Real-Time Optimization by Extremum Seeking Control

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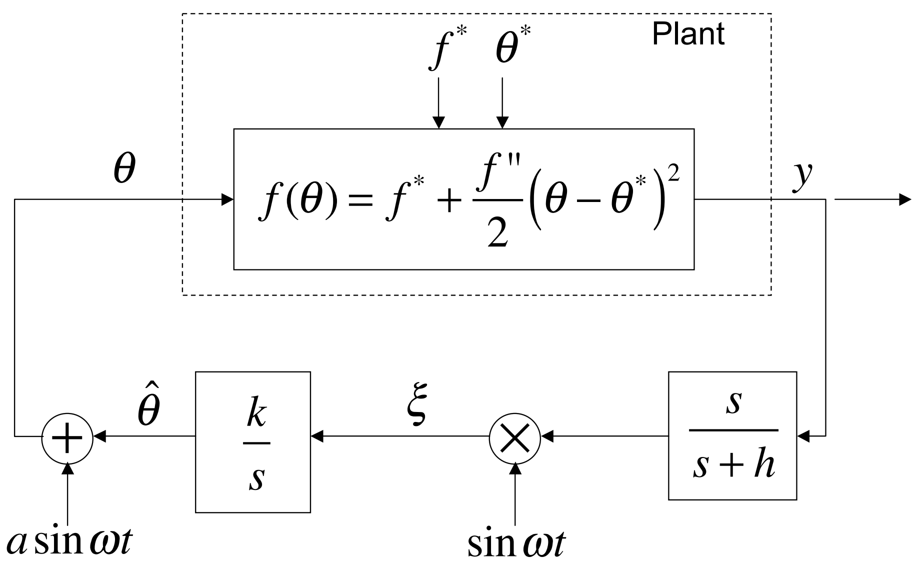
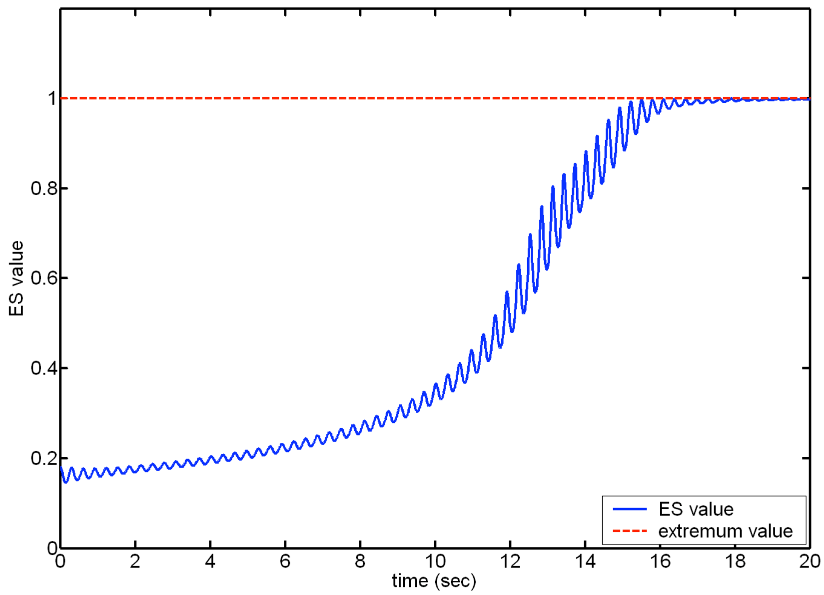
American Control Conference, June 2006

Example of Single-Parameter Maximum Seeking



Example of Single-Parameter Maximum Seeking



Topics - Theory

- History
- Single parameter ES, how it works, and stability analysis by averaging
- ES with plant dynamics and compensators for performance improvement
- Internal model principle for tracking parameter changes
- Multi-parameter ES
- Slope seeking
- ES in discrete time
- Limit cycle minimization via ES

Topics - Applications

- Anti-skid braking
- Compressor instabilities in jet engines
- Combustion instabilities
- Flow separation control in diffusers
- Thermoacoustic coolers
- Automotive engine mapping
- Beam matching in particle accelerators
- Formation flight
- Bioreactors
- PID tuning
- Autonomous vehicles without position sensing

Speaking Schedule

- Introduction, history, single-parameter stability analysis (Krstic)
- Anti-skid braking (Ariyur)
- Bioreactor (Krstic)
- Plant dynamics, compensators, and IMC for tracking parameter changes (Ariyur)
- Limit cycle minimization via ES (Krstic)
- Multi-parameter ES (Ariyur)
- Thermoacoustic coolers (Rotea)
- Automotive engine optimization (Jankovic)
- Formation flight (Ariyur)
- Autonomous vehicles without position sensing (Krstic)
- ES in discrete time (Ariyur)
- Beam matching in particle accelerators (Schuster)
- PID tuning (Krstic)
- Slope seeking (Ariyur)
- Compressor instabilities in jet engines (Krstic, Ariyur)
- Combustion instabilities (Banaszuk)
- Flow separation control in diffusers (Banaszuk)

History

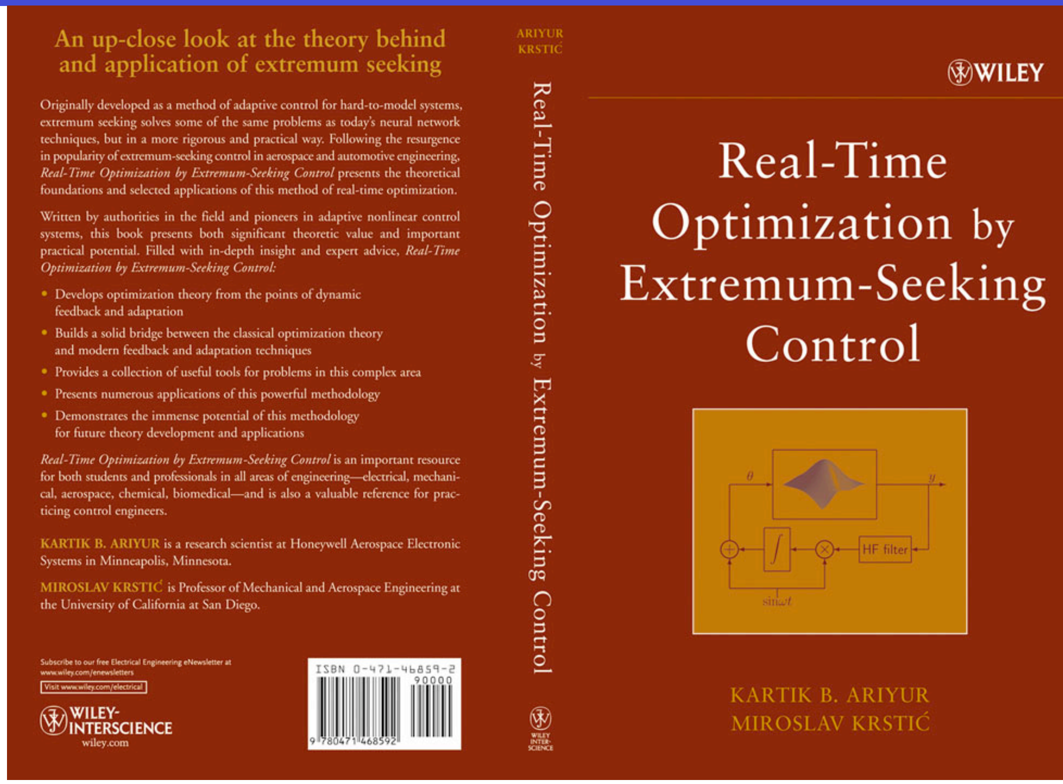
- **Leblanc (1922)** - electric railways
- **Early Russian literature (1940's)** - many papers
- **Drapper and Li (1951)** - application to IC engine spark timing tuning
- **Tsien (1954)** - a chapter in his book on Engineering Cybernetics
- **Feldbaum (1959)** - book *Computers in Automatic Control Systems*
- **Blackman (1962 chap. in book by Westcott)** - nice intuitive presentation of ES
- **Wilde (1964)** - a book
- **Chinaev (1969)** - a handbook on self-tuning systems
- Papers by[Morosanov], [Ostrovskii], [Pervozvanskii], [Kazakevich], [Frey, Deem, and Altpeter], [Jacobs and Shering], [Korovin and Utkin] - late 50s - early 70's
- **Meerkov (1967, 1968)** - papers with averaging analysis
- **Sternby (1980)** - survey
- **Astrom and Wittenmark (1995 book)** - rates ES as one of the most promising areas for adaptive control

Recent Developments

- **Krstic and Wang (2000, *Automatica*)** - stability proof for single-parameter general dynamic nonlinear plants
- **Choi, Ariyur, Wang, Krstic** - discrete-time, limit cycle minimization, IMC for parameter tracking, etc.
- **Rotea; Walsh; Ariyur** - multi-parameter ES
- **Ariyur** - slope seeking
- **Tan, Nesic, Mareels (2005)** - semi-global stability of ES
- Other approaches: **Guay, Dochain, Titica**, and coworkers; **Zak, Ozguner**, and coworkers; **Banavar, Chichka, Speyer**; **Popovic, Teel**; etc.

- **Applications** not presented in this workshop:
  - Electromechanical valve actuator (Peterson and Stephanopoulou)
  - Artificial heart (Antaki and Paden)
  - Exercise machine (Zhang and Dawson)
  - Chemical reactors and petroleum processing (several research groups)
  - Shape optimization for magnetic head in hard disk drives (UCSD)

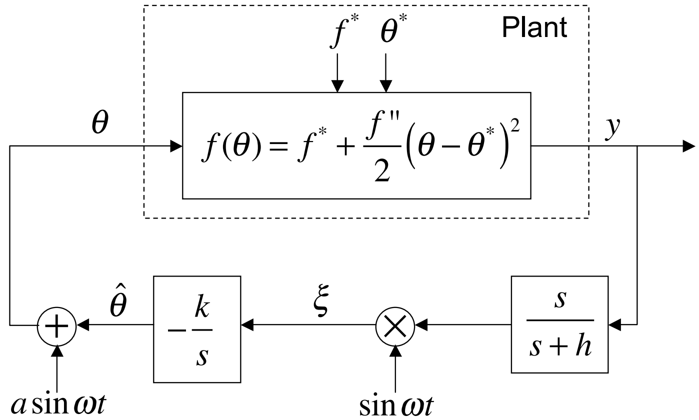
ES Book



Workshop Topics Covered in the Book

- Introduction, history, single-parameter stability analysis (Krstic)
- Anti-skid braking (Ariyur)
- Bioreactor (Krstic)
- Plant dynamics, compensators, and IMC for tracking parameter changes (Ariyur)
- Limit cycle minimization via ES (Krstic)
- Multi-parameter ES (Ariyur)
- Thermoacoustic coolers (Rotea)
- Automotive engine mapping (Jankovic)
- Formation flight (Ariyur)
- Autonomous vehicles without position sensing (Krstic)
- ES in discrete time (Ariyur)
- Beam matching in particle accelerators (Schuster)
- PID tuning (Killingsworth)
- Slope seeking (Ariyur)
- Compressor instabilities in jet engines (Krstic, Ariyur)
- Combustion instabilities (Banaszuk)
- Flow separation control in diffusers (Banaszuk)

Basic Extremum Seeking - Static Map



y = output to be minimized

f^* = minimum of the map

f'' = second derivative (positive - $f(\theta)$ has a min.)

θ^* = unknown parameter

$\hat{\theta}$ = estimate of θ^*

k = adaptation gain (positive) of the integrator $\frac{1}{s}$

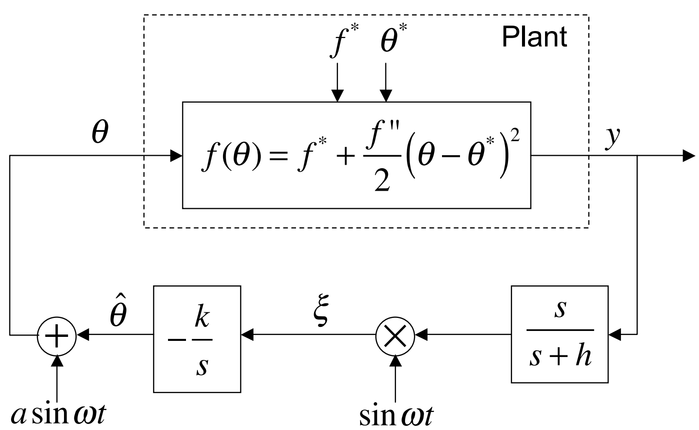
a = amplitude of the probing signal

ω = frequency of the probing signal

h = cut-off frequency of the "washout filter" $\frac{s}{s+h}$

$+/\times$ = modulation/demodulation

How Does It Work?



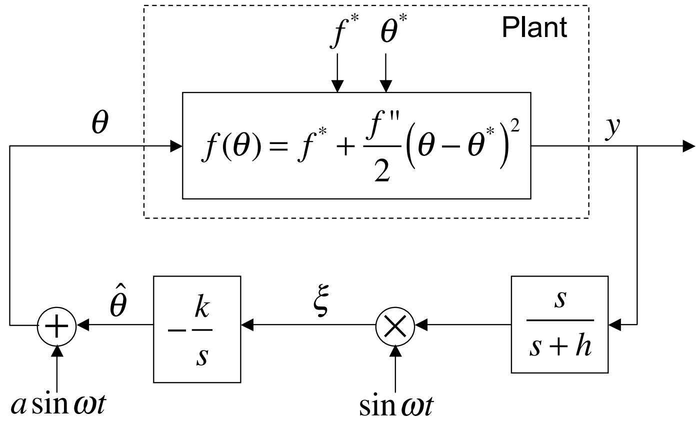
Estimation error: $\tilde{\theta} = \theta^* - \hat{\theta}$

$$y = f^* + \frac{a^2 f''}{4} + \frac{f''}{2}\tilde{\theta}^2 - af''\tilde{\theta}\sin\omega t + \frac{a^2 f''}{4}\cos 2\omega t$$

Loc. Analysis - neglect quadratic terms:

$$y \approx f^* + \frac{a^2 f''}{4} - af''\tilde{\theta}\sin\omega t + \frac{a^2 f''}{4}\cos 2\omega t$$

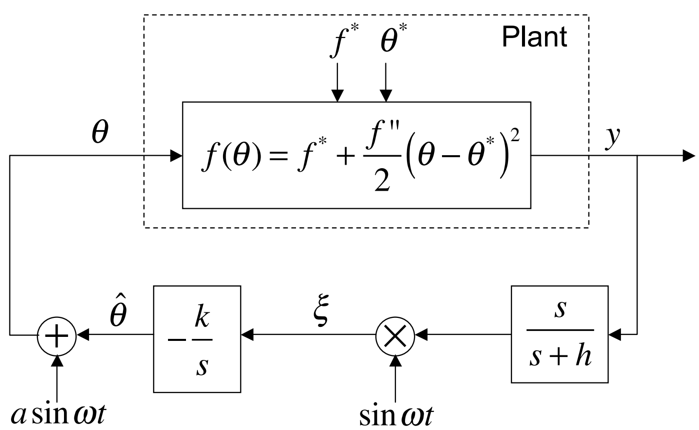
How Does It Work?



$$y \approx f^* + \frac{a^2 f''}{4} - a f'' \tilde{\theta} \sin \omega t + \frac{a^2 f''}{4} \cos 2\omega t$$

$$\frac{s}{s+h}[y] \approx - a f'' \tilde{\theta} \sin \omega t + \frac{a^2 f''}{4} \cos 2\omega t$$

How Does It Work?

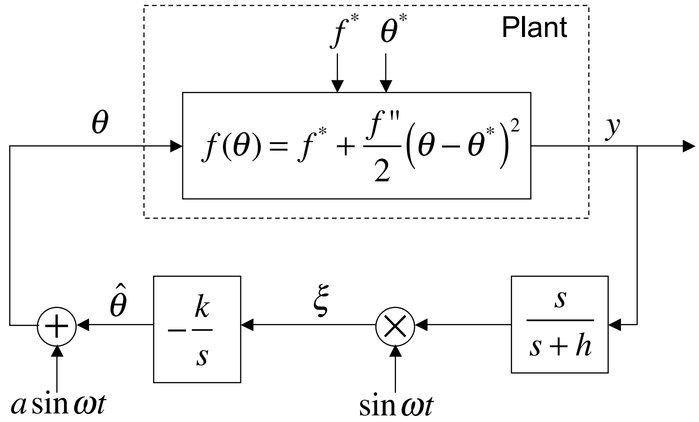


Demodulation:

$$\xi = \sin \omega t \frac{s}{s+h}[y] \approx -a f'' \tilde{\theta} \sin^2 \omega t + \frac{a^2 f''}{4} \cos 2\omega t \sin \omega t$$

$$\xi \approx -\frac{a^2 f''}{4} \tilde{\theta} + \frac{a^2 f''}{4} \tilde{\theta} \cos 2\omega t + \frac{a^2 f''}{8} \left(\sin \omega t - \sin 3\omega t\right)$$

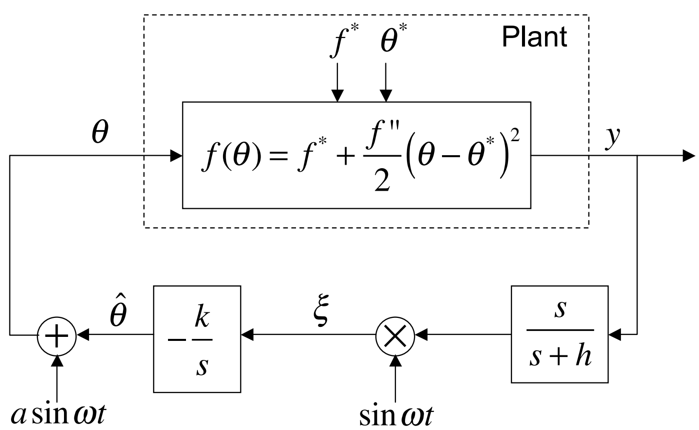
How Does It Work?



Since $\tilde{\theta} = \theta^* - \hat{\theta}$

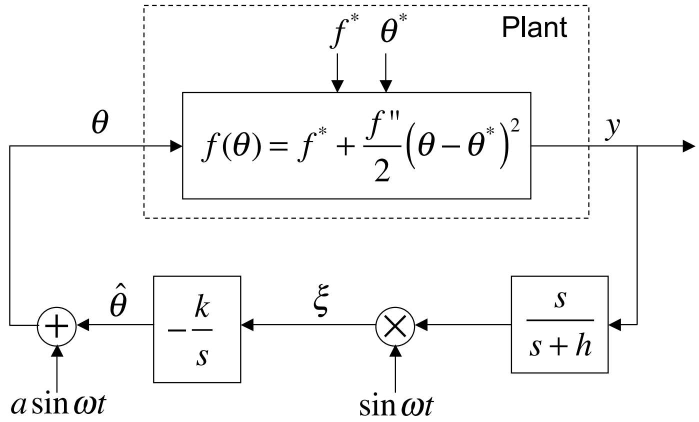
then $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$

How Does It Work?



$$\tilde{\theta} \approx \frac{k}{s}\left[-\frac{a^2 f''}{4}\tilde{\theta} + \underbrace{\frac{a^2 f''}{4}\tilde{\theta}\cos 2\omega t + \frac{a^2 f''}{8}(\sin \omega t - \sin 3\omega t)}_{\text{high frequency terms - attenuated by integrator}} \right]$$

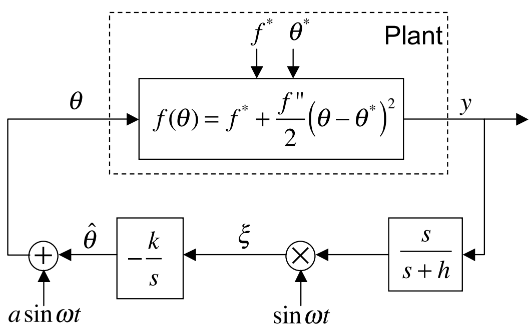
How Does It Work?



$$\boxed{\dot{\tilde{\theta}} \approx -\frac{ka^2 f''}{4}\tilde{\theta}}$$

Stable because $k, a, f'' > 0$

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

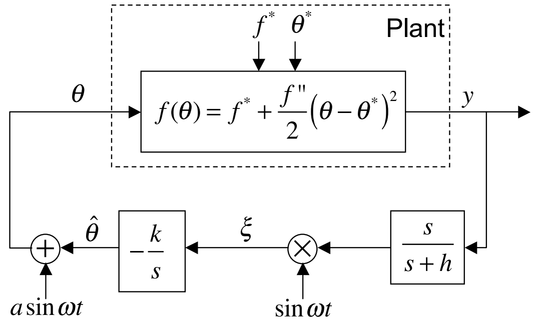
$$\tau = \omega t$$

Full nonlinear time-varying model:

$$\frac{d}{d\tau}\tilde{\theta} = \frac{k}{\omega}\left(\frac{f''}{2}\left(\tilde{\theta} - a\sin\tau\right)^2 - e\right)\sin\tau$$

$$\frac{d}{d\tau}e = \frac{h}{\omega}\left(-e - \frac{f''}{2}\left(\tilde{\theta} - a\sin\tau\right)^2\right)$$

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

$$\tau = \omega t$$

Average system:

$$\frac{d}{d\tau}\tilde{\theta}_{\text{av}} = -\frac{kaf''}{2\omega}\tilde{\theta}_{\text{av}}$$

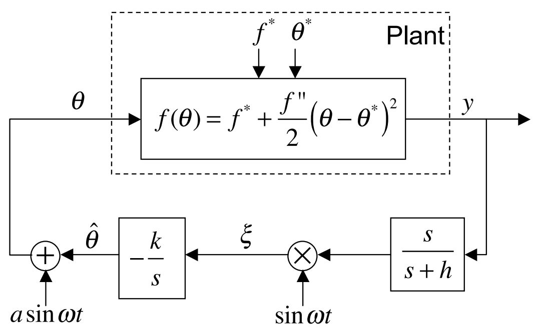
$$\frac{d}{d\tau}e_{\text{av}} = \frac{h}{\omega}\left(-e_{\text{av}} - \frac{f''}{2}\left(\tilde{\theta}^2_{\text{av}} + \frac{a^2}{2}\right)\right)$$

Average equilibrium:

$$\tilde{\theta}_{\text{av}} = 0$$

$$e_{\text{av}} = -\frac{a^2 f''}{4}$$

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

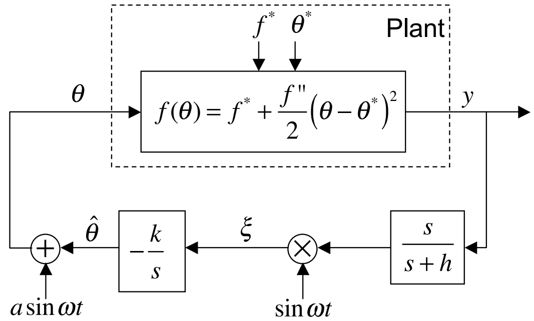
$$e = f^* - \frac{h}{s+h}[y]$$

$$\tau = \omega t$$

Jacobian of the average system:

$$J_{\text{av}} = \begin{bmatrix} -\frac{kaf''}{2\omega} & 0 \\ 0 & -\frac{h}{\omega} \end{bmatrix}$$

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

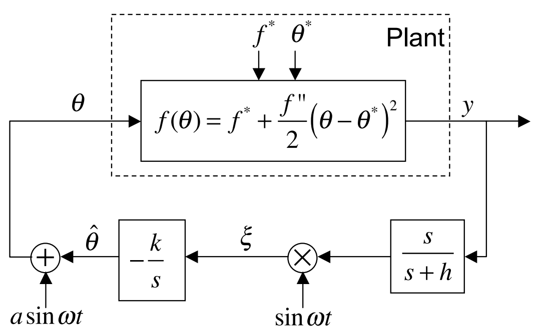
$$\tau = \omega t$$

Theorem. *For sufficiently large* ω *there exists a unique exponentially stable periodic solution of period* 2π/ω *and it satisfies*

$$\left|\tilde{\theta}_{2\pi/\omega}(t)\right| + \left|e_{2\pi/\omega}(t) - \frac{a^2 f''}{4}\right| \le O\left(\frac{1}{\omega}\right), \qquad \forall t \ge 0$$

Speed of convergence proportional to $1/\omega$, a^2, k, f''

Stability Proof by Averaging



$$\tilde{\theta} = \theta^* - \hat{\theta}$$

$$e = f^* - \frac{h}{s+h}[y]$$

$$\tau = \omega t$$

Output performance:

$$y - f^* \to f''O\left(\frac{1}{\omega^2} + a^2\right)$$