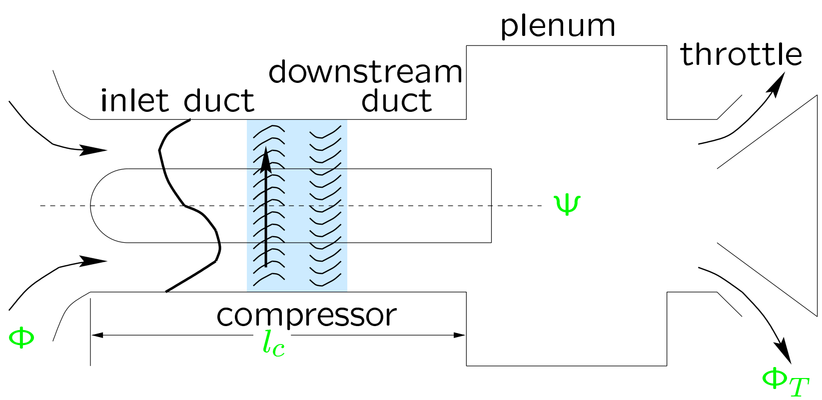
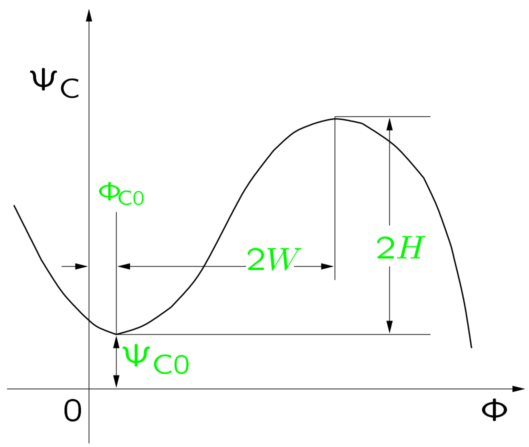


Compression system



Compressor characteristic

Moore-Greitzer model:

$$\begin{aligned}
\dot{A} &= \frac{\sigma}{3\pi}\int_0^{2\pi} \Psi_C(\Phi + A\sin\theta)\sin\theta d\theta \\
\dot{\Phi} &= -\Psi + \frac{1}{2\pi}\int_0^{2\pi} \Psi_C(\Phi + A\sin\theta)\, d\theta \\
\dot{\Psi} &= \frac{1}{\beta^2}(\Phi - \Phi_T)
\end{aligned}$$

Cubic characteristic:

$$\Psi_C(\Phi) = \Psi_{C0} + 1 + \frac{3}{2}\Phi - \frac{1}{2}\Phi^3$$

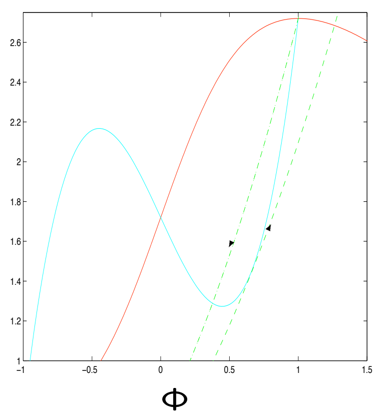
Family of "deep-hysteresis" characteristics:

$$\boxed{\Psi_C(\Phi) = \Psi_{C0} + 1 + (1-\epsilon)\left(\frac{3}{2}\Phi - \frac{1}{2}\Phi^3\right) + \epsilon\frac{2\Phi}{1+\Phi^2}}$$

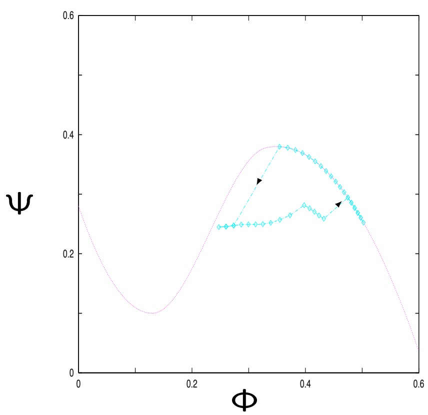
$0 \le \epsilon \le 1$

A new parameterization

Deep vs. small hysteresis



small hysteresis



deep hysteresis

The ϵ-MG3 Model:

$$
\begin{aligned}
\dot{R} &= \sigma\Big\{(1-\epsilon)\,R\,(1-\Phi^2-R) \\
&\quad +\epsilon\frac{2}{3}\Bigg[1-\frac{1}{\sqrt{2}\left[(\Phi^2-4R-1)^2+4\Phi^2\right]^{1/2}} \\
&\quad \times\Big(\Big(\big((\Phi^2-1)(\Phi^2-4R-1)+4\Phi^2\big)^2+64\Phi^2R^2\Big)^{1/2} \\
&\quad +(\Phi^2-1)(\Phi^2-4R-1)+4\Phi^2\Big)^{1/2}\Bigg]\Big\} \\
\dot{\Phi} &= -\Psi+\Psi_{C0}+1+(1-\epsilon)\left(\frac{3}{2}\Phi-\frac{1}{2}\Phi^3-3\Phi R\right) \\
&\quad +\epsilon\frac{\sqrt{2}\,\mathrm{sgn}(\Phi)}{\left[(\Phi^2-4R-1)^2+4\Phi^2\right]^{1/2}} \\
&\quad \times\left\{\left[(\Phi^2-4R-1)^2+4\Phi^2\right]^{1/2}+(\Phi^2-4R-1)\right\}^{1/2} \\
\dot{\Psi} &= \frac{1}{\beta^2}(\Phi-\Phi_T)\,.
\end{aligned}
$$

Open-Loop Bifurcation Diagrams

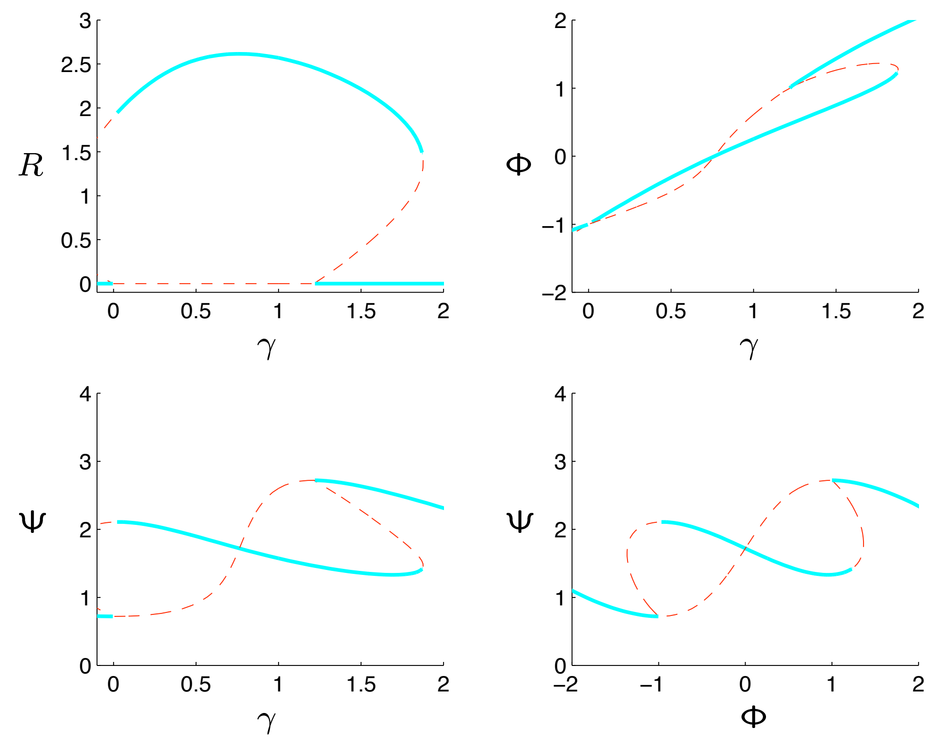
Throttle characteristic:

$$\Psi = \frac{1}{\gamma^2}(1 + \Phi_{C0} + \Phi_T)^2$$

γ—bifurcation parameter

Using values for a three stage compressor from Eveker et al.

$\epsilon = 0.9$ and $\beta = 1.42$



Critical Slopes

$$S_1 = \left. \frac{d\Phi_{R+}(R)}{dR} \right|_{R=0}$$
$$S_2 = \left. \frac{d\Psi_{R+}(R)}{dR} \right|_{R=0}$$

S_1 –skewness

Bifurcation control for aeroengine compressors

Moore-Greitzer model with general comp. char.:

$$\begin{aligned} \dot{R} &= \sigma R \mathcal{F}(R, \Phi) \\ \dot{\Phi} &= -\Psi + \mathcal{G}(R, \Phi) \\ \dot{\Psi} &= \frac{1}{\beta^2}(\Phi - \Phi_T) \end{aligned}$$

where

$$\begin{aligned} \mathcal{F}(R, \Phi) &= \frac{1}{3\pi\sqrt{R}} \int_0^{2\pi} \Psi_C \left(\Phi + 2\sqrt{R} \sin\theta \right) \sin\theta d\theta \\ \mathcal{G}(R, \Phi) &= \frac{1}{2\pi} \int_0^{2\pi} \Psi_C \left(\Phi + 2\sqrt{R} \sin\theta \right) d\theta \end{aligned}$$

Throttle characteristic

$$\Psi = \frac{1}{\gamma^2}(1 + \Phi_{C0} + \Phi_T)^2$$

Backstepping control law:

$$\gamma = \frac{\Gamma + \bar{\beta}^2 \left(c_\Psi \Psi - c_\Phi \Phi\right)}{\sqrt{\Psi}}$$

Full-state feedback control law:

$$\boxed{\gamma = \frac{\Gamma + \bar{\beta}^2 \left(c_\Psi \Psi - c_\Phi \Phi + c_R R - d_\Phi \dot{\Phi}\right)}{\sqrt{\Psi}}}$$

set-point/disturbance parameter

Control Objectives:

- supercritical bifurcation w.r.t. Γ
- minimize sensitivity of R to Γ
- maximize the interval of Γ beyond stall inception with stable stall equilibria

Theorem. *Suppose the following cond's are satisf'd:*

$c_R - S_1 c_* + S_2 c_\Psi > 0$
$c_* > 0$
$c_\Psi + d_\Phi > 0$

Then, an interval of stall equilibria sufficiently close to the stall inception point are locally exponentially stable. The stall inception point is locally asymptotically stable.

Proof. Linearization + Center Manifold Theorem.

Control design with actuator dynamics

Actuator with a Lag:

$$\boxed{\gamma = \frac{1}{\tau s + 1} u}$$

Full-state feedback control law:

$$\boxed{u = \frac{\Gamma + \bar{\beta}^2 \left(c_\Psi \Psi - c_\Phi \Phi + c_R R - d_\Phi \dot{\Phi} \right)}{\sqrt{\Psi}}}$$

Γ–set-point/disturbance parameter

Theorem. *Suppose the following cond's are satisf'd:*

$c_R - S_1 c_* + S_2 c_\Psi > 0$
$c_* > 0$
$c_\Psi + d_\Phi + \frac{\tau}{\beta^2} > \left(\frac{1}{\tau} + \frac{2+\Phi_{C0}}{2\beta^2(2+\Psi_{C0})}\right)^{-1}\left(c_\Phi + \frac{1}{\beta^2}\right)$

Then, an interval of stall equilibria sufficiently close to the stall inception point are locally exponentially stable. The stall inception point is locally asymptotically stable.

Proof. Linearization + Center Manifold Theorem.

Closed-Loop Bifurcation Diagrams

Γ—bifurcation parameter

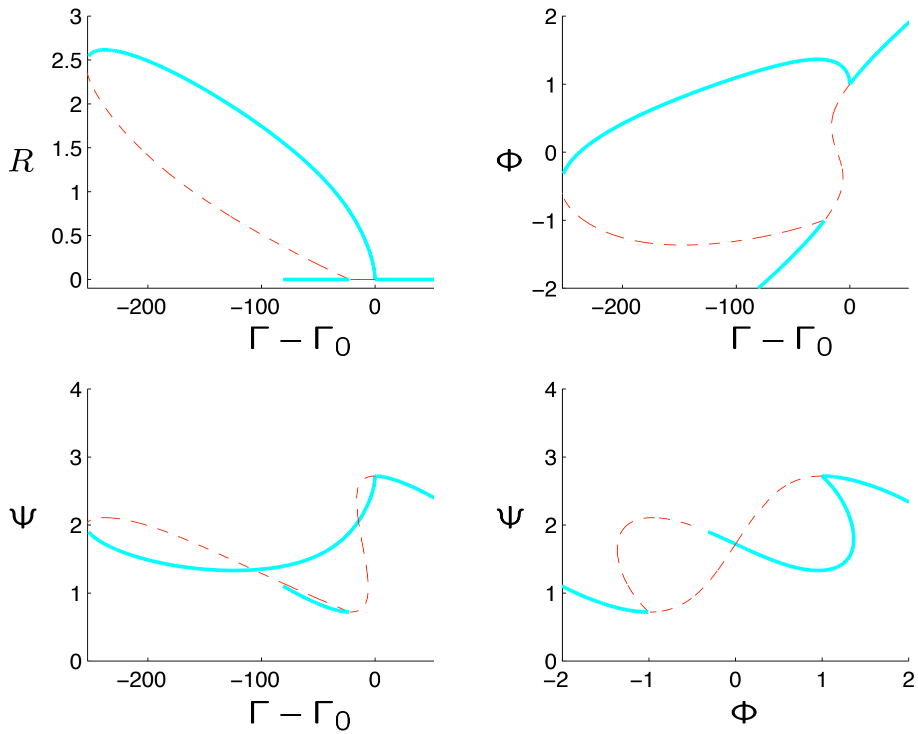
Using values for a three stage compressor from Eveker et. al.

Right-Skew Case ($\epsilon = 0.9$)

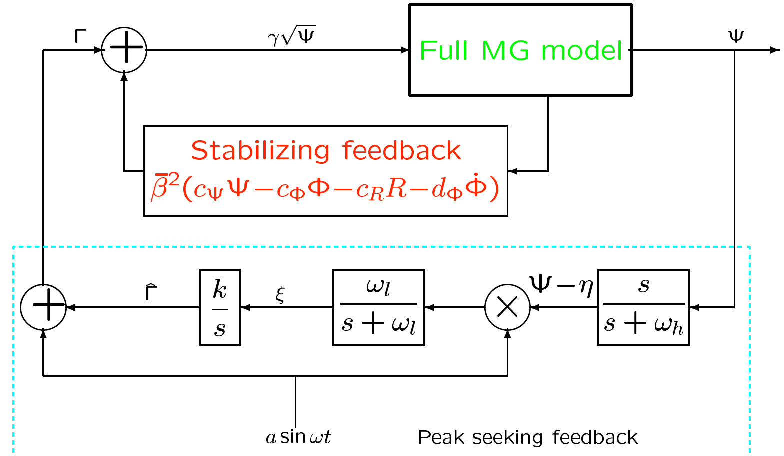
Full-state (Ψ, R, Φ)**-controller**

$\beta = 1.42, \tau = 0.44$

$c_R = 43,\ c_\Psi = 17,\ c_\Phi = 22$

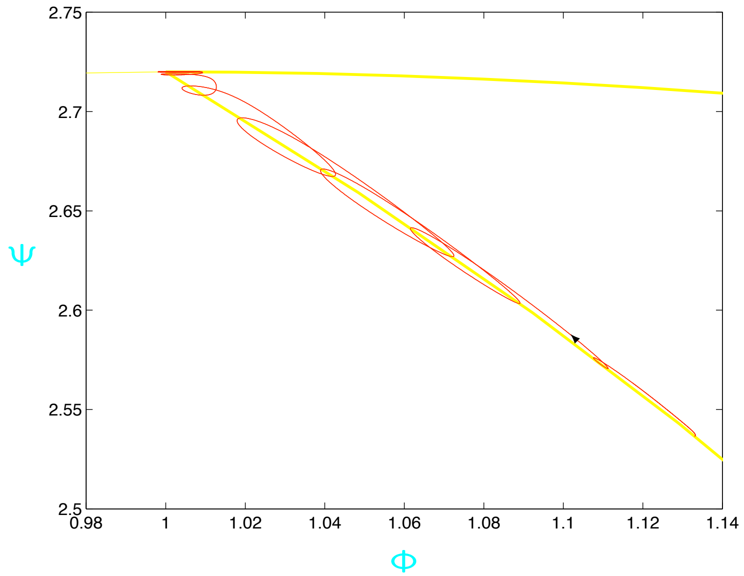


Peak seeking for compressors

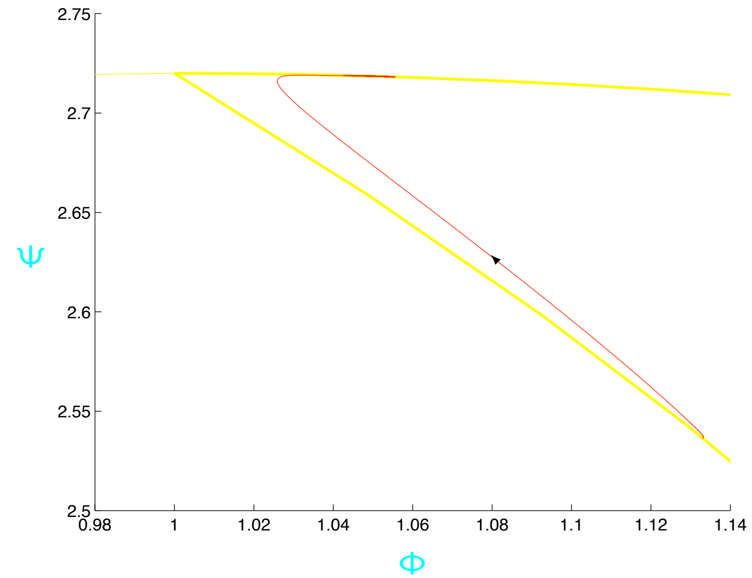


Peak seeking scheme for the full Moore-Greitzer model

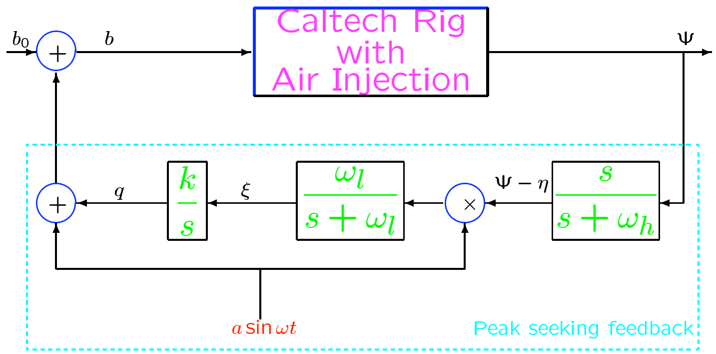
A trajectory under the peak seeking feedback



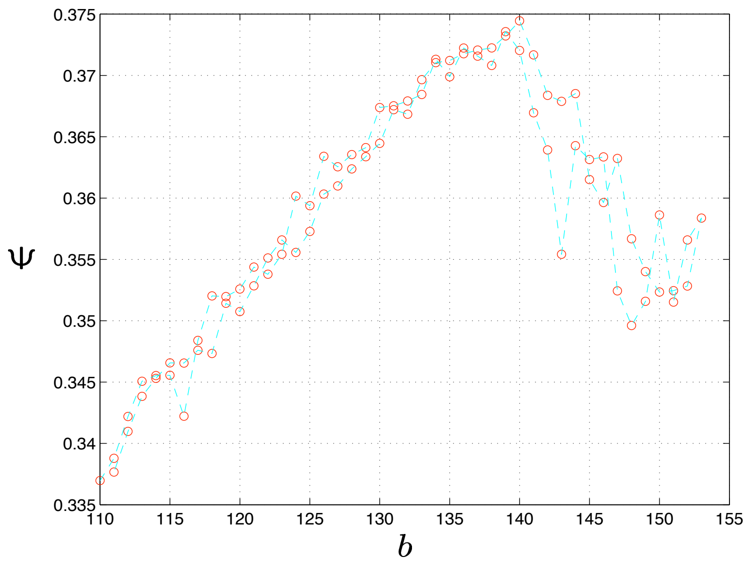
A trajectory under the peak seeking feedback



Peak seeking scheme for the Caltech rig



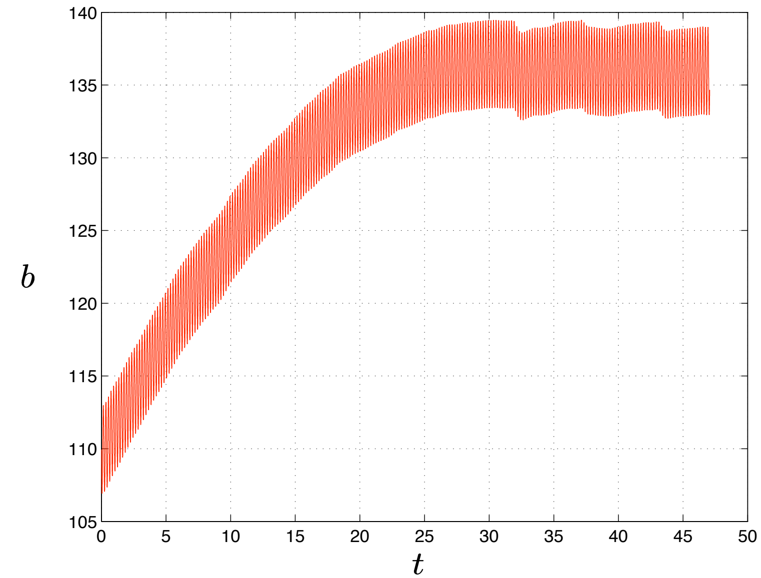
Ψ and bleed valve angle relationship



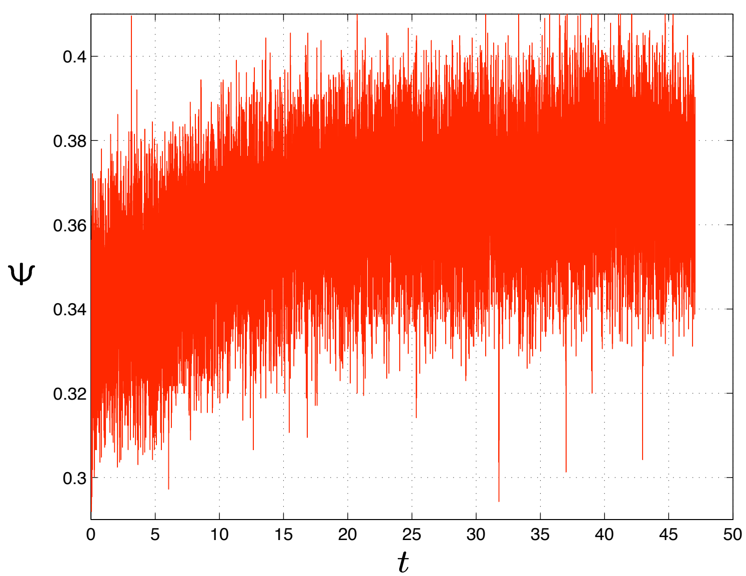
Two Experiments

- Initial point at axisymmetric characteristic
  - integrator gain = 600
  - frequency of perturbation : 5 Hz
  - amplitude of perturbation : 3°
  - initial bleed angle : 110°
- Initial point at nonaxisymmetric characteristic
  - the settings are same as above except integrator gain = 400
  - initial bleed angle : 150°

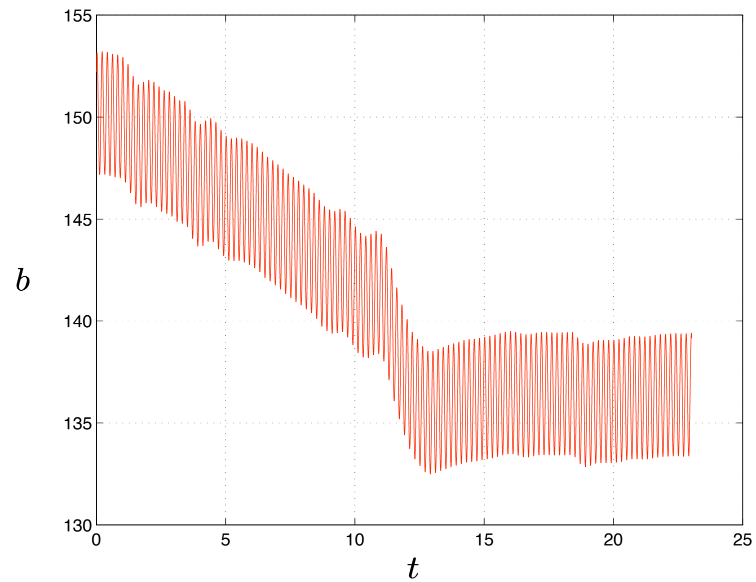
The time response of the bleed angle initiating from the axisymmetric characteristic.



The time response of the pressure rise initiating from the axisymmetric characteristic.



The time response of the bleed angle initiating from the stall characteristic.



The time response of the pressure rise initiating from the stall characteristic.