Extremum-Seeking Control of Flow Separation in a Planar Diffuser

Andrzej Banaszuk

United Technologies Research Center, E. Hartford, CT, U.S.A.

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Extremum-seeking control used to optimize performance & explore beneficial flow structures in separated flows

Outline:

1. Problem motivation:
   - Multi-frequency control creates “beneficial” vortex interactions
   - Need algorithm to automatically select optimal parameters

2. Extremum-seeking control
   - Principle
   - Stability

3. Demonstration of extremum-seeking for optimization of diffuser pressure recovery

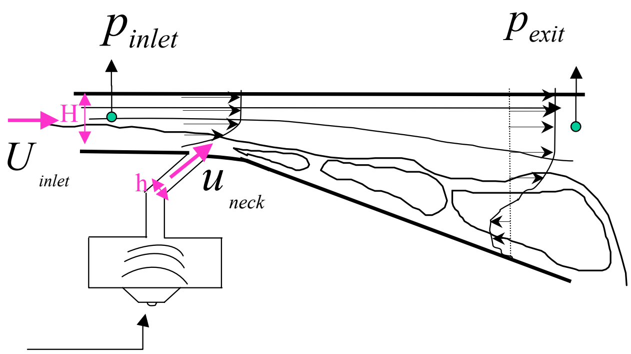
Objective of pressure recovery control

Performance

$$C_p(t) = \frac{p_{exit} - p_{inlet}}{\frac{1}{2}\rho U^2_{inlet}}$$

Control effort

$$C_\mu(t) = \frac{u^2_{neck}\, h}{U^2_{inlet}\, H}$$



$$\frac{\overline{C}_p}{\overline{C}_\mu} = \frac{\text{bang}}{\text{buck}}$$

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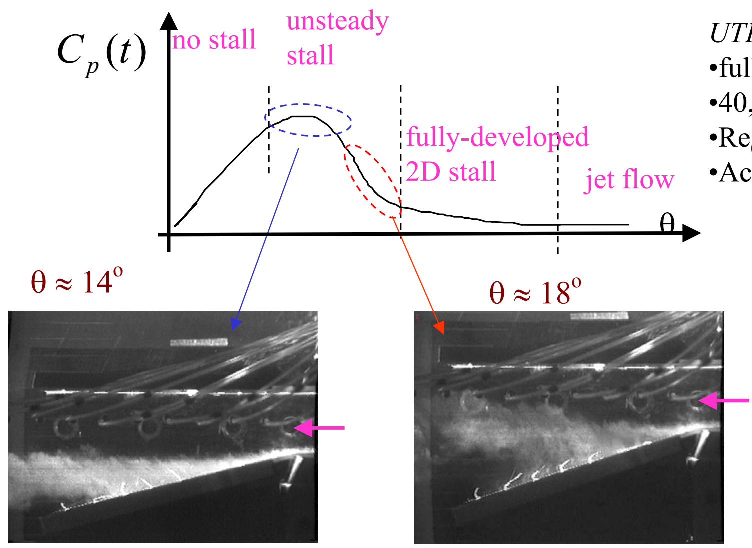
Experimental Setup

Pressure recovery as function of diffuser angle (no control)



UTRC diffuser rig

- fully turbulent BL
- $40{,}000 < Re_H < 140{,}000$
- $Re_{\theta e} > 300$; $M < 0.1$
- Actuation: $C_\mu \sim 0.001$

- Optimum uncontrolled performance
- Insignificant improvement with control

- Poor uncontrolled performance
- Significant improvement with control

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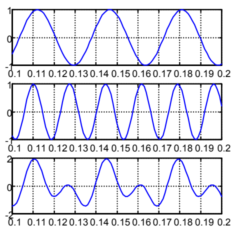
Two frequency control creates “beneficial” vortex interaction

Control signal is $U(t)=A_1\sin(2\pi\ f\ t)+A_2\ \sin(2\pi\ 2f\ t-\theta)$

Construction of control waveform



$A_1\sin(2\pi\ f\ t)$

$+$

$A_2\sin(2\pi\ 2f\ t-\theta)$

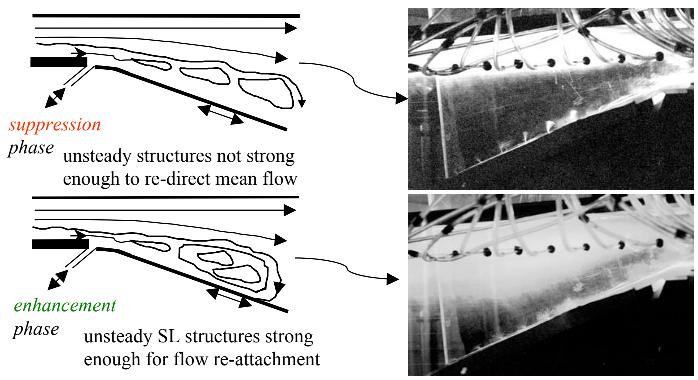
||

$U(t)$

$A_1=A_2$ =const => constant “power”

Adjustable parameters: f & θ

with appropriate choice of control phase one can suppress or enhance vortex interaction



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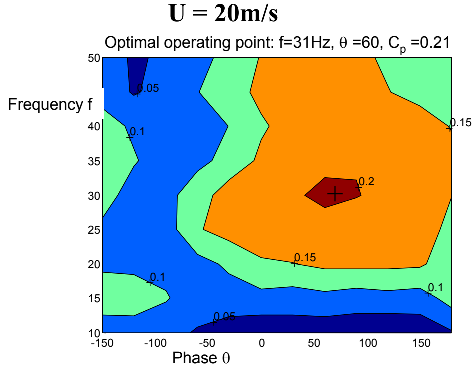
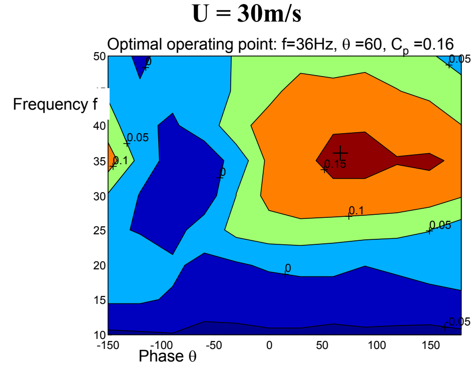
Need: control algorithm to optimize performance

Two frequency control law: $U(t)=A_1*(\sin(2*\pi*f*t) + \sin(2*\pi*2f*t-\theta))$

U = 20m/s



U = 30m/s



Objective:

- **Optimize performance without exhaustive search**

Challenges:

- **Noisy measurement**
- **Flow transients**
- **Keeping up with operating condition change**

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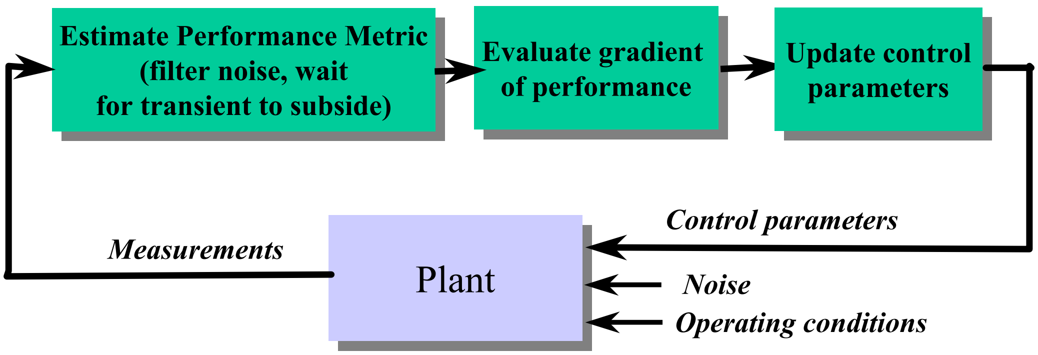
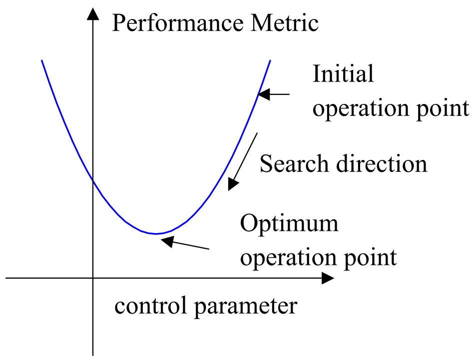
Extremum-Seeking Control: Principle





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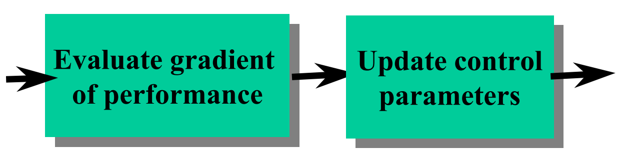
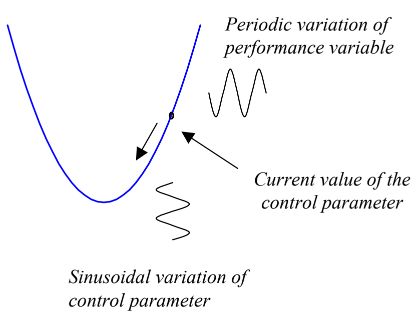
Extremum-Seeking Control: Options



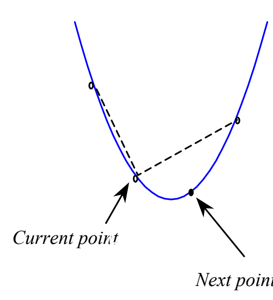
Classical algorithm

Correlation of variations determines direction of change of mean control variable



Triangular search (Y. Zhang)

New control parameter determined by subdivision of an interval determined by previous control parameters



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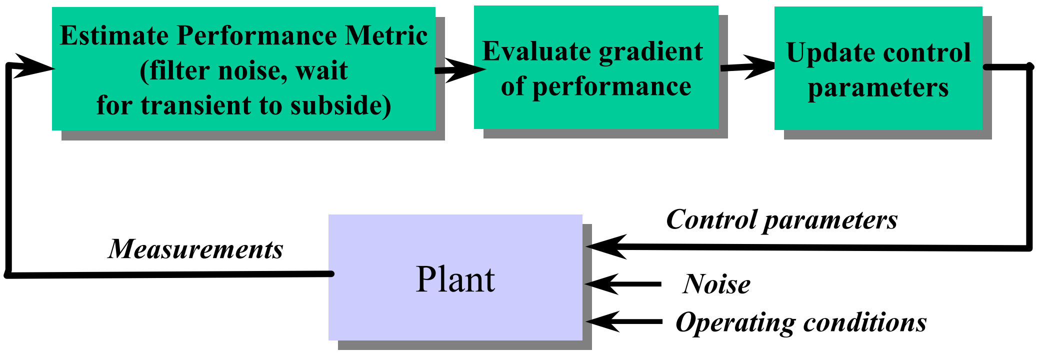
Extremum Seeking Control: Stability



- Stability guaranteed for time scale separation
  - fastest: plant dynamics + noise averaging
  - middle: rate of change in control parameters
  - slowest: operating conditions
- More challenging if time scales not separated (Aryiur and Krstic 1998-2002)

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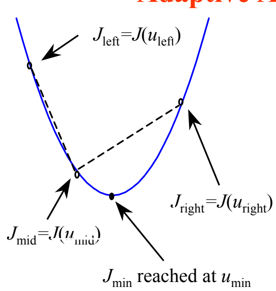
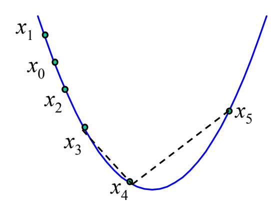
Adaptive Algorithm used in flow control experiments (Y. Zhang)

Concept



$u_{left} < u_{mid} < u_{right}$ and $J_{left} > J_{mid}, J_{right} > J_{mid}$ $\xRightarrow{\text{convexity}}$ $u_{left} < u_{min} < u_{right}$

Feature
- •Infinite memory
- •Allow forgetting



- Wrong direction: flip direction immediately
$(x_2 - x_0)(x_1 - x_0) < 0$

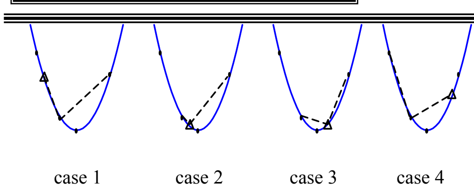
- Right direction: increment step size to accelerate triangle points location:
$(x_5 - x_4) = k(x_4 - x_3)$, $(x_3 - x_2) = k(x_2 - x_0)$, $k > 1$

Feature
- •At most one step wrong
- •Exponential acceleration
- •At most

$$n = \left[\log_k\left(\frac{x}{\Delta}(k-1)+1\right)\right]$$

steps to cover a length x interval with initial step size Δ & maximum step size $\Delta_{max}=k^{n-1}\Delta$,
- •Allow adjustable Δ, Δ_{max}

Initialization



Next point Δ chosen to maximize shrinking of interval

Feature
- •Exponential speed of interval shrinking

$$k = \frac{\sqrt{5}-1}{2}$$

Searching

Adaptation

Feature
- •Automatic adaptation of step size to distance to minimum and noise level
- •Accelerate search when far from minimum
- •Decelerate search when close to minimum
- •Keep on minimum shaking of the system to track variation
- •No external excitation necessary

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

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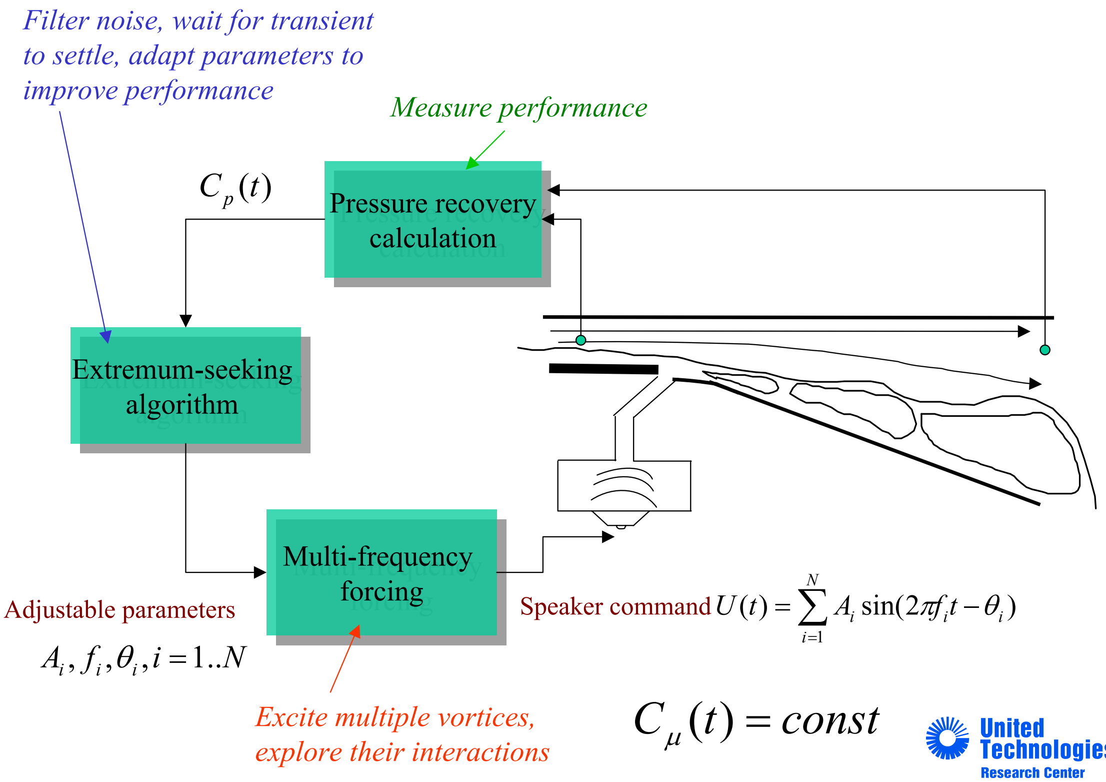
Adaptive control used to optimize performance



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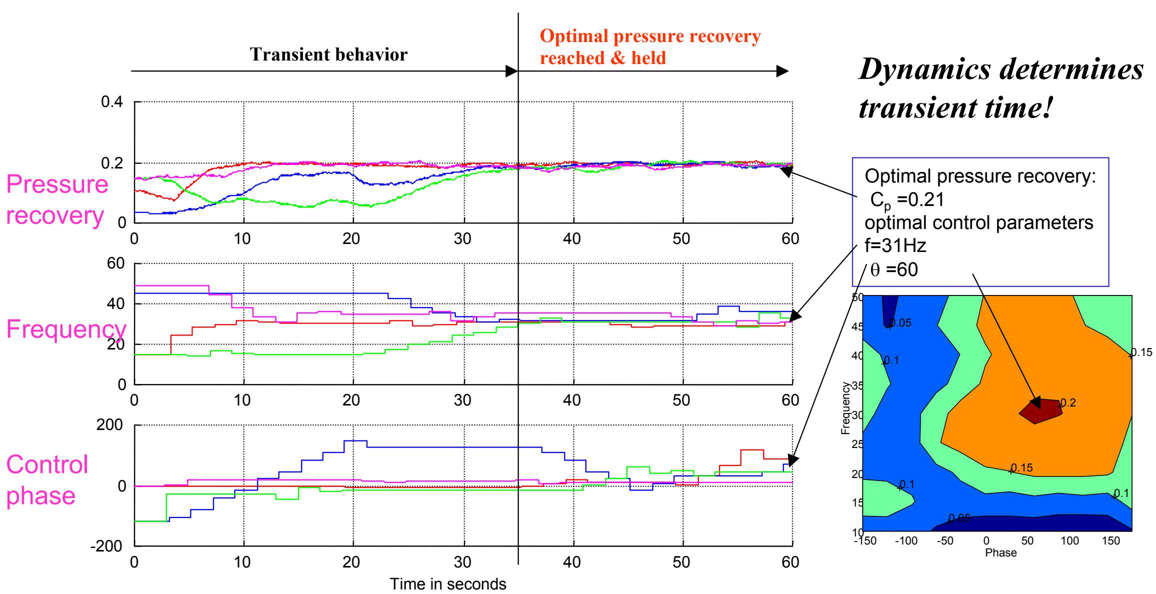
Automatic Control Parameter Tuning to Optimum Values

On-line optimization of pressure recovery using extremum-seeking algorithm demonstrated.



- **Mean pressure recovery, control frequency, and phase in four independent adaptive control experiments.**
- **The control frequency and phase initialized away from the optimal values.**

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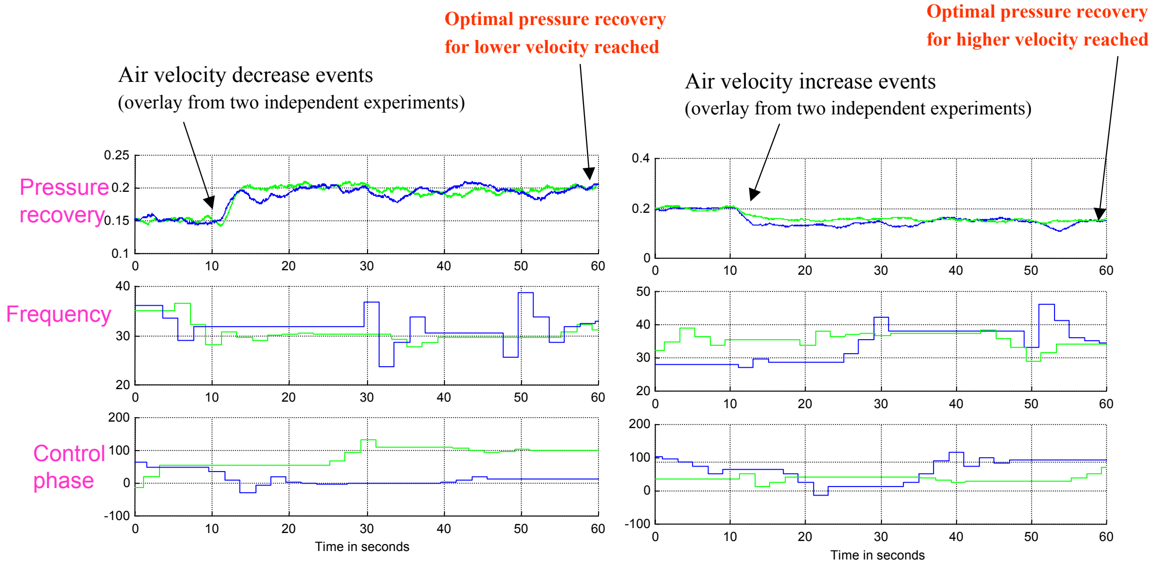
Automatic Parameter Tuning for Operating Condition Changes

Adaptive algorithm tunes control frequency & phase during abrupt changes in operating conditions.



Mean pressure recovery & control frequency & phase during abrupt changes in air velocity between 20m/sec & 30m/sec in two independent experiments.

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Adaptive Control to Find "Beneficial" Coherent Structures

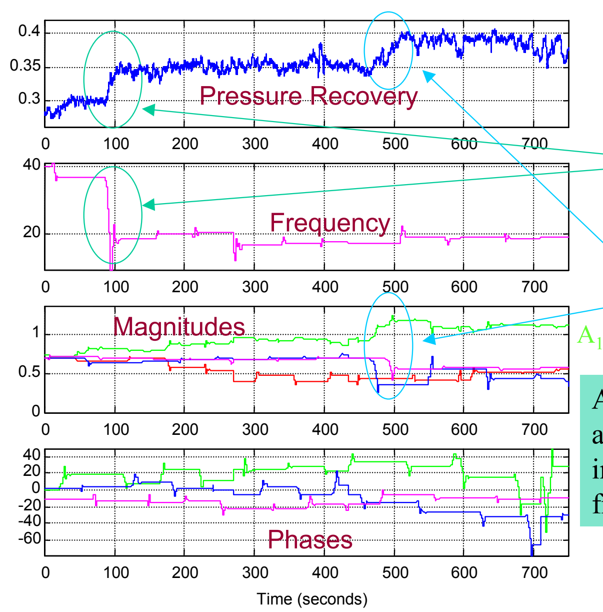
Extremum-seeking control algorithm with four harmonics to explore multiple vortex interactions at U_{inlet} = 10m/s

$$U(t)=A_1 \sin(2\pi f t)+A_2 \sin(2\pi 2f t-\theta_2)+A_3 \sin(2\pi 3f t-\theta_3)+A_4 \sin(2\pi 4f t-\theta_4)$$

$f, A_2, A_3, A_4, \theta_2, \theta_3, \theta_4$ are tuned.

A_1 is determined from:

$A_1^2 + A_2^2 + A_3^2 + A_4^2$ = const.



A_1

First beneficial event: optimal frequency found

Second beneficial event: optimal ratio of magnitudes found

Adaptive tuning of 4-frequency control algorithm achieved 10% performance improvement over manually tuned 2-frequency algorithm

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Closed-loop flow control options

Approach	Model requirements	Sensor bandwidth requirements	Actuator bandwidth requirements
Multi-harmonic forcing + extremum-seeking *(low hanging fruit)*	Crude	Low	High
High bandwidth control *(highest pole in tent)*	Accurate	High	High

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Adaptive control used to optimize performance & explore beneficial flow structures in separated flows

Summary:

- **Multi-frequency control creates "beneficial" vortex interactions**
- **"Benefit" (pressure recovery) depends on control parameters**
- **Extremum-seeking: a simple way to automatically tune control parameters**
- **Minimum model requirement**
- **Stability vs performance tradeoff exist**
- **Demonstrated in diffuser flow**

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Appendix: Zhang's triangular search algorithm:

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

Triangular Search Minimum Seeking

- Discrete time algorithm
- Self-driving switching system: no constant external perturbation necessary
- Uses the local convexity property of the static tunable parameter to cost function map to form triangle locator of the minimum.

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

Generic Minimum Seeking Problem

$$z_n = T\left(\theta_n, z^{-1}\right)u_n + v_n$$

- z_n: output at sampling time n
- u_n : input at time n, constrained in a compact region $u_n \in \mathbb{U}$
- v_n : zero mean measurement noise
- $T(\bullet, \bullet)$: nonlinear difference operator, asymptotic stable $\forall u \in \mathbb{U}$, $\forall \theta$ fixed.

  $$T\left(\theta_n = \theta^*, z^{-1}\right)\left(u_n = u^*\right) \xrightarrow{n\to\infty} T\left(\theta_n = \theta^*, z^{-1} = 1\right)$$

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

Data Smoothing I: Low Pass Filtering

$$y_n = \frac{1-\rho}{1-\rho z^{-1}}\left[T\left(\theta_n, z^{-1}\right)u_n + v_n\right]$$

- y_n: smoothed output
- $\rho \in (0,1)$: a constant determines the bandwidth and high frequency attenuation. The close ρ is to 1, the narrower the bandwidth (and hence slower response) and larger the high frequency attenuation.

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

Data Smoothing II: Down-sampling

$$\bar{y}_k = y_{Nk}$$
$$\bar{u}_k = u_{Nk} = u_{Nk-1} = \cdots = u_{Nk-N+1}$$
$$\bar{y}_k = \frac{1-\rho}{1-\rho^N z^{-N}} \sum_{n=0}^{N-1} \rho^n T\left(\theta_{Nk-n}, z^{-1}\right)\bar{u}_k + \frac{1-\rho}{1-\rho z^{-1}} v_{Nk}$$

- $\bar{u}_k, \bar{y}_k$ are the input, output in the down-sampled domain, N is the number of down samples.

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

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Input-Output Model in Down-sampled Domain

$$\bar{y}_k = \bar{T}_k\left(\theta_k^*\right)\bar{u}_k + \underbrace{\frac{(1-\rho)\rho^N\left(z^{-N}-1\right)}{\left(1-\rho^N z^{-N}\right)\left(1-\rho^N\right)} \sum_{n=0}^{N-1} \rho^n T\left(\theta_{Nk-n}, z^{-1}\right)\bar{u}_k}_{A}$$
$$+ \underbrace{\frac{1-\rho}{1-\rho^N} \sum_{n=0}^{N-1} \rho^n \Delta_{k,n}^{\theta} \bar{u}_k}_{B} + \underbrace{\frac{1-\rho}{1-\rho^N} \sum_{n=0}^{N-1} \rho^n \Delta_{k,n}^{z} \bar{u}_k}_{C} + \underbrace{\frac{1-\rho}{1-\rho z^{-1}} v_{Nk}}_{D}$$
$$\Delta_{k,n}^{\theta} = T\left(\theta_{Nk-n}, z^{-1}\right) - T_k\left(\theta_k^*, z^{-1}\right)$$
$$\Delta_{k,n}^{z} = T_k\left(\theta_k^*, z^{-1}\right) - \bar{T}_k$$
$$\theta_k^* = \arg\min_{\theta} \max_{0\le n<N} \sup_{u\in U} \left\| \frac{1-\rho}{1-\rho^N z^{-N}} \sum_{n=0}^{N-1} \rho^n \left[T\left(\theta_{Nk-n}, z^{-1}\right)u - T\left(\theta, z^{-1}\right)u\right] \right\|_{\infty}$$

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

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Analysis of Perturbation Terms

- A term: $\left\| \dfrac{(1-\rho)\rho^N \left(z^{-N}-1\right)}{\left(1-\rho^N z^{-N}\right)\left(1-\rho^N\right)} \right\|_\infty = \dfrac{4(1-\rho)\rho^N}{\left(1-\rho^N\right)\left(1+\rho^N\right)^2} \xrightarrow{\rho\to 0 \text{ or } N\to\infty} 0$
- B term: decreases with $\Delta^{\theta}_{k,n}$ (difference due to plant time variation) and grows with N.
- C term: $\left|\Delta^z_{k,n} u\right| \le \beta\alpha^{N-1-n}, \forall 0 \le n \le N, u \in U \xrightarrow{\alpha << \rho < 1}$

$$\left| \frac{1-\rho}{1-\rho^N} \sum_{n=0}^{N-1} \rho^n \Delta^z_{k,n} \bar{u}_k \right| \le \frac{(1-\rho)\beta}{1-\rho^N} \rho^{N-1} \xrightarrow{\rho\to 0 \text{ or } N\to\infty} 0$$

- D term: $\displaystyle\int_{-\pi}^{\pi} \frac{S_D(j\omega)}{S_v(j\omega)} d\omega = \frac{2(1-\rho)}{1+\rho}\pi \xrightarrow{\rho\to 1} 0$

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

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Data Smoothing: Summary

- Bandwidth parameter ρ:
  - closer to 1 for maximum noise attenuation
  - closer to 0 for maximum filter transient and plant dynamics suppression
- Down-samples N:
  - large as possible for maximum filter transient and plant dynamics suppression
  - small as possible for minimum plant time variation effect

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

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Triangular Search Algorithm

$$y_k = f_k(u_k) + v_k = f(\theta_k^*, u_k) + v_k$$

- Assumption: $f_k(u)$ has a unique minimum reached by the "best" control $u=u^*$, $f'_k(u^*)=0$, and $\exists\delta>0$ such that $f'_k(u^*+\delta_u)>0$, $f'_k(u^*-\delta_u)<0$, $\forall\ \delta_u \in (0,\ \delta)$

$$(p_k, q_k) = \begin{cases} (u_k, p_{k-1}) & \text{if } y_k \le y_{p_{k-1}} \\ (p_{k-1}, u_k) & \text{if } y_k > y_{p_{k-1}} \end{cases}$$

$$u_{k+1} = p_k + \mu_{k+1}\,\mathrm{sgn}(p_k - q_k)$$

- $y_{p_{k-1}}$ represents the output y_n associated with the control input $u_n=p_{k-1}$
- $\mu_{k+1}>0$ is the step size

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

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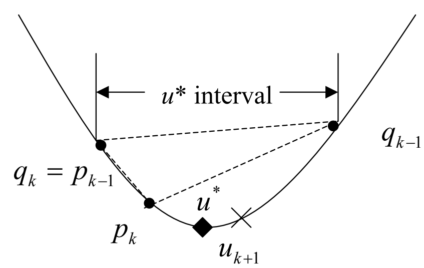
Graphical Representation of the Triangular Search Algorithm



Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

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Convergence in the Ideal Case

- Region of attraction: algorithm guarantees convergence to [u^*-2$\mu_{\max}$,u^*+2$\mu_{\max}$]
- Convergence to optimum point ($u \to u^*$) with contracting step size: Once entering the region of attraction, we shrink the step size as follows:

$$\mu_{k+1} = \gamma \mu_k, \ \gamma = \frac{\sqrt{5}-1}{2}$$

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

Full Model

- Assumptions:
  - Bounded variation: $\left| \frac{\partial f}{\partial \theta}(\vartheta)(\theta_i - \theta_j) \right| \le \varepsilon_\theta, \forall \vartheta, \theta_i, \theta_j \in \Theta$
  - Bounded perturbation: $|v_k| \le \varepsilon_v, \forall k$
  - Dominance of convexity: the map $\mathcal{J}$ is locally convex around $u^* \in \mathcal{J}$, and the noise level and the parameter variation are small compared to the variation of the map due to the change in u.
- Region of Attraction (for $\mu_k \in [\mu_{\min}, \mu_{\max}] \subset (0,\infty)$)

$$D(\mu_{\min}, \mu_{\max}, \varepsilon) = \left\{ [u_l - 2\mu_{\max}, u_r + 2\mu_{\max}] \middle| |f'(x)| \le \frac{2\varepsilon_v + \varepsilon_\theta}{\mu_{\min}} + \varepsilon, \forall x \in [u_l, u_r] \right\}$$

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

Trade-off in Step Size

- As $\mu_{\min}$ increase, the inner region defined by $[u_l,u_r]$ decreases, but the outer ring $[u_l - 2\mu_{\max}, u_l] \cup [u_r, u_r + 2\mu_{\max}]$ increases as $\mu_{\max}$ has to increase.
- As $\mu_{\min}$ increase, the convergence rate (to the region of attraction) is faster, but there is more oscillation due to the large outer ring.

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

Additional Modifications for Robustness w.r.t. Time Variation

- Slow variation assumption (vs bounded variation):

$$\left|\frac{\partial f}{\partial \theta}(\vartheta)(\theta_i - \theta_j)\right| \le \varepsilon_\theta, \forall \vartheta, \theta_i, \theta_j \in \Theta, |i-j| \le N_\theta$$

- <u>Age test</u>: keep age tag n_{pk} on the point p_k. When p_k has been the up-to-current minimum for more than L_0 times, we set $u_{k+1} = p_k$ and re-evaluate the corresponding output y_k +1 instead of using the y_{pk}
- <u>Convexity test</u>: suppose (u_l,u_r) is an interval covering u^*. Then $\forall u \in (u_l,u_r)$, if $y(u) > y(u_l)$ or $y(u) > y(u_r)$, we set $\mu_{\min} > \max\{|u-u_l|,|u-u_r|\}$

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

Summary

- Triangular search is a simple, intuitive, cheap algorithm practical for real applications.
- Requires data smoothing prior to applying the search.
- Guarantees stability, convergence, and robustness under reasonable assumption.
- Has been used in application such as combustion instability control (see separate paper).

Ref: Youping Zhang, "Stability and Performance Tradeoff with Discrete Time Triangular Search Minimum Seeking", Proc. of American Control Conference, Chicago 2000

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