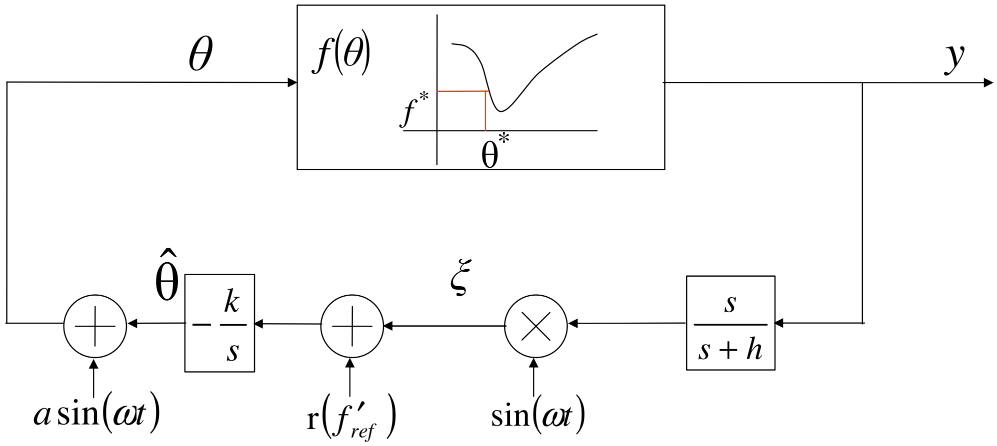
Slope Seeking

Why Slope Seeking?

- Extremum of plant reference-to-output map susceptible to destabilization:
  - Compressor instability
  - Antiskid Braking
  - Formation flight
- Need to operate at a particular slope of plant operating characteristic
  - Nuclear fusion

Slope Seeking on a Static Map

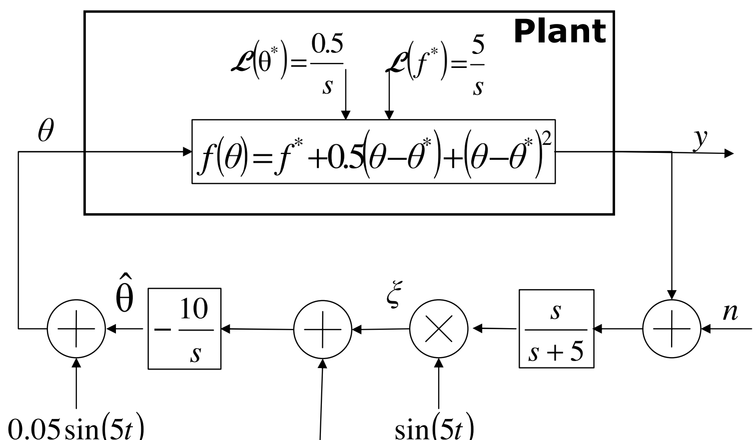


Stability Test:

y converges to an $f^* + O(a + 1/\omega)$ if $\dfrac{1}{1+L(s)}$ is a.s.,

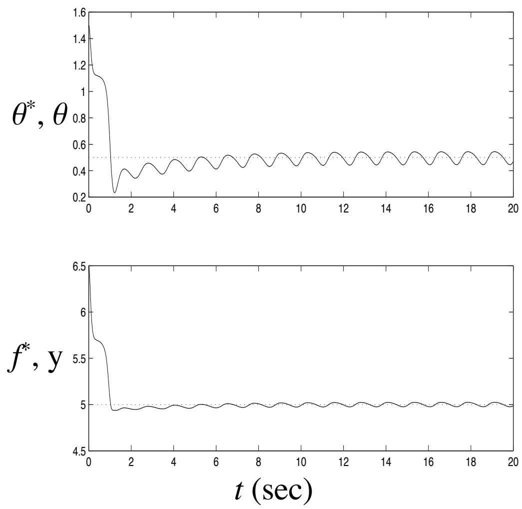
$$L(s) = \frac{kaf''}{2s}, \text{ and } r(f'_{ref}) = -\frac{af'_{ref}}{2}\operatorname{Re}\left\{\frac{j\omega}{j\omega + h}\right\}$$

Example

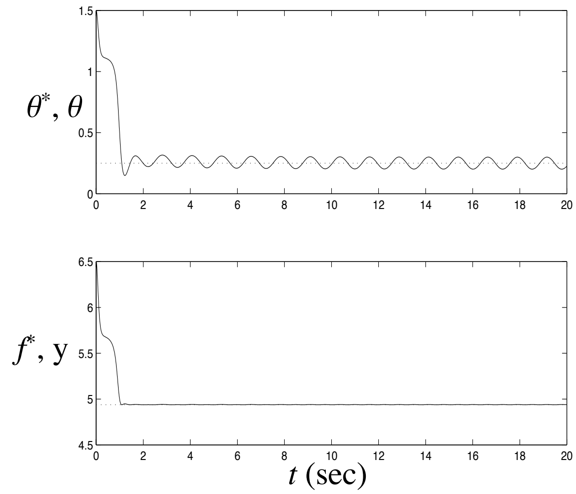


$$r(f'_{ref}) = \frac{0.05 \times 0.5}{2}\operatorname{Re}\left\{\frac{j5}{j5+5}\right\}$$

Simulation

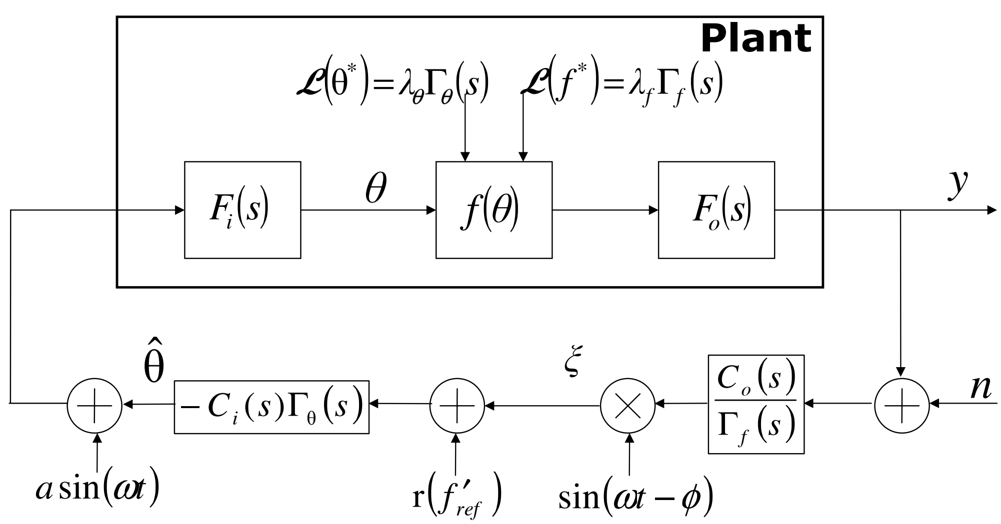


Slope seeking: $r(f'_{ref})=0.5$



Extremum seeking: $r(f'_{ref})=0$

Generalized Slope Seeking



Slope setting :

$$r(f'_{ref}) = \frac{a f'_{ref}}{2} \mathrm{Re}\left\{ e^{j\varphi} H_o(j\omega) F_i(j\omega) \right\}$$

Single Parameter Stability Theorem…

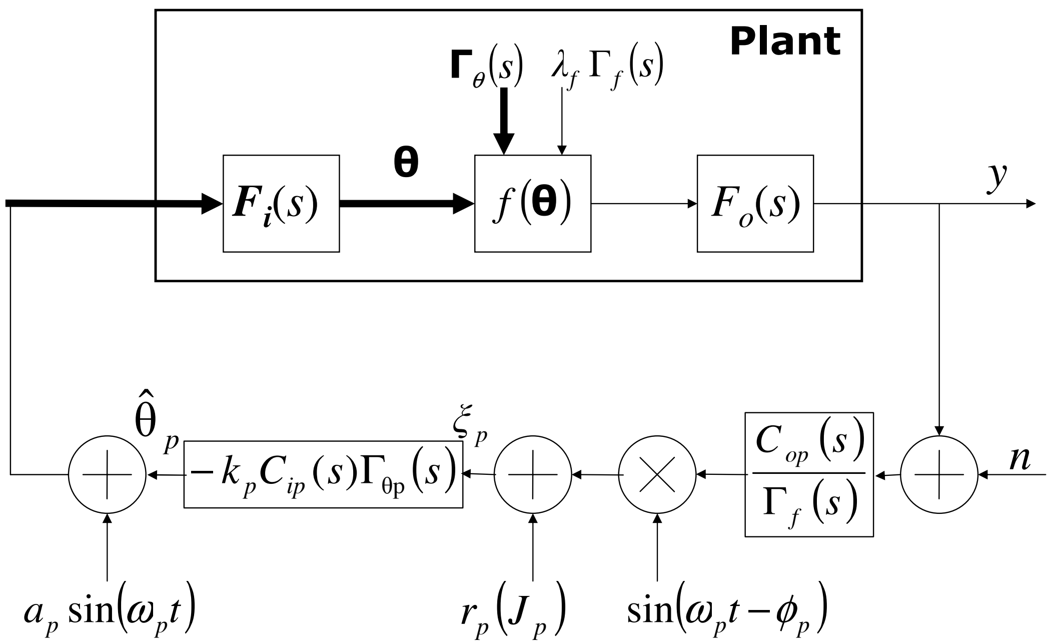
Output error $\tilde{y} = y - F_o(s)[f^*(t)]$ achieves local exponential convergence to an $O(a+1/\omega)$ neighbourhood of the origin provided $n = 0$ and:

1. Perturbation frequency ω is sufficiently large compared to dynamics in $H_{obp}(s)$ and $F_i(s)$, sufficiently small compared to dynamics in $H_{osp}(s)$, and $\pm j\omega$ is not a zero of $F_i(s)$.
2. Zeros of $\Gamma_f(s)$ that are not asymptotically stable are also zeros of $C_o(s)$.
3. Poles of $\Gamma_\theta(s)$ that are not asymptotically stable are not zeros of $C_i(s)$.

…Single Parameter Stability Theorem

4. $C_o(s)$ and $\dfrac{1}{1+L(s)}$ are asymptotically stable, where
   $$L(s) = \frac{af''}{4}\mathrm{Re}\{e^{j\phi}F_i(j\omega)\}H_i(s),$$
   and $H_i(s) = C_i(s)\Gamma_\theta(s)F_i(s)$.
5. $$r(f'_{ref}) = -\frac{af'_{ref}}{2}\mathrm{Re}\{e^{j\varphi}H_o(j\omega)F_i(j\omega)\}.$$

Gradient Seeking



Multiparameter Stability Test...

Output error $\tilde{y} = y - F_o(s)\left[f^*(t)\right]$ achieves local exponential convergence to an $O\left(1/\omega_1 + l\sum_{p=1}^{l} a_p\right)$ neighbourhood of the origin provided $n = 0$ and:

1. Perturbation frequencies $\omega_1 < \omega_3 < \cdots < \omega_l$ are rational, sufficiently large compared to dynamics in $H_{obp}^p(\mathrm{s})$ and $F_i(\mathrm{s})$, sufficiently small compared to dynamics in $H_{osp}^p(\mathrm{s})$, and $\pm j\omega_p$ is not a zero of $F_{ip}(s)$.
2. Zeros of $\Gamma_f(s)$ that are not asymptotically stable are also zeros of $C_{op}(s)$ for all $p = 1,\ldots,l$.
3. Poles of $\Gamma_{\theta p}(s)$ that are not asymptotically stable are not zeros of $C_{ip}(s)$, for any $p = 1,\ldots,l$.

…Multiparameter Stability Test

4. $C_{op}(s)$ are asymptotically stable for all $p = 1, \ldots, l$

and $\dfrac{1}{\det(\mathrm{I}_l + \mathbf{X}(s))}$ is asymptotically stable, where

$$\mathbf{X}(\mathbf{s}) = \begin{pmatrix} X_{11}(s) & X_{12}(s) & \cdots & X_{1l}(s) \\ X_{21}(s) & X_{22}(s) & \cdots & X_{2l}(s) \\ \vdots & \ddots & \ddots & \vdots \\ X_{l1}(s) & X_{l2}(s) & \cdots & X_{ll}(s) \end{pmatrix}, \text{ and } X_{pq}(s) = P_{pq} a_p L_p(s)$$

$$L_p(s) = \frac{1}{2} \mathrm{Re}\left\{e^{j\phi_p} F_{ip}(j\omega_p)\right\} H_{ip}(s)$$

and $H_{ip}(s) = C_{ip}(s)\Gamma_{\theta p}(s) F_{ip}(s)$.

5. $\mathrm{r}_p(J_p) = -\dfrac{a_p J_p}{2} \mathrm{Re}\left\{\mathrm{e}^{j\varphi_p} H_{op}(j\omega) F_{ip}(j\omega)\right\}$.