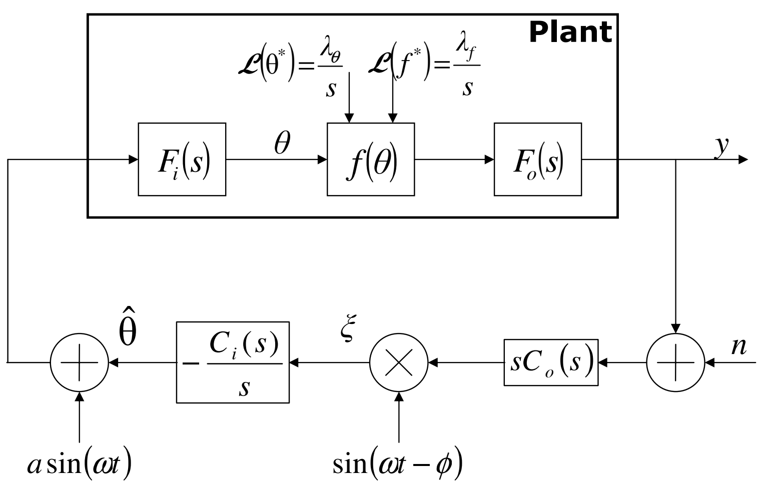
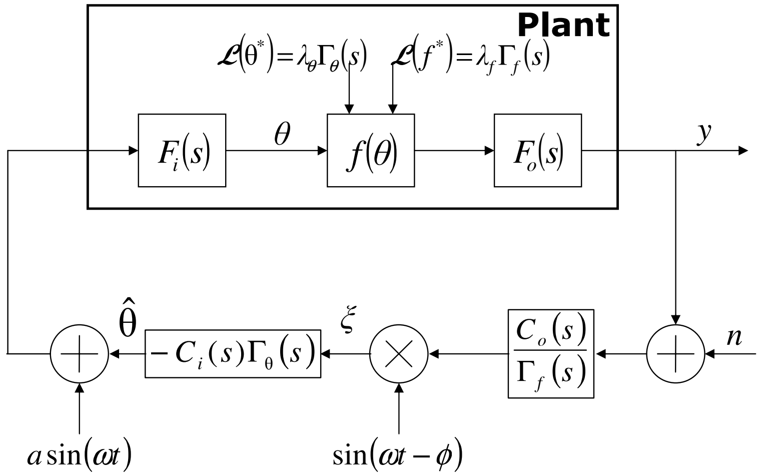
Extremum Seeking with Plant Dynamics and Parameter Tracking

Plant with Dynamics



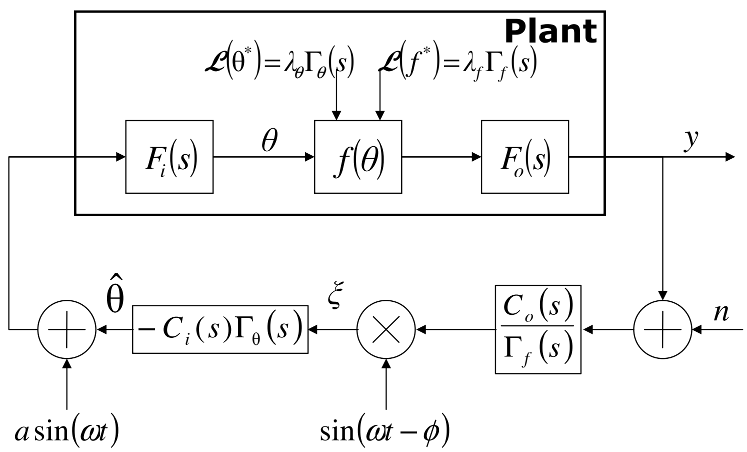
- $f(\theta)=f^*+\frac{f''}{2}(\theta-\theta^*)^2$
- $F_i(s)$ and $F_o(s)$ are asymptotically stable and proper
- $sC_o(s)$ and $\frac{C_i(s)}{s}$ are proper

Parameter Tracking



1. $f(\theta)=f^*(t)+\frac{f''}{2}\left(\theta-\theta^*(t)\right)^2$
2. $\Gamma_\theta(s)$ and $\Gamma_f(s)$ are strictly proper rational functions
3. $\frac{C_o(s)}{\Gamma_f(s)}$ and $C_i(s)\Gamma_\theta(s)$ are proper

Example



Plant : $F_i(s)=\frac{s-1}{s^2+3s+2},\ F_o(s)=\frac{1}{s+1},\ f(\theta)=f^*+\left(\theta-\theta^*\right)^2$

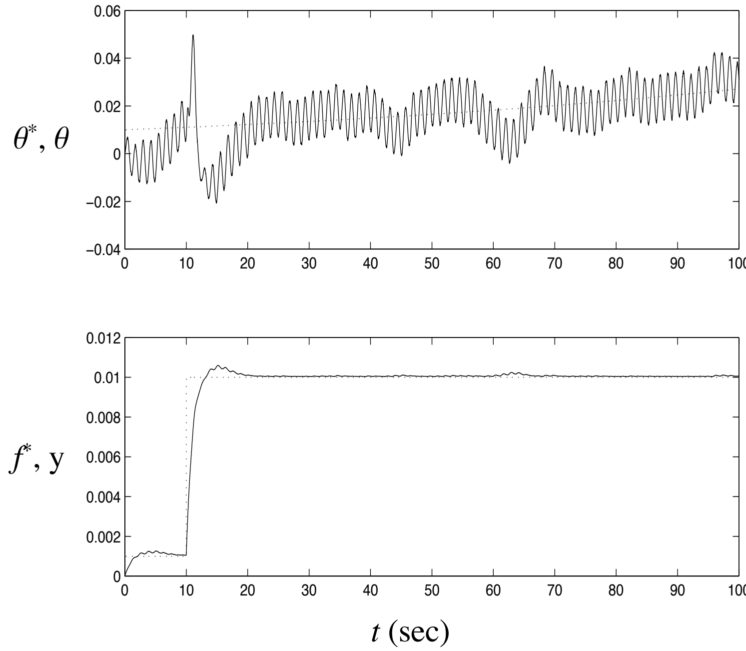
$$f^*=0.01u(t-\tau),\ \theta^*=0.01e^{0.01t},\ \tau=10\text{ sec}$$

Design :

$$\omega=5\text{ rad/ sec},\ a=0.05,\ C_o(s)=\frac{1}{s+5},\ \phi=0.7955,\ C_i(s)=s-4$$

$k=107.7$

Simulation Results



A Singular Perturbation Argument

Let $H_o(s) = k\dfrac{C_o(s)}{\Gamma_f(s)}F_o(s) \equiv H_{osp}(s)H_{obp}(s)$, where $H_{osp}(s)$ denotes the strictly proper part and $H_{obp}(s) = 1 + H_{obp}^{sp}(\mathrm{s})$ denotes the biproper part. Let k be chosen such that $\lim\limits_{s\to 0} H_{osp}(s) = 1$.

Assumption:

Let the smallest in absolute value among the real parts of all the poles of $H_{osp}(s)$ be denoted by a. Let the largest among the moduli of all the poles of $\mathrm{F}_i(s)$ and $H_{obp}(s)$ be denoted by b. The ratio $M = a/b$ is sufficiently large.

Fast 'Output' Dynamics: An Example

$$F_i(s)=\frac{1}{s+1}\,\Gamma_\theta(s)=\frac{1}{s},\Gamma_f(s)=\frac{1}{s},F_o(s)=\frac{1}{(s+1)(2s+3)}$$

$$C_o(s)=\frac{k(s+4)}{(s+5)(s+6)},k=60$$

$$H_o(s)=60\frac{(s+4)s}{(s+5)(s+6)(s+1)(2s+3)}$$

$$=\frac{30}{(s+5)(s+6)}\left(1+\frac{1.5(s-1)}{(s+1)(s+1.5)}\right)$$

$$H_{osp}(s)=\frac{30}{(s+5)(s+6)},\lim_{s\to 0}H_{osp}(s)=1,H_{obp}(s)=\left(1+\frac{1.5(s-1)}{(s+1)(s+1.5)}\right),$$

$$H_{obp}^{sp}(\mathrm{s})=\frac{1.5(s-1)}{(s+1)(s+1.5)},M=\frac{5}{1.5}=3.33$$

Single Parameter Stability Theorem...

Output error $\tilde{y}=y-F_o(s)\left[f^*(t)\right]$ achieves local exponential convergence to an $O\left(a^2+\delta^2\right)$ neighbourhood of the origin, where $\delta=1/\omega+1/M$ provided $n=0$ and :

1. Perturbation frequency ω is sufficiently large compared to dynamics in $H_{obp}(\mathrm{s})$ and $F_i(\mathrm{s})$, and $\pm j\omega$ is not a zero of $F_i(s)$.
2. Zeros of $\Gamma_f(s)$ that are not asymptotically stable are also zeros of $C_o(s)$.
3. Poles of $\Gamma_\theta(s)$ that are not asymptotically stable are not zeros of $C_i(s)$.

…Single Parameter Stability Theorem

4. $C_o(s)$ and $\dfrac{1}{1+L(s)}$ are asymptotically stable, where

$$L(s) = \frac{af''}{4}\mathrm{Re}\{e^{j\phi}F_i(j\omega)\}H_i(s),$$

and $H_i(s) = C_i(s)\Gamma_\theta(s)F_i(s)$.

Governing Equations

$$y = F_o(s)\left[f^*(t) + \frac{f''}{2}\left(\theta - \theta^*(t)\right)^2 \right]$$

$$\theta = F_i(s)[a \sin \omega t - C_i(s)\Gamma_\theta(s)[\xi]]$$

$$\xi = \sin(\omega t - \phi)\frac{C_o(s)}{\Gamma_f(\mathrm{s})}[y + n]$$

Definitions:

$$\theta_0 = F_i(s)[a \sin \omega t]$$

$$\tilde{\theta} = \theta^*(t) - \theta + \theta_0$$

$$\tilde{y} = y - F_o(s)[f^*(t)]$$

$$= F_o(s)\left[\frac{f''}{2}\left(\theta - \theta^*(t)\right)^2 \right]$$

Outline of Proof…

Key steps:

- Linearization
- Singular perturbation reduction of strictly proper part of output dynamics
- Averaging of time varying terms

$$\tilde{\theta} = \theta^*(t) + H_i(s)\left[\sin(\omega t - \varphi) H_o(s)\left[f^*(t) + \frac{f''}{2}\left(\theta - \theta^*(t)\right)^2\right]\right]$$

$$= \theta^*(t) + H_i(s)\left[\sin(\omega t - \varphi) H_o(s)\left[f^*(t) + \frac{f''}{2}\left(\theta_0 - \tilde{\theta}\right)^2\right]\right]$$

Reduction via Singular Perturbation

Using properties of the Laplace transform, we get

$$\tilde{\theta} = \theta^*(t) + H_i(s)\left[\sin(\omega t - \varphi) H_o(s)\left[\frac{f''}{2}\theta_0^2\right] + \varepsilon^{-t}\right]$$
$$+ H_i(s)\left[\sin(\omega t - \varphi) H_{osp}(s)\left(1 + H_{obp}^{sp}(s)\right)\left[\frac{f''}{2}\tilde{\theta}^2 - f''\theta_0\tilde{\theta}\right]\right]$$

Using $\lim_{s \to 0} H_{osp}(s) = 1$, and the fact that its dynamics are sufficiently fast

$$\tilde{\theta} = \theta^*(t) + H_i(s)\left[\sin(\omega t - \varphi)\left(1 + H_{obp}^{sp}(s)\right)\left[\frac{f''}{2}\theta_0^2\right] + \varepsilon^{-t}\right]$$
$$+ H_i(s)\left[\sin(\omega t - \varphi)\left(1 + H_{obp}^{sp}(s)\right)\left[\frac{f''}{2}\tilde{\theta}^2 - f''\theta_0\tilde{\theta}\right]\right]$$

Averaging

Using the fact ω is sufficiently fast relative to the rest of the dynamics, averaging analysis yields

$$\tilde{\theta} = \frac{1}{1 + \frac{af''}{4}\operatorname{Re}\left\{e^{j\phi}F_i(j\omega)\right\}H_i(s)}\left[\varepsilon^{-t}\right]$$

which is the required stability test.

Compensator Design

1. Select the perturbation frequency ω sufficiently large. Also, ω should not equal any frequency in noise or any imaginary axis zero of $F_i(s)$.

2. Set perturbation amplitude a small so as to obtain small steady state error $\tilde{y}$.

3. Design $C_o(s)$ asymptotically stable, with zeros of $\Gamma_f(s)$ that are not asymptotically stable as its zeros, and such that $\frac{C_o(s)}{\Gamma_f(s)}$ is proper.

4. Design $C_i(s)$ by any linear SISO design technique such that it does not include poles of $\Gamma_\theta(s)$ that are not asymptotically stable as its zeros, $C_i(s)\Gamma_\theta(s)$ is proper, and $\frac{1}{1+L(s)}$ is asymptotically stable.

Design of $C_i(s)$ for Robustness

1. Design $C_i(s)$ to minimize $\left\| \dfrac{P}{1+P} \right\|_{H_\infty}$,

   which maximizes the allowable $\Delta f'' = \dfrac{\hat{f}''}{\left\| \dfrac{P}{1+P} \right\|_{H_\infty}}$

   under which the system is still asymptotically stable,

   where $P(s) = \dfrac{\hat{f}''}{f''} L(s)$, $\Delta f'' = f'' - \hat{f}''$.

2. Uncertainties in $F_i(s)$:

   Several methods available in standard texts