Target Seeking Without Position Measurement: Application to Nonholonomic Vehicles

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Outline

- Introduction
- Overview of Extremum Seeking (ES)
- Introductory Example: Point Mass
- Nonholonomic Vehicle: Unicycle Model
    - Tuning Surge Velocity (collocated sensor)
    - Tuning Angular Velocity (non-collocated sensor)
- Summary
- Conclusions

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Introduction

- Motivation
  - Control a vehicle to locate the source of an unknown signal
- Previous Work
  - Porat and Nehorai - vehicle has position information
  - Ogren, Fiorelli and Leonard - “group” gradient estimation
  - Justh and Krishnaprasad - convergent vehicle formation
  - Klein and Morgansen - trajectory tracking
  - Marshal, Broucke and Francis - cyclic pursuit problem

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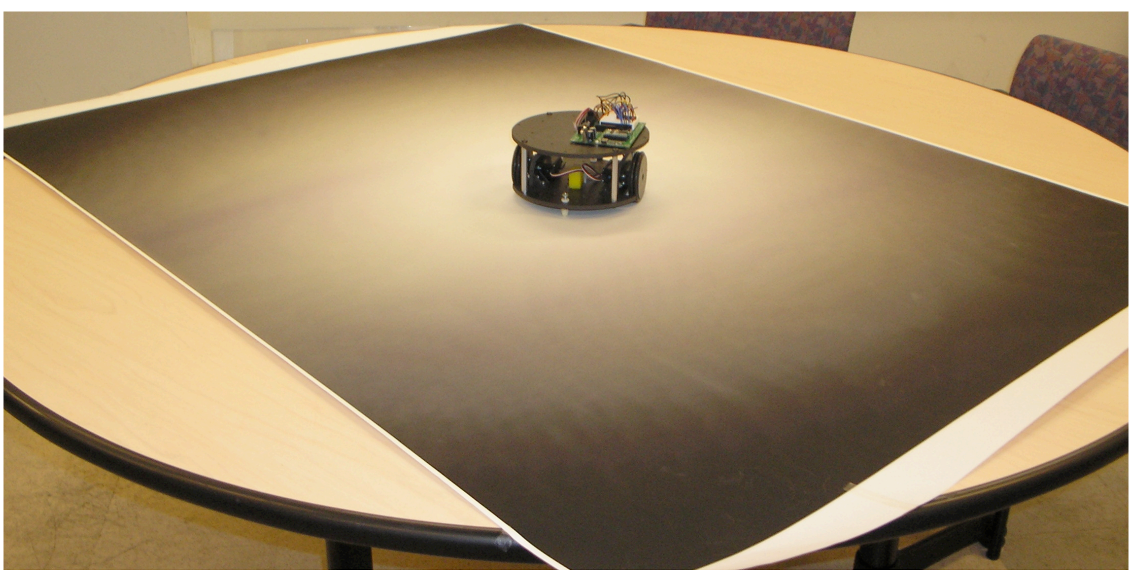
Robot on Map



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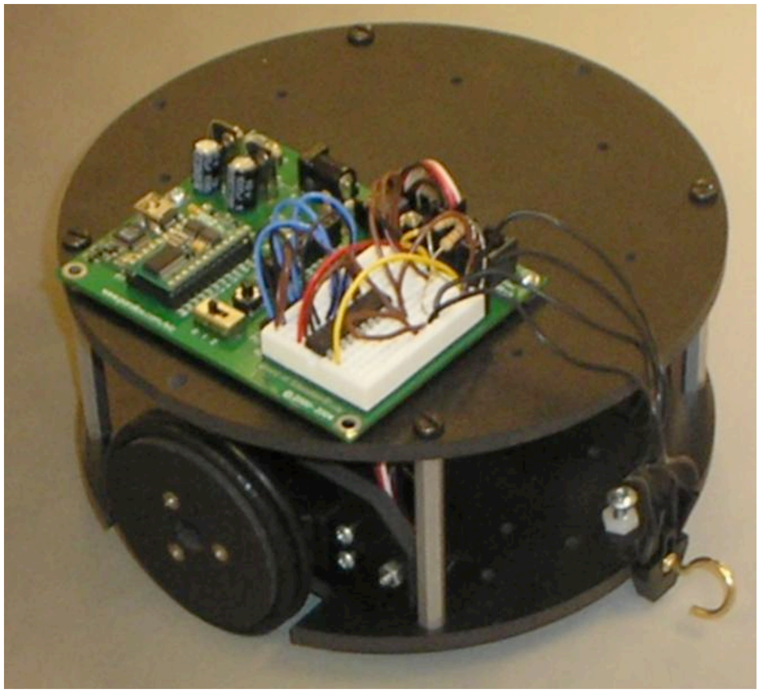
Robot



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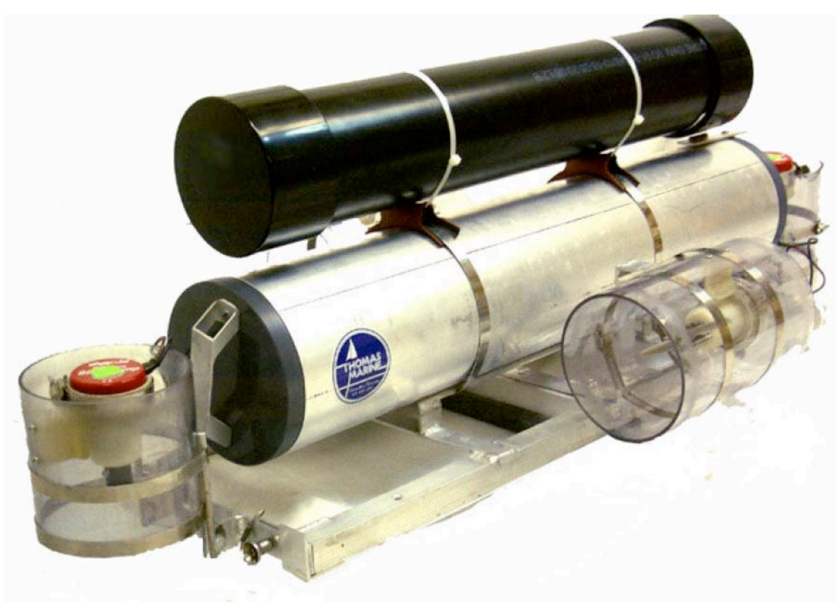
Submarine



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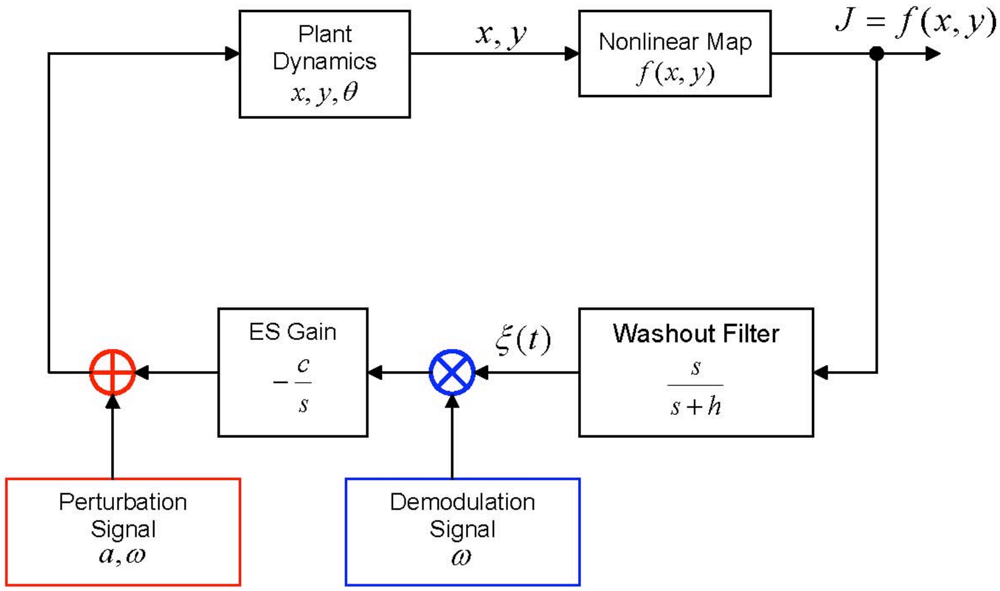
Overview of ES



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Overview of ES

- The designer will choose a, ω, h and the gain c
- *ES* will tune $\xi(t)$, essentially tuning the control inputs to the plant
- In simulations, the nonlinear map has the form:

$$f(x, y) = \frac{f^*}{1 + q_x(x - x^*)^2 + q_y(y - y^*)^2}$$

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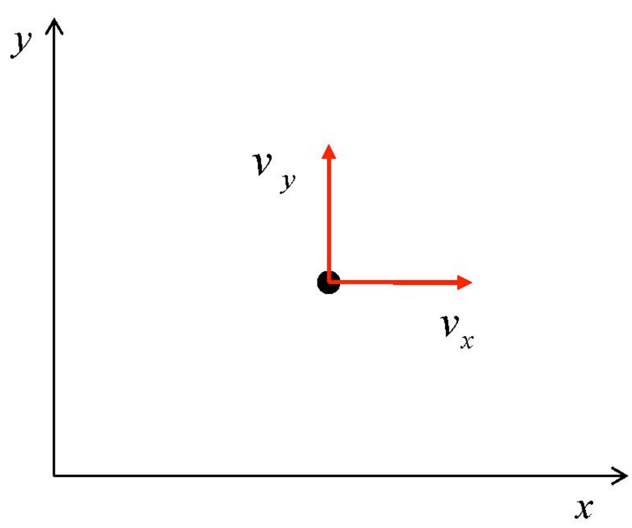
Introductory Example: Point Mass

Model

Point Mass



Dynamics

$$\dot{x} = v_x$$
$$\dot{y} = v_y$$

Inputs

$$v_x, v_y$$

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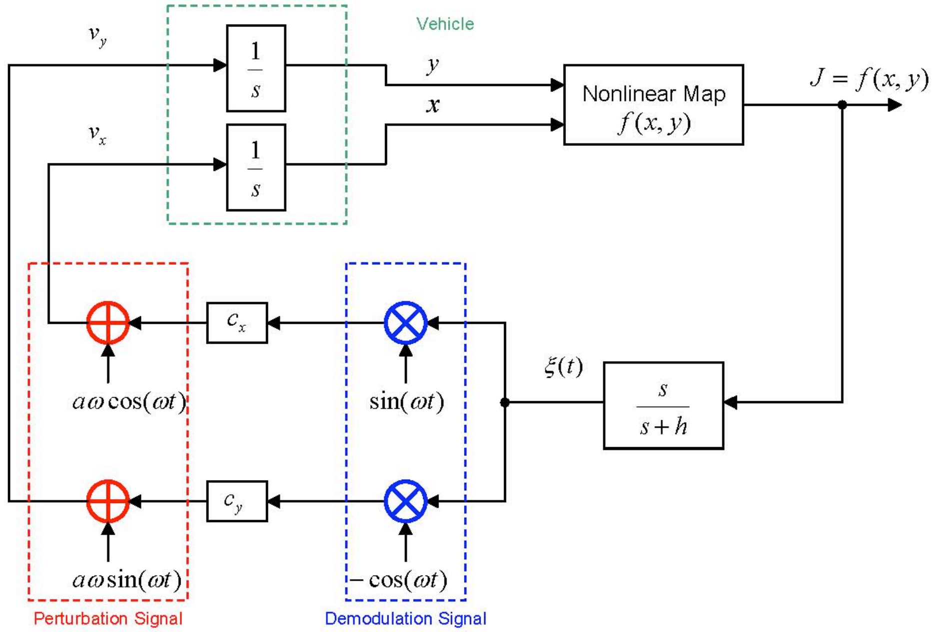
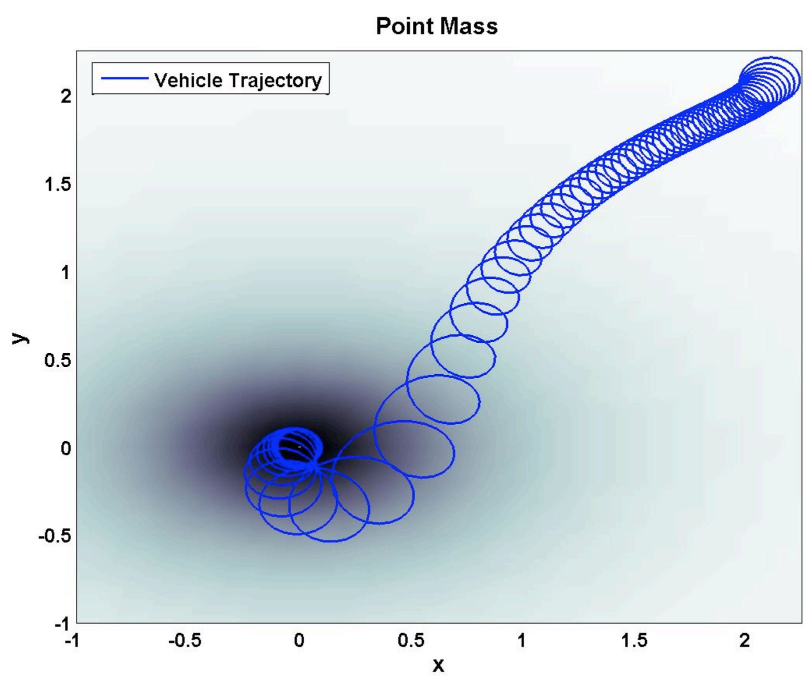
Block Diagram

Point Mass

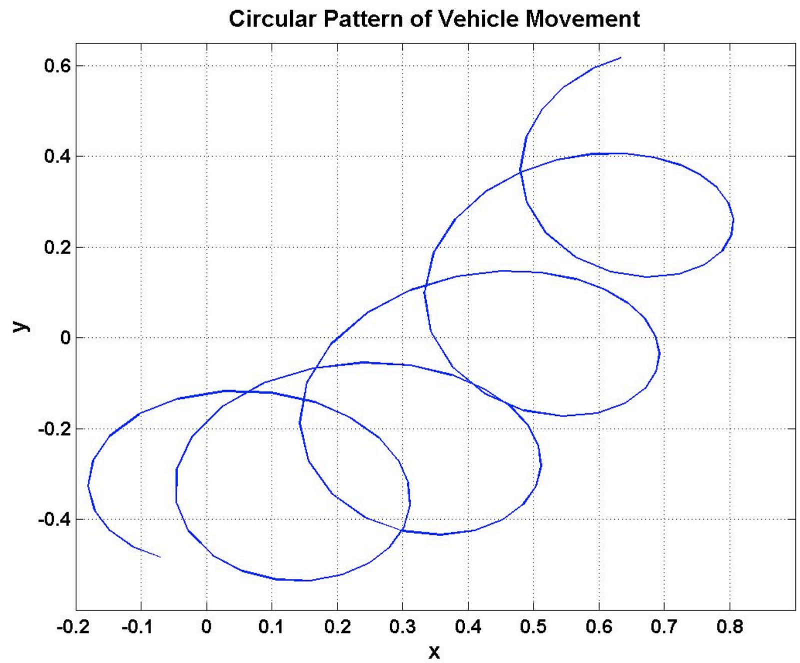


Simulation Results

Point Mass



Simulation Results



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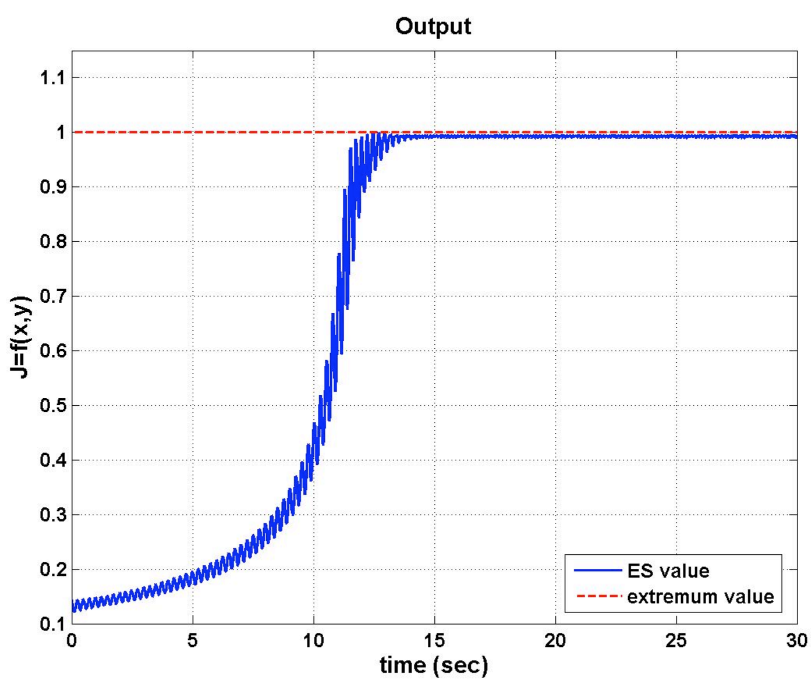
Simulation Results

Point Mass



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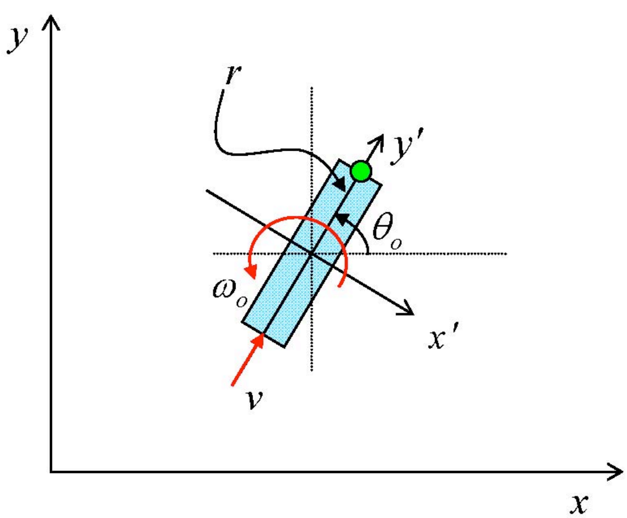
Nonholonomic Vehicle: Unicycle Model

Model

Unicycle: non-collocated



Sensor Dynamics

$$\dot{x}_s = v\cos\theta_o - r\dot{\theta}_o\sin\theta_o$$

$$\dot{y}_s = v\sin\theta_o + r\dot{\theta}_o\cos\theta_o$$

$$\dot{\theta}_o = \omega_o = \frac{d}{dt}\theta_o$$

Inputs

v, ω_o

System is linearly **uncontrollable** (from inputs v, ω_o) and **unobservable** (from the output $f(x,y)$ at its peak)

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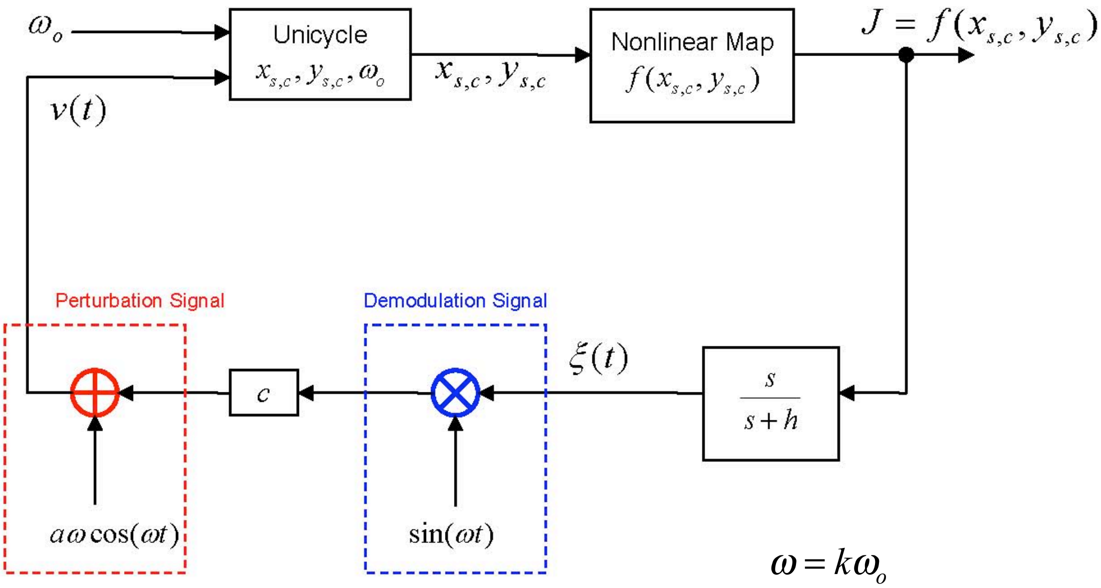
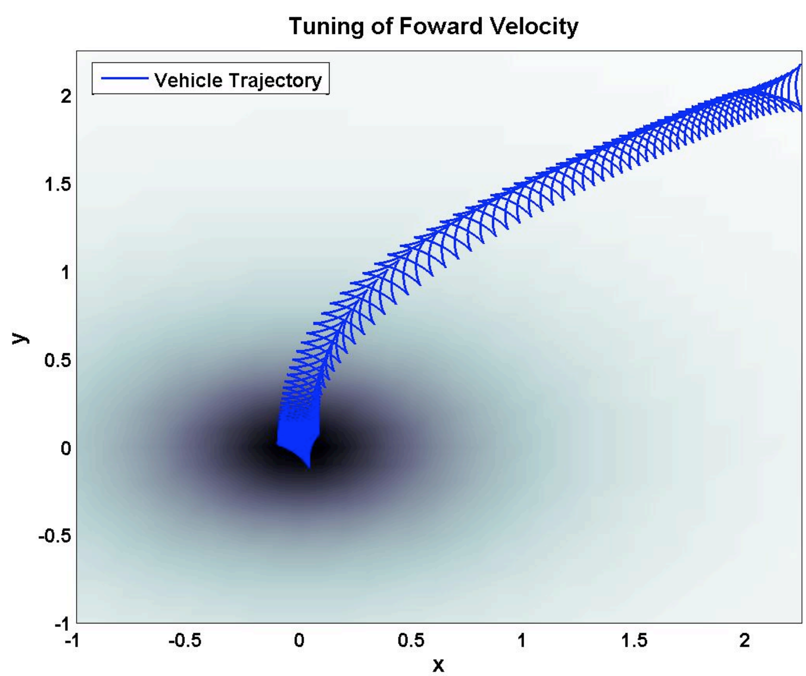
Tuning the Forward Velocity

Unicycle: collocated



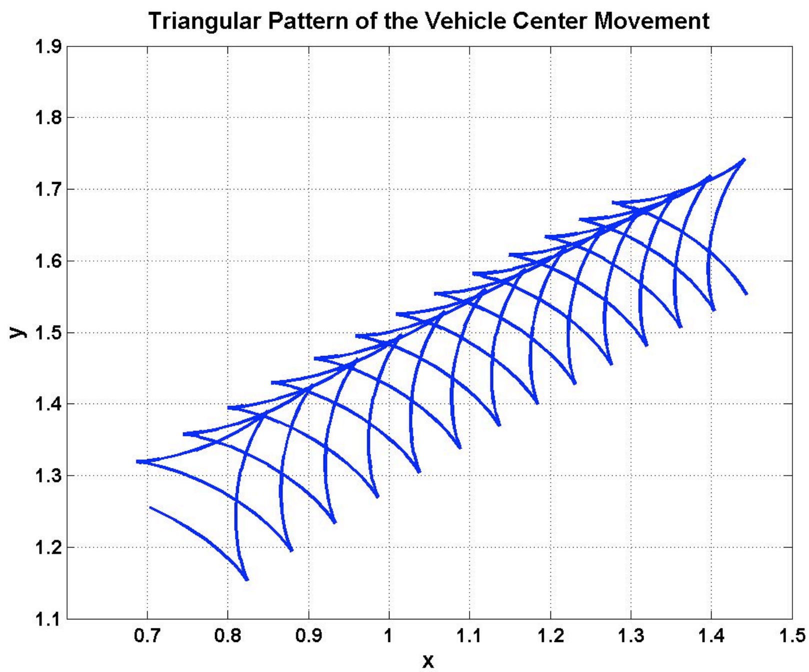
Simulation Results

Unicycle: collocated



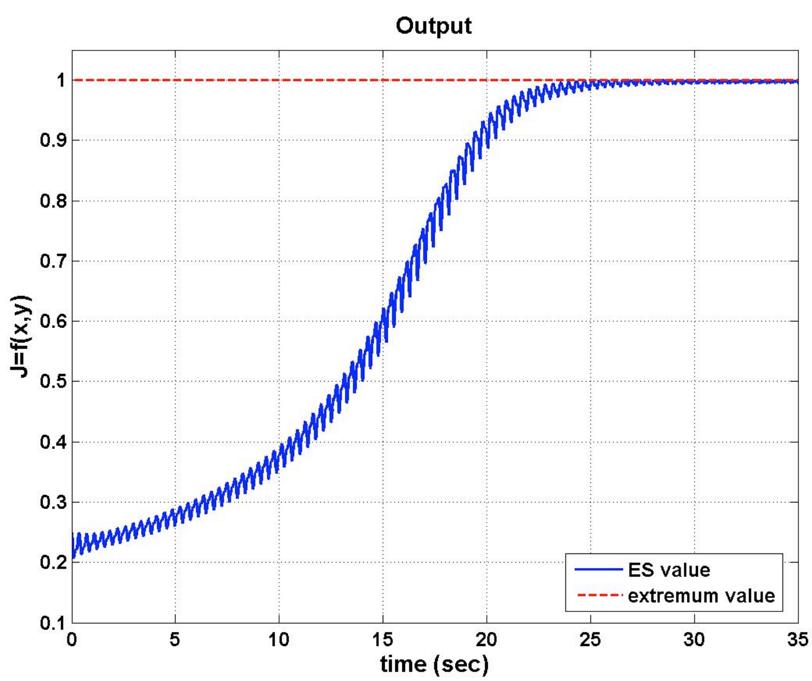
Simulation Results

Unicycle: collocated



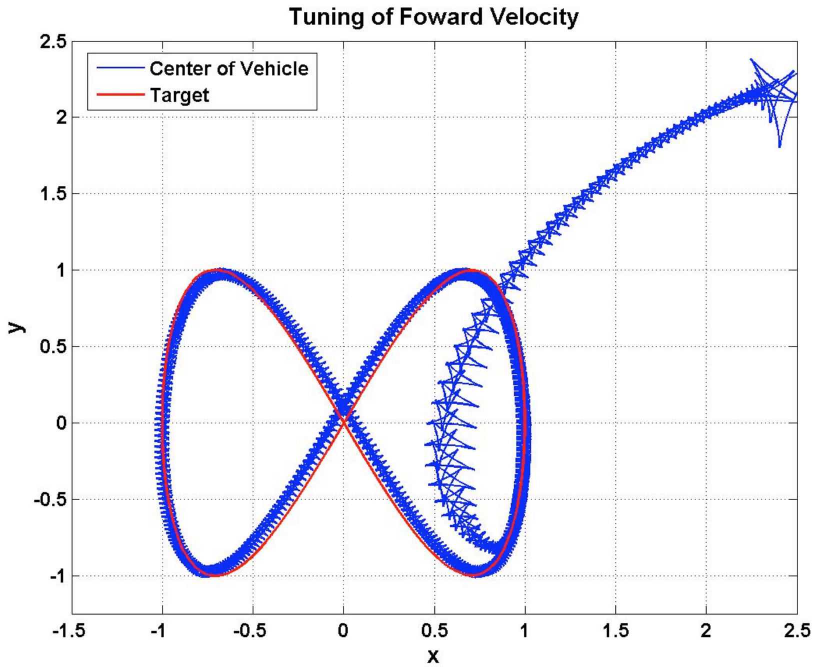
Simulation Results

Unicycle: collocated



Simulation Results

Unicycle: collocated



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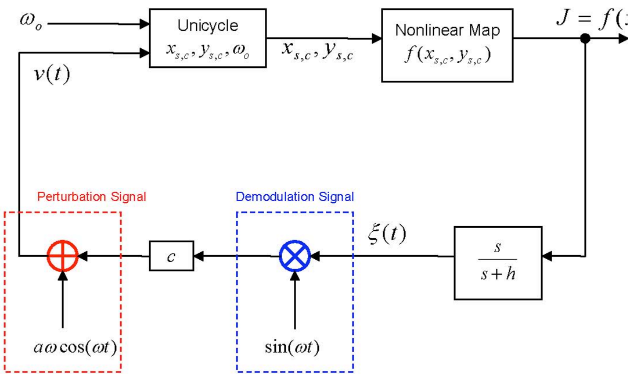
Stability Proof by Averaging Tuning of Forward Velocity

Stability Proof by Averaging

Full Nonlinear Time-Varying Model



$$\tilde{x} = x_c - x^* - a\sin(\omega t)\cos(\omega_o t)$$
$$\tilde{y} = y_c - y^* - a\sin(\omega t)\sin(\omega_o t)$$
$$e = \frac{h}{s+h}[J] - f^*$$
$$\xi = \frac{s}{s+h}[J] = J - f^* - e$$
$$\tau = \omega t$$

$$\frac{d}{d\tau}\tilde{x} = \frac{1}{\omega}\left[c\sin\tau\cos\left(\frac{\tau}{k}\right)\xi + a\omega_o\sin\tau\sin\left(\frac{\tau}{k}\right)\right]$$
$$\frac{d}{d\tau}\tilde{y} = \frac{1}{\omega}\left[c\sin\tau\sin\left(\frac{\tau}{k}\right)\xi - a\omega_o\sin\tau\cos\left(\frac{\tau}{k}\right)\right]$$
$$\frac{d}{d\tau}e = \frac{h}{\omega}\xi$$

where

$$\xi = -q_x\left(\tilde{x} + a\sin\tau\cos\left(\frac{\tau}{k}\right)\right)^2 - q_y\left(\tilde{y} + a\sin\tau\sin\left(\frac{\tau}{k}\right)\right)^2 - e$$

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Stability Proof by Averaging

Applying Averaging

$$\frac{d}{d\tau}\tilde{x}_{avg} = \frac{1}{\omega}\frac{1}{2\pi}\int_0^{2\pi}\left[c\sin(k\tau)\cos\tau\,\xi + a\omega_o\sin(k\tau)\sin\tau\right]d\tau$$
$$= -\frac{1}{\omega}\frac{1}{2k\pi}\int_0^{2\pi} c\sin(k\tau)\cos\tau\left(q_x\tilde{x}^2 + q_y\tilde{y}^2 + e\right)d\tau$$
$$-\frac{1}{\omega}\frac{1}{2k\pi}\int_0^{2\pi} c\sin(k\tau)\cos\tau\left[2q_x\tilde{x}a\sin(k\tau)\cos\tau + 2q_y\tilde{y}a\sin(k\tau)\sin\tau\right]d\tau$$
$$-\frac{1}{\omega}\frac{1}{2k\pi}\int_0^{2\pi} c\sin(k\tau)\cos\tau\left[q_x a^2\sin^2(k\tau)\cos^2\tau + 2q_y a^2\sin^2(k\tau)\sin^2\tau\right]d\tau$$
$$+\frac{1}{\omega}\frac{1}{2k\pi}\int_0^{2\pi} a\omega_o\sin(k\tau)\sin\tau\,d\tau$$
$$= -\frac{1}{2\omega}acq_x\tilde{x}_{avg}$$

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Stability Proof by Averaging

Equilibrium of Average System

$$\frac{d}{d\tau}\tilde{x}_{avg} = -\frac{1}{2\omega} acq_x \tilde{x}_{avg}$$
$$\frac{d}{d\tau}\tilde{y}_{avg} = -\frac{1}{2\omega} acq_y \tilde{y}_{avg}$$
$$\frac{d}{d\tau}e_{avg} = -\frac{h}{\omega}\left[q_x \tilde{x}_{avg}^2 + q_y \tilde{y}_{avg}^2 + e_{avg} + \left(\frac{a^2}{4}\right)(q_x + q_y)\right]$$

The equilibrium of the average model is:

$$\tilde{x}_{avg}^e = 0,$$
$$\tilde{y}_{avg}^e = 0,$$
$$e_{avg}^e = -\frac{a^2}{4}(q_x + q_y)$$

$$J_{avg}\Big|_{equil.} = \frac{1}{2\omega}\begin{bmatrix} -acq_x & 0 & 0 \\ 0 & -acq_y & 0 \\ 0 & 0 & -2h \end{bmatrix}$$

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Stability Proof by Averaging

Theorem

For sufficiently large ω there exists a unique exponentially stable periodic solution of period 2π/ω and it satisfies

$$\left\| \begin{bmatrix} \tilde{x}^{\frac{2\pi}{\omega}} \\ \tilde{y}^{\frac{2\pi}{\omega}} \\ e^{\frac{2\pi}{\omega}} + \frac{a^2}{4}(q_x + q_y) \end{bmatrix} \right\| \le O(1/\omega), \qquad \forall \ t \ge 0$$

Speed of convergence proportional to $1/\omega$, a^2, c, q_x, q_y

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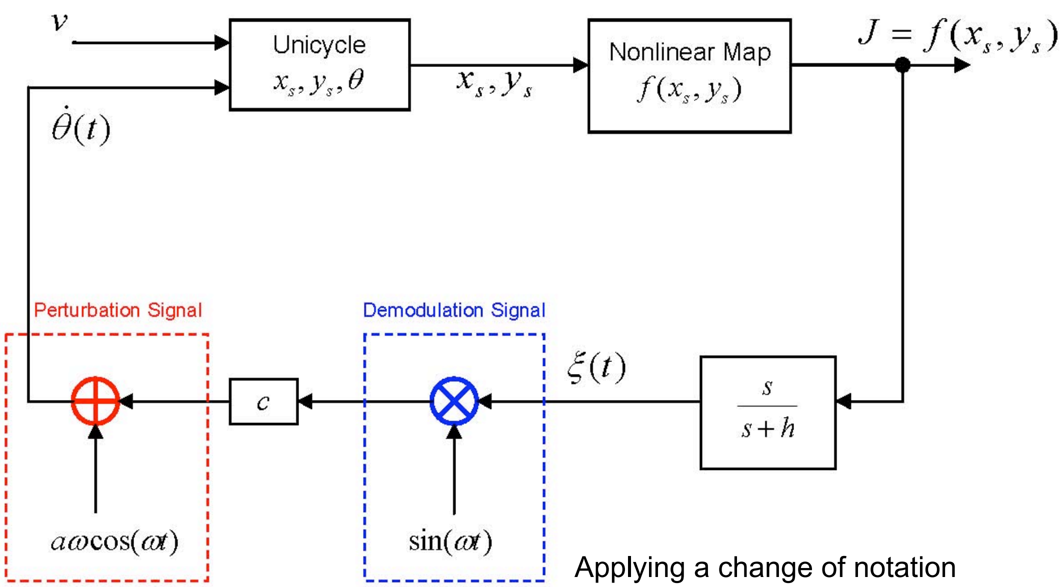
Tuning the Angular Velocity

Unicycle: non-collocated



Applying a change of notation

$$\theta = \theta_o$$

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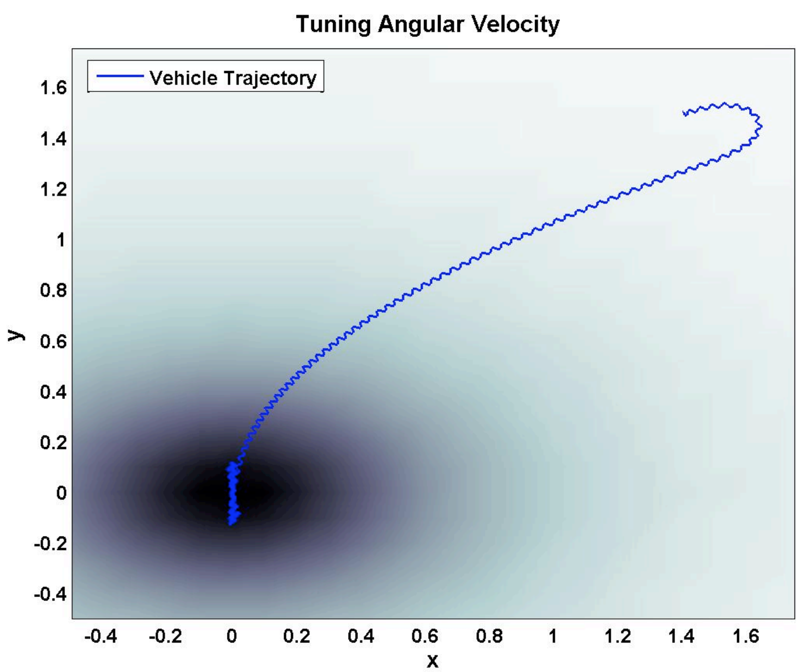
Simulation Results

Unicycle: non-collocated



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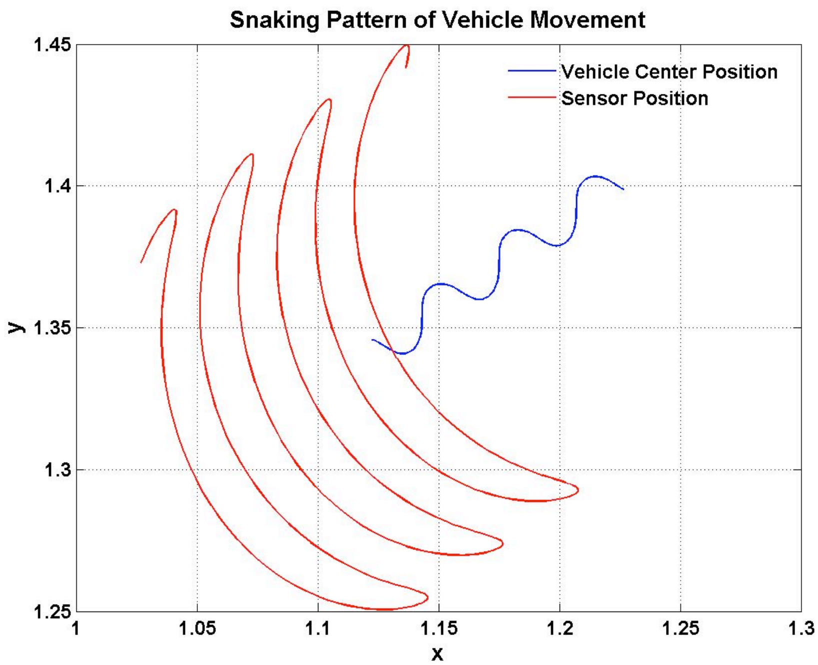
Simulation Results

Unicycle: non-collocated



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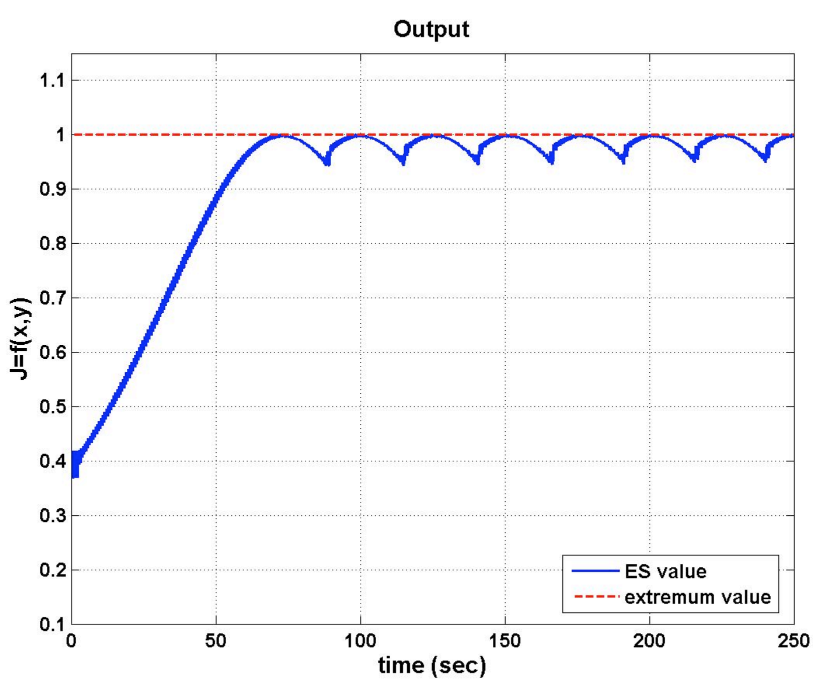
Simulation Results

Unicycle: non-collocated



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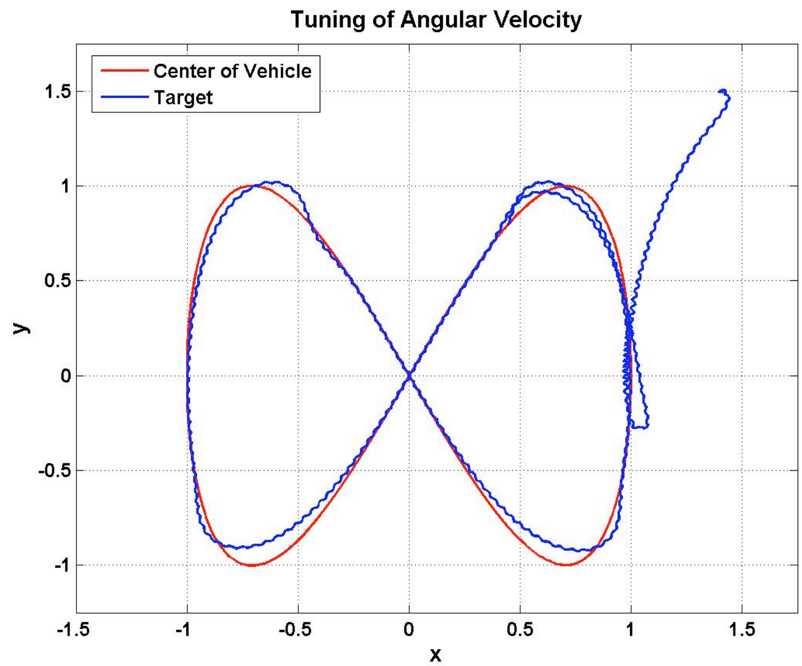
Simulation Results

Unicycle: non-collocated



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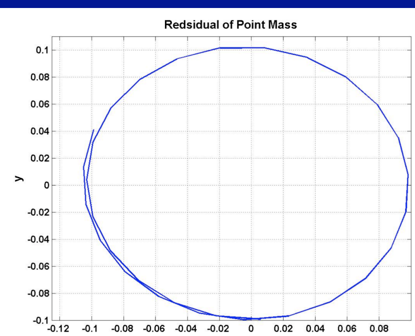
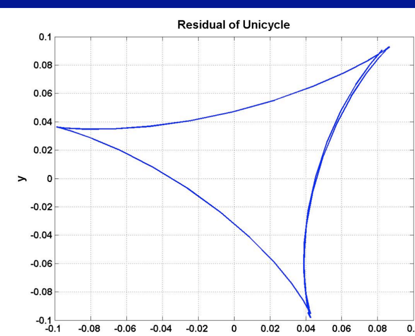
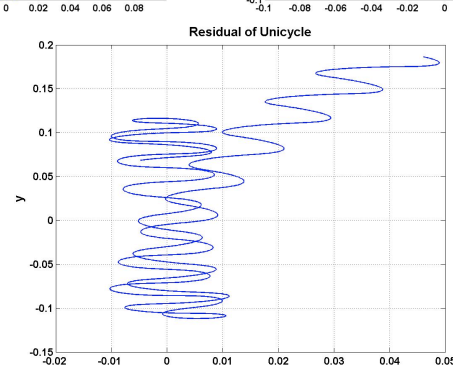
Summary







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Conclusion

- ES has been used to…
  - Control a nonholonomic autonomous vehicle to locate the source of an unknown signal

- Future work includes…
  - Extending simulations to include diffusive sources
  - Continuing work on robots
  - Expanding the capabilities of the GUI

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