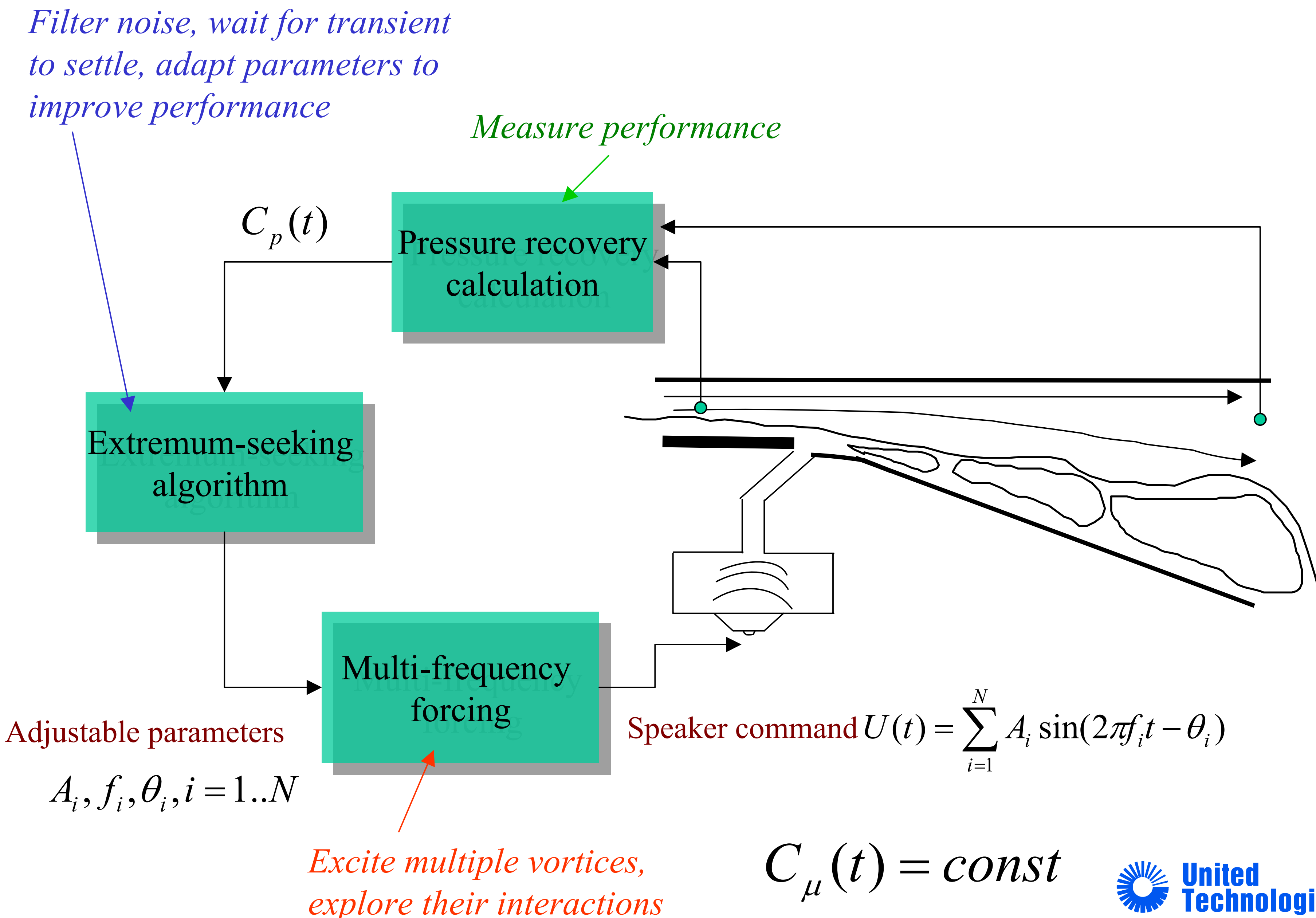
Adaptive control used to optimize performance



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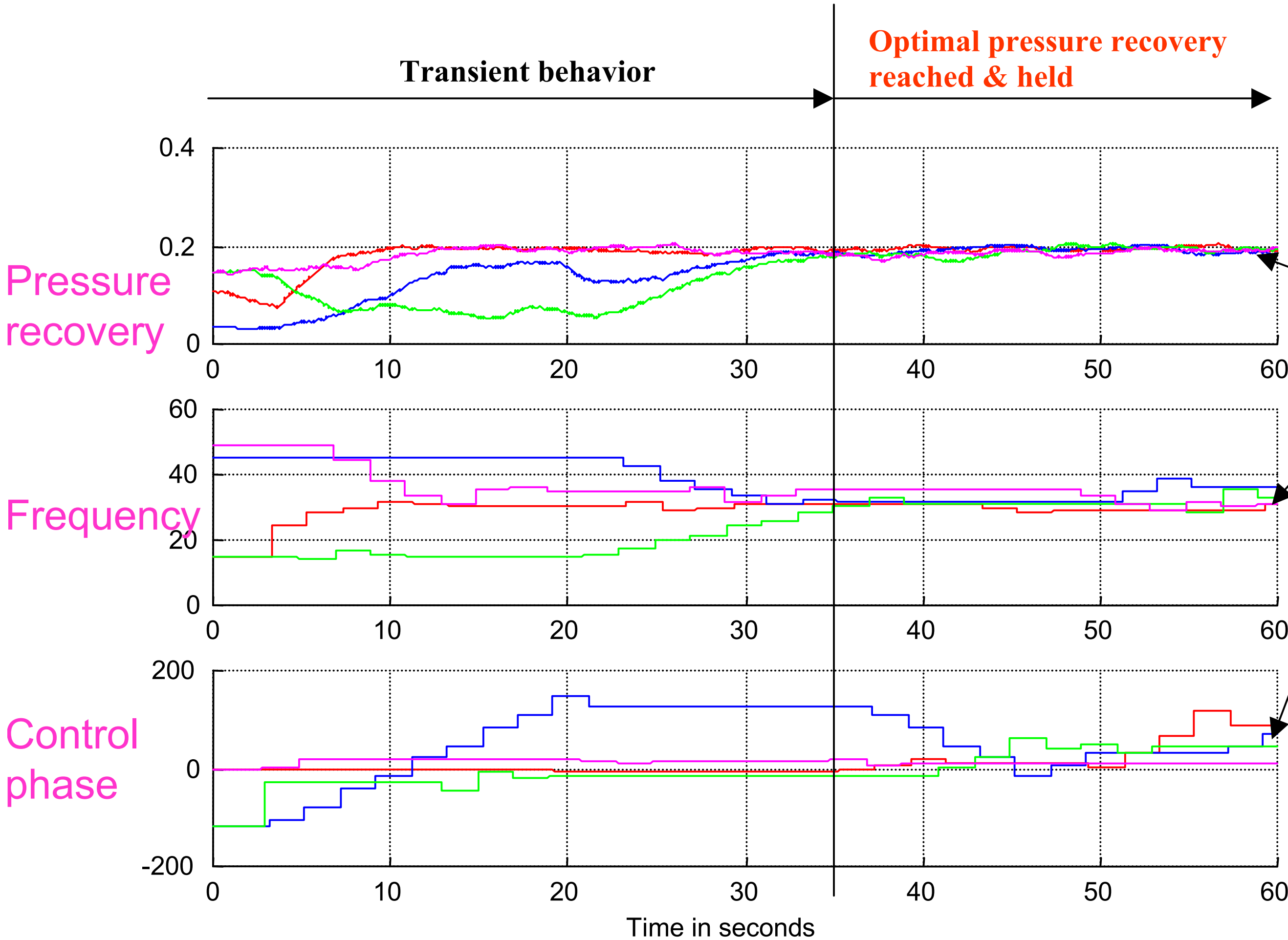
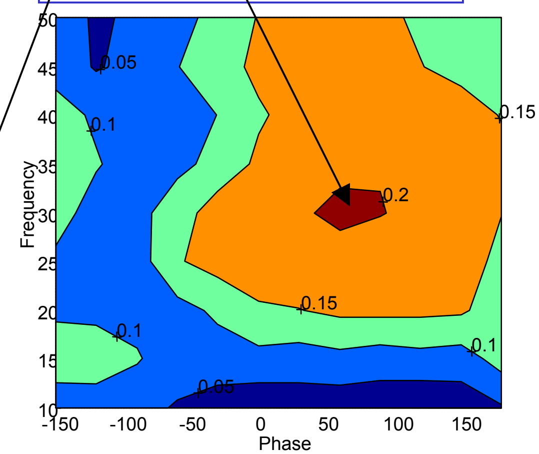
Automatic Control Parameter Tuning to Optimum Values

On-line optimization of pressure recovery using extremum-seeking algorithm demonstrated.



Dynamics determines transient time!

Optimal pressure recovery:
C_p =0.21
optimal control parameters
f=31Hz
θ =60



- **Mean pressure recovery, control frequency, and phase in four independent adaptive control experiments.**
- **The control frequency and phase initialized away from the optimal values.**

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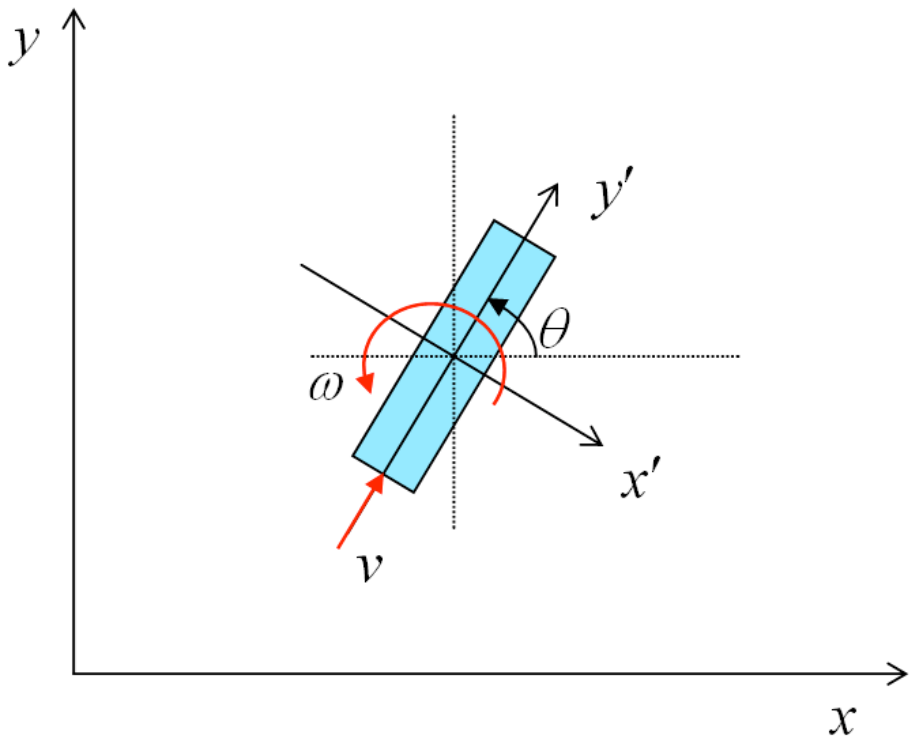
Autonomous Vehicle Control via Extremum Seeking

Antranik A. Siranosian and Miroslav Krstic

Model

Unicycle



Unicycle Dynamics

$$\dot{x} = v\cos(\theta)$$

$$\dot{y} = v\sin(\theta)$$

$$\dot{\theta} = \omega$$

Unicycle Inputs

$$v, \omega$$

System is linearly Uncontrollable (from inputs *v,w*) and Unobservable (from output *f(x,y)* at its peak)

Control Inputs

Unicycle

- Let the angular velocity input ω be constant

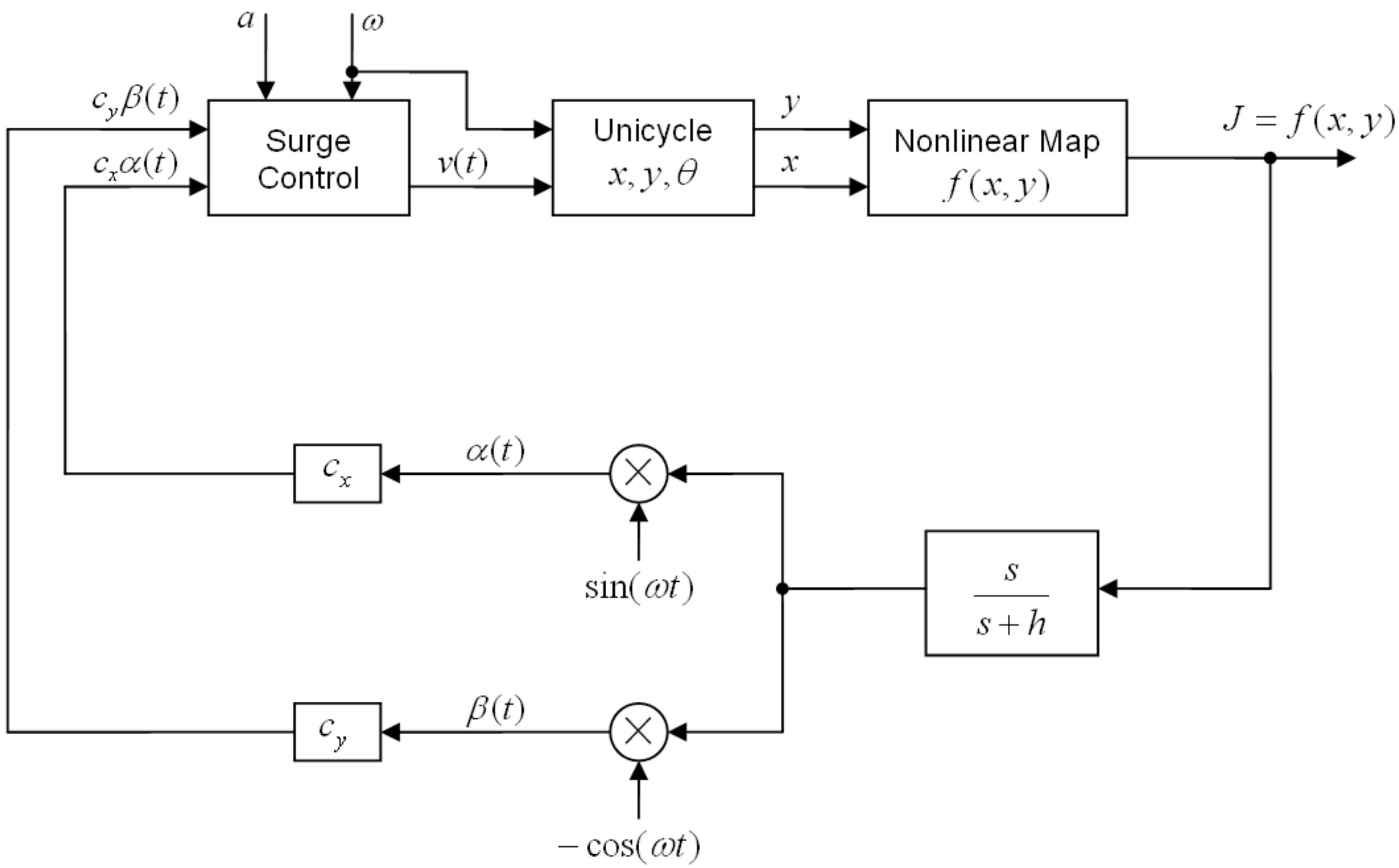
- The surge velocity input is composed of a constant, mean velocity ($a\omega$) and a sinusoidal velocity perturbation

$$v(t) = a\omega + 2\sqrt{\alpha(t)^2 + \beta(t)^2}\,\sin\left(\omega t + \tan^{-1}\left(\frac{\alpha(t)}{\beta(t)}\right)\right)$$

- $\alpha(t)$ and $\beta(t)$ are the parameters tuned by ES

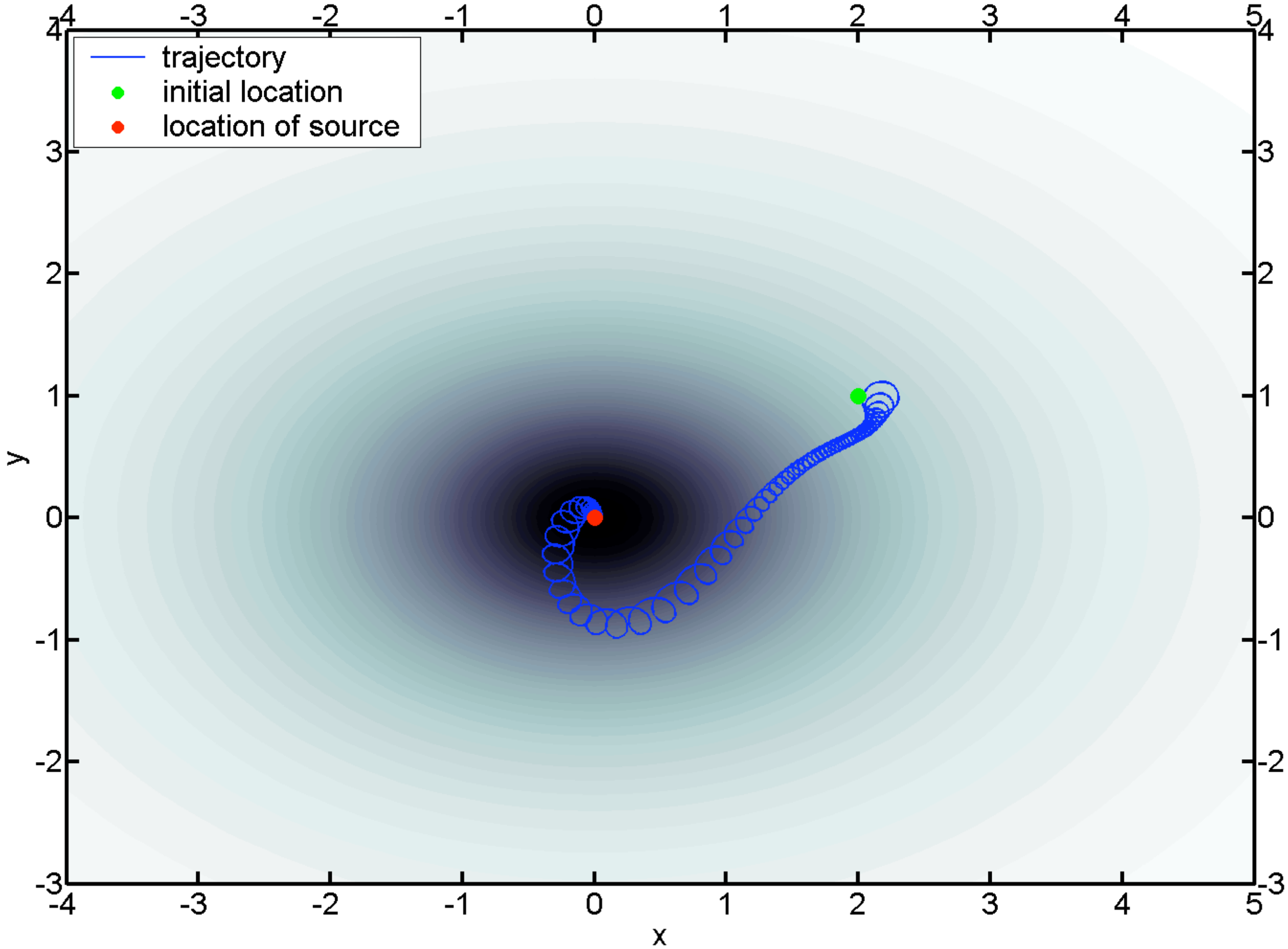
Block Diagram

Unicycle



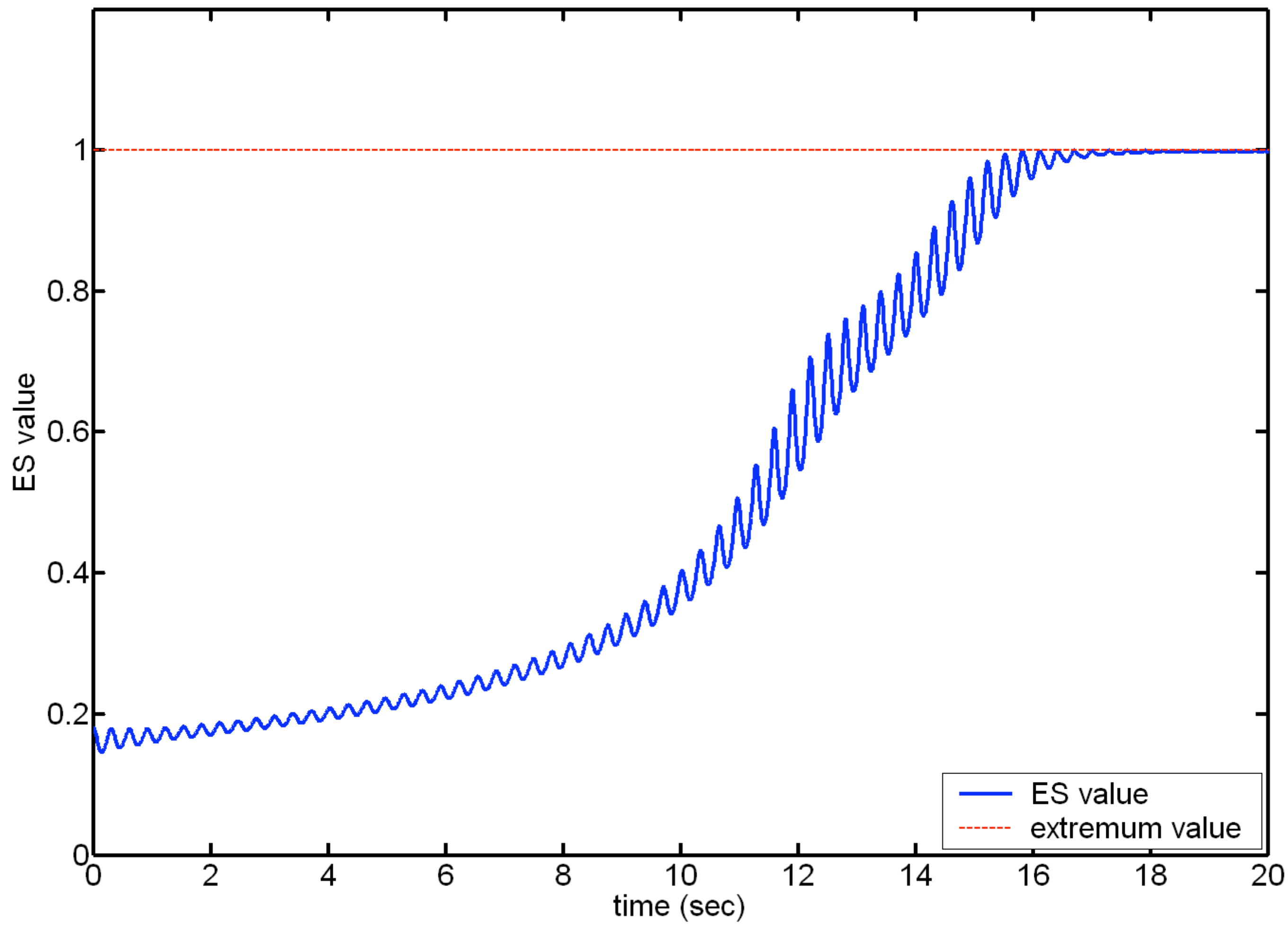
Simulation Results – Trajectory

Unicycle



Simulation Results – ES Value

Unicycle



BEAM MATCHING IN PARTICLE ACCELERATORS

Eugenio Schuster

Complex Control Systems Laboratory
Mechanical Engineering and Mechanics

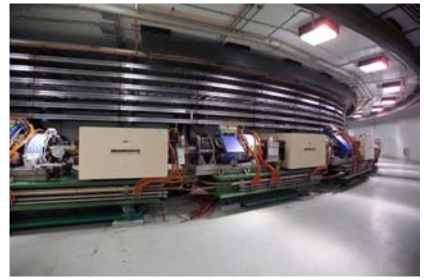
LEHIGH
UNIVERSITY

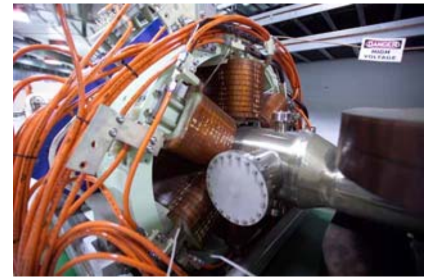
Workshop: Real Time Optimization by Extremum Seeking Control
American Control Conference 2005
June 7, 2005

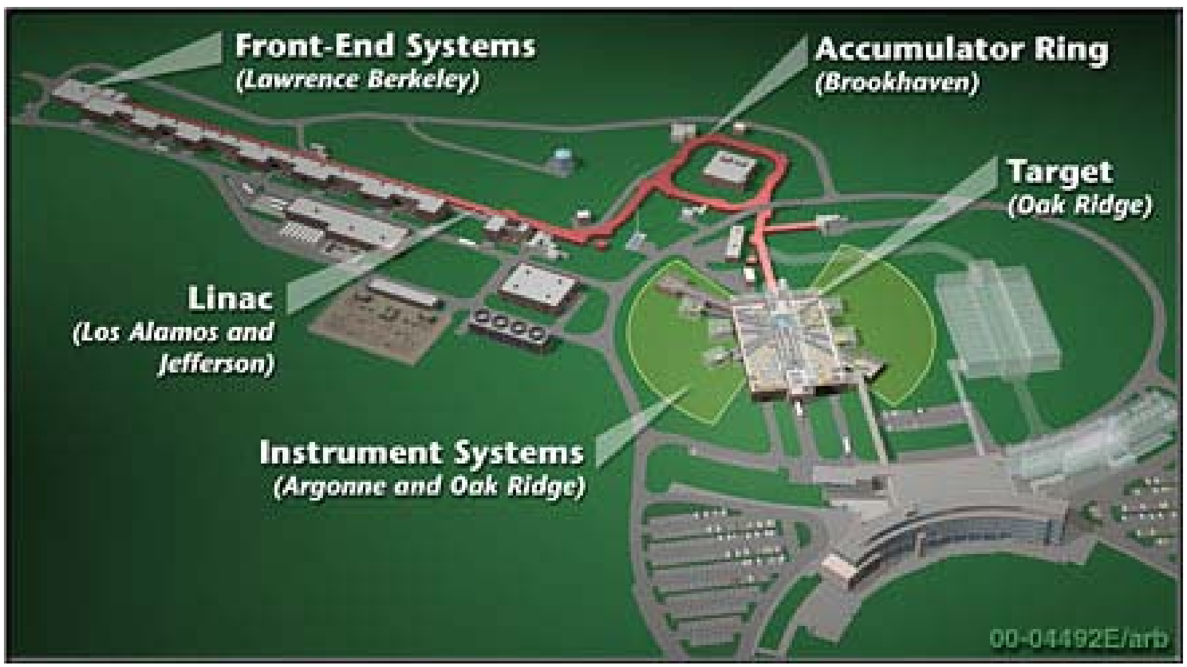
PARTICLE ACCELERATORS

The Spallation Neutron Source (SNS), being built in Oak Ridge, Tennessee, by the U.S. Department of Energy with a cost of $1.4 billion, is the most intense accelerator-based neutron source in the world.

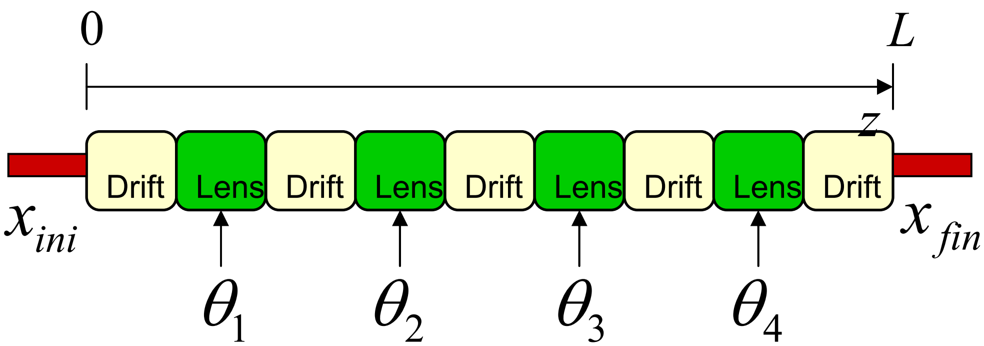
SPALLATION NEUTRON SOURCE





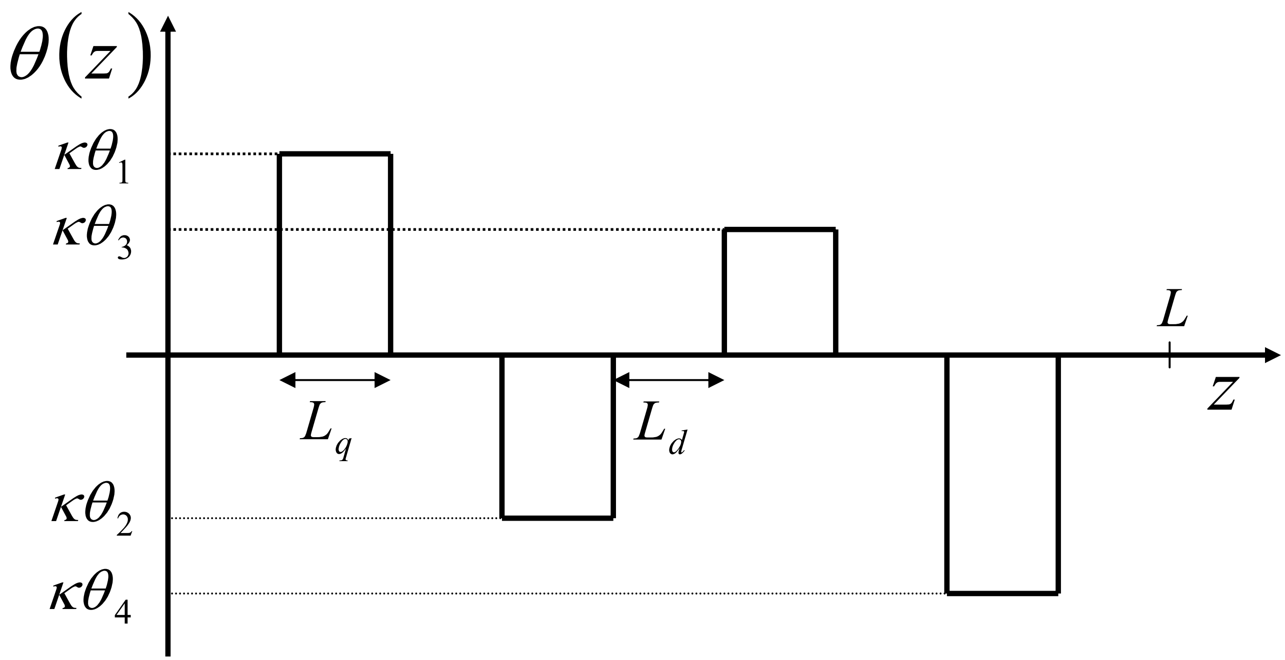


BEAM MATCHING CHANNEL

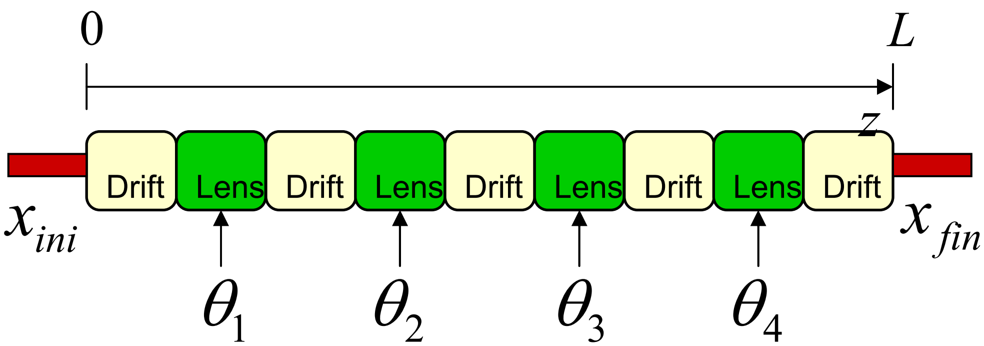


$$X'' - \theta(z)X - \frac{2K}{X+Y} - \frac{\varepsilon_X^2}{X^3} = 0$$

$$Y'' + \theta(z)Y - \frac{2K}{X+Y} - \frac{\varepsilon_Y^2}{Y^3} = 0$$



BEAM MATCHING OPTIMIZATION



$$x_{ini} = \begin{bmatrix} X_{ini} \\ X'_{ini} \\ Y_{ini} \\ Y'_{ini} \end{bmatrix}, x_{fin} = \begin{bmatrix} X_{fin} \\ X'_{fin} \\ Y_{fin} \\ Y'_{fin} \end{bmatrix}, x_{tar} = \begin{bmatrix} X_{tar} \\ X'_{tar} \\ Y_{tar} \\ Y'_{tar} \end{bmatrix}$$

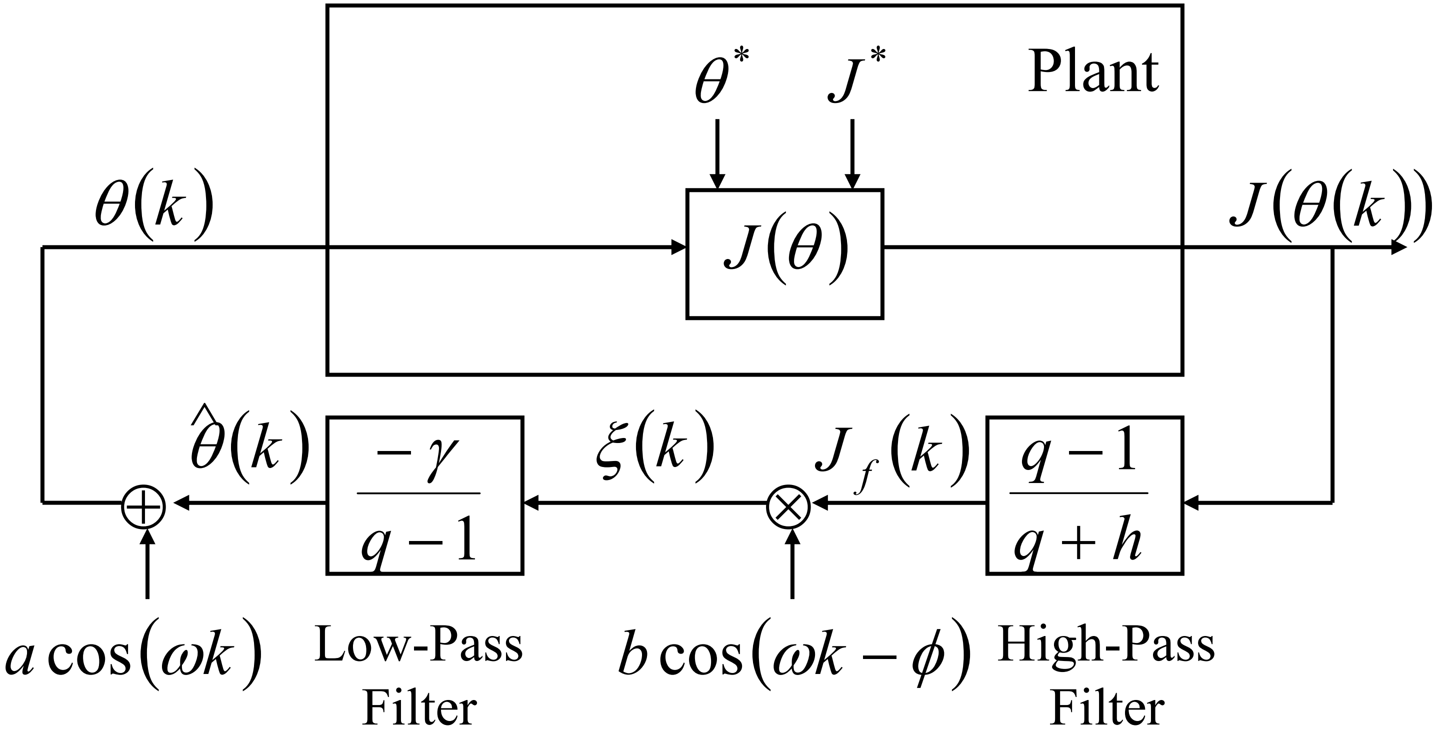
$$\boxed{J = k_1 J_1 + k_2 J_2 + k_3 J_3}$$

$$J_1 = K_X \left(X_{fin} - X_{tar}\right)^2 + K_Y \left(Y_{fin} - Y_{tar}\right)^2$$

$$J_2 = K_{dX} \left(X'_{fin} - X'_{tar}\right)^2 + K_{dY} \left(Y'_{fin} - Y'_{tar}\right)^2$$

$$J_3 = \int_o^L w(z) \left[K_{iX} \left(X(z) - X_{des}(z)\right)^2 + K_{iY} \left(Y(z) - Y_{des}(z)\right)^2\right] dz$$

BEAM MATCHING OPTIMIZATION



$$J_f(k) = -hJ_f(k-1) + J(k) - J(k-1)$$

$$\xi(k) = J_f(k) b\cos(\omega k - \phi)$$

$$\hat{\theta}(k+1) = \hat{\theta}(k) - \gamma\xi(k)$$

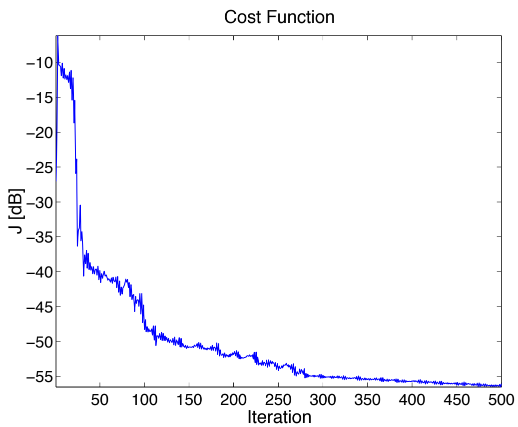
$$\theta(k+1) = \hat{\theta}(k+1) + a\cos(\omega(k+1))$$

BEAM MATCHING OPTIMIZATION – 4D

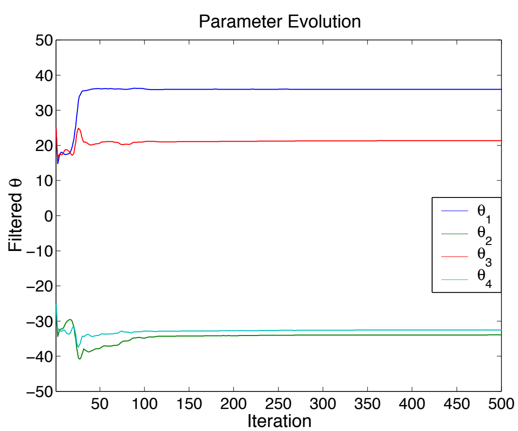
$$x_{tar} = \begin{bmatrix} 0.001094 \\ -0.007864 \\ 0.003290 \\ 0.011726 \end{bmatrix} \quad x_{fin} = \begin{bmatrix} 0.001070 \\ -0.006730 \\ 0.003289 \\ 0.011034 \end{bmatrix}$$

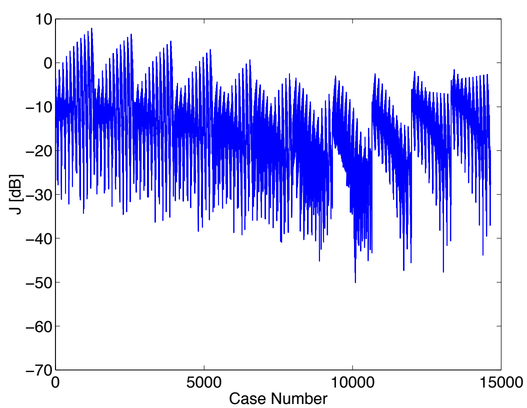
TERMINAL CONSTRAINTS ONLY

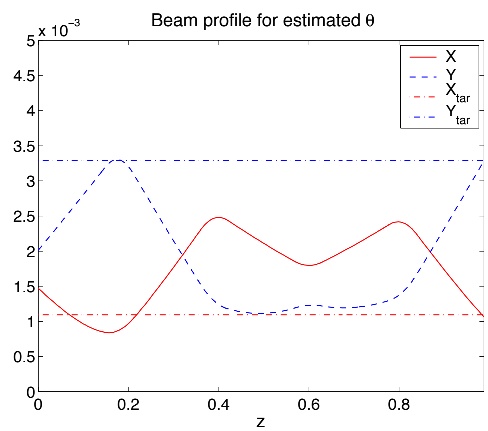
$$K_X = 2000,\ K_Y = 1000,\ K_{dX} = 1,\ K_{dY} = 1, K_{iX} = K_{iY} = 0,$$
$$k_1 = 1,\ k_2 = 1,\ k_3 = 0.$$



$$\boxed{\begin{array}{l}
a_{1,2,3,4} = [2.25 \quad 2 \quad 1.75 \quad 1.5] \ \text{if} \ \text{-}40dB \le J \\
a_{1,2,3,4} = [0.25 \quad 0.75 \quad 0.5 \quad 0.5] \ \text{if} \ \text{-}50dB \le J < \text{-}40dB \\
a_{1,2,3,4} = [0.1 \quad 0.25 \quad 0.1 \quad 0.1] \ \text{if} \ \text{-}55dB \le J < \text{-}50dB \\
a_{1,2,3,4} = [0.05 \quad 0.05 \quad 0.05 \quad 0.05] \ \text{if} \ \text{-}60dB \le J < \text{-}55dB \\
a_{1,2,3,4} = [0.01 \quad 0.01 \quad 0.025 \quad 0.025] \ \text{if} \ J < \text{-}60dB
\end{array}}$$

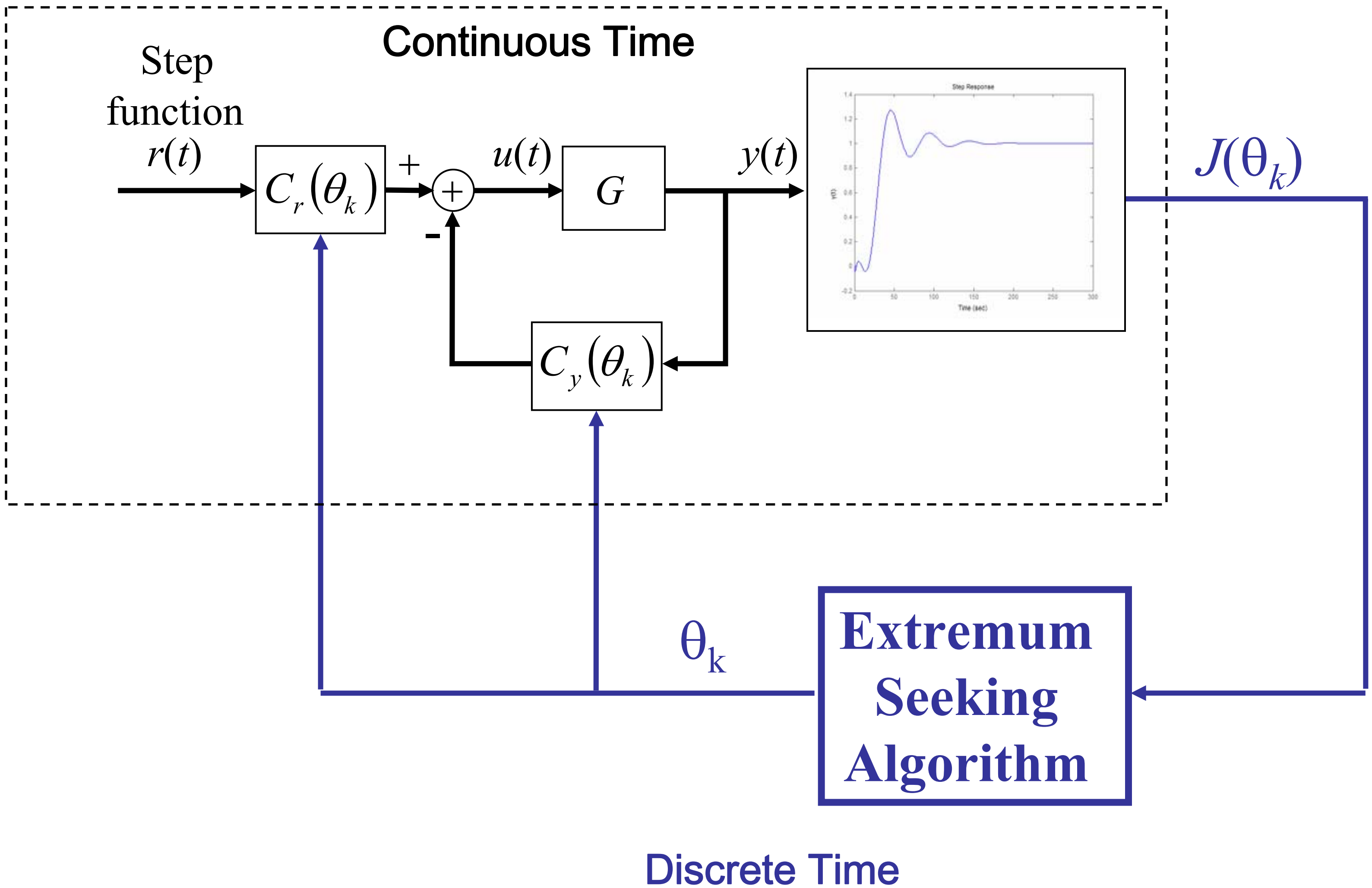






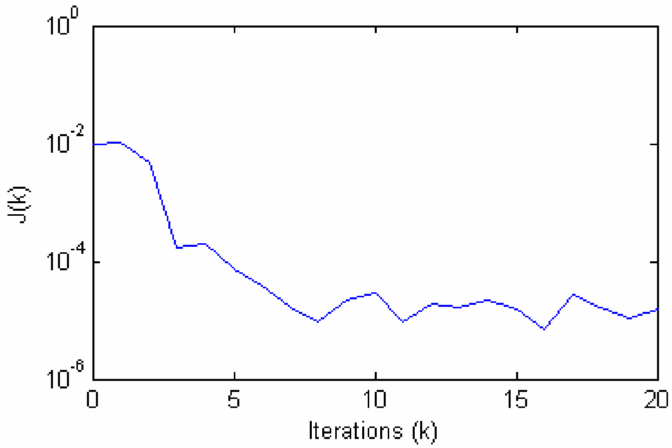
PID Tuning using Extremum Seeking

Extremum Seeking Tuning Scheme

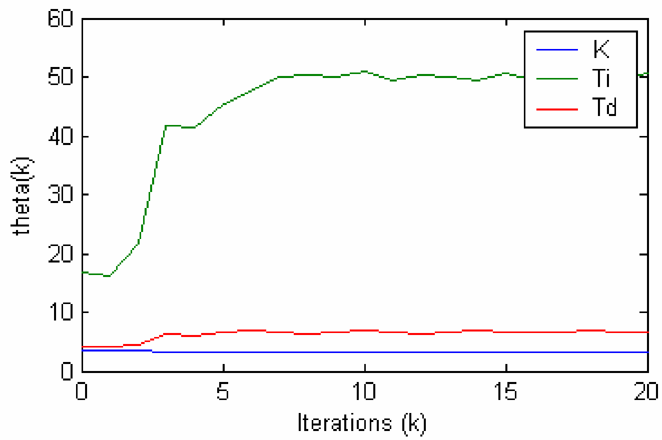


$$G_4(s) = \frac{1-5s}{(1+10s)(1+20s)}$$

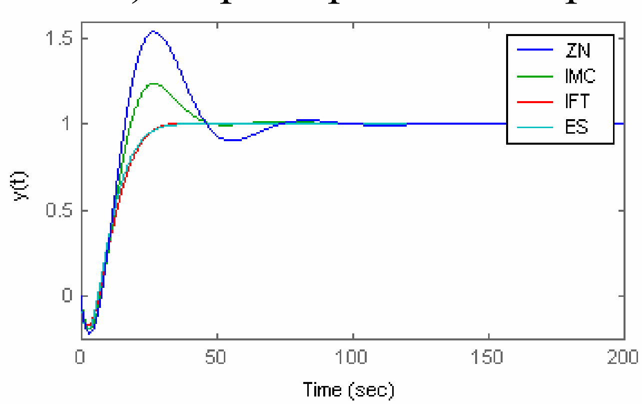
a) Evolution of Cost Function



b) Evolution of PID Parameters



c) Step Response of output



d) Step Response of controller