Near-Optimal Compressor Operation via Slope Seeking

The Moore-Greitzer Model

$\dot{R} = \sigma R \mathcal{F}(R, \Phi)$; where $\mathcal{F}(R, \Phi) = \dfrac{1}{3\pi\sqrt{R}} \displaystyle\int_0^{2\pi} \Psi_C \left(\Phi + 2\sqrt{R}\sin\theta\right) \sin\theta d\theta$

$\dot{\Phi} = -\Psi + \mathcal{G}(R, \Phi)$; where $\mathcal{G}(R, \Phi) = \dfrac{1}{2\pi} \displaystyle\int_0^{2\pi} \Psi_C \left(\Phi + 2\sqrt{R}\sin\theta\right) d\theta$

$\dot{\Psi} = \dfrac{1}{\beta^2}(\Phi - \Phi_T)$

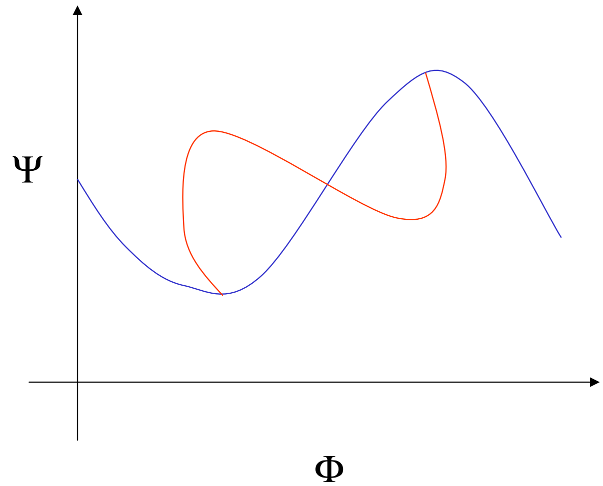
$\Psi_C(\Phi)$: compressor characteristic,

$\Psi = \dfrac{1}{\gamma^2}(1 + \Phi_{C0} + \Phi_T)^2$: throttle characteristic,

γ : throttle opening

Notation in the Moore-Greitzer Model

$$\Phi = \hat{\Phi}/W - 1 - \Phi_{C0}$$
$$\Phi_T = \dot{m}_T/W - 1$$
$$\Psi = \hat{\Psi}/H$$
$$A = \hat{A}/W, R = (A/2)^2$$
$$\beta = \frac{2H}{W}B;\ \sigma = \frac{3l_c}{m+\mu}$$
$$t = \frac{H}{Wl_c}\hat{t};\ \hat{t} = \Omega\tau$$



The ε-MG3 Parametrization

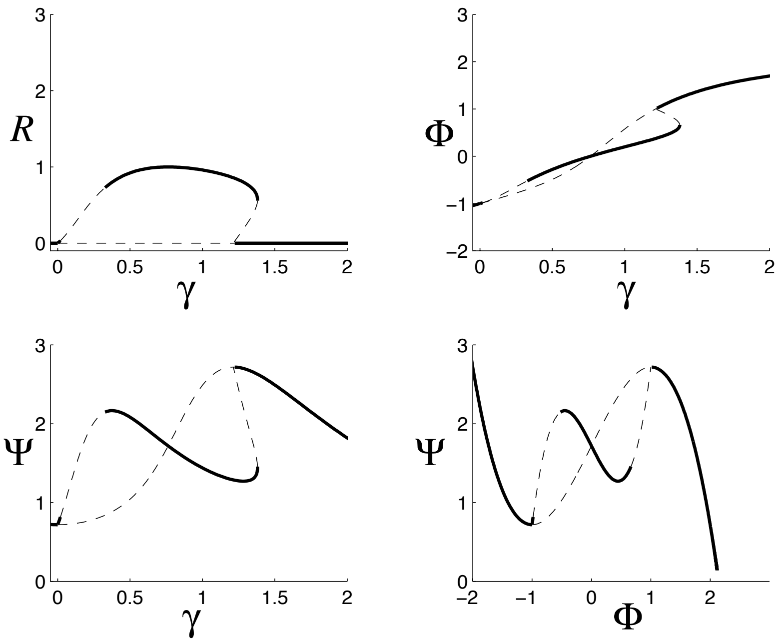
Using $\Psi_C(\Phi) = \Psi_{C0} + 1 + (1-\varepsilon)\left(\frac{3}{2}\Phi - \frac{1}{2}\Phi^3\right) + \varepsilon\frac{2\Phi}{1+\Phi^2}$ gives

$$\dot{R} = \sigma\left\{(1-\varepsilon)R\left(1-\Phi^2-R\right) + \frac{2\varepsilon}{3}\left[1 - \frac{1}{\sqrt{2}\left[\left(\Phi^2-4R-1\right)^2+4\Phi^2\right]^{1/2}}\right.\right.$$
$$\times\left(\left(\left(\left(\Phi^2-1\right)\left(\Phi^2-4R-1\right)+4\Phi^2\right)^2+64\Phi^2R^2\right)^{1/2}\right.$$
$$\left.\left.\left.+\left(\Phi^2-1\right)\left(\Phi^2-4R-1\right)+4\Phi^2\right)^{1/2}\right]\right\}$$

$$\dot{\Phi} = -\Psi + \Psi_{C0} + 1 + (1-\varepsilon)\left(\frac{3}{2}\Phi - \frac{1}{2}\Phi^3 - 3\Phi R\right)$$
$$+\varepsilon\frac{\sqrt{2}\,\mathrm{sgn}(\Phi)}{\left[\left(\Phi^2-4R-1\right)^2+4\Phi^2\right]^{1/2}}\left\{\left[\left(\Phi^2-4R-1\right)^2+4\Phi^2\right]^{1/2}+\left(\Phi^2-4R-1\right)\right\}^{1/2}$$

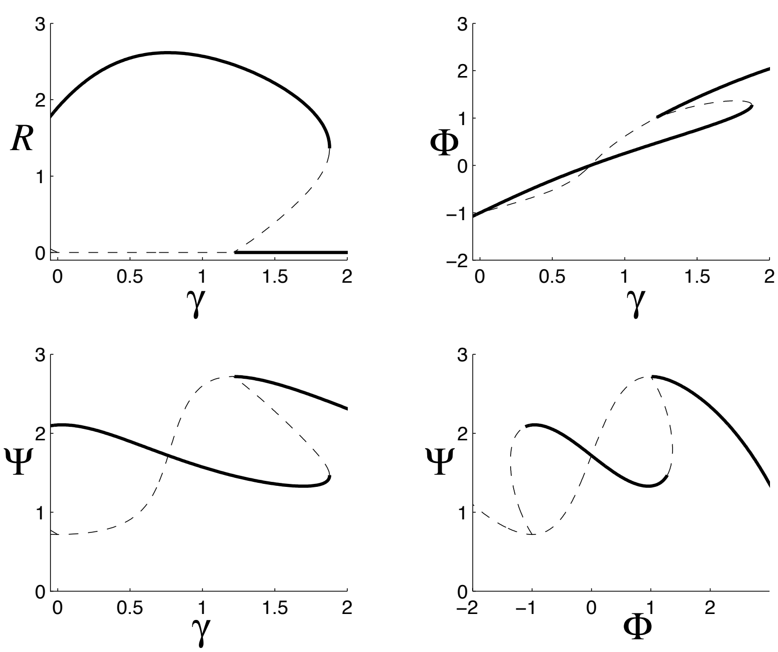
$$\dot{\Psi} = \frac{1}{\beta^2}(\Phi - \Phi_T)$$

Equilibria and Bifurcation Diagrams



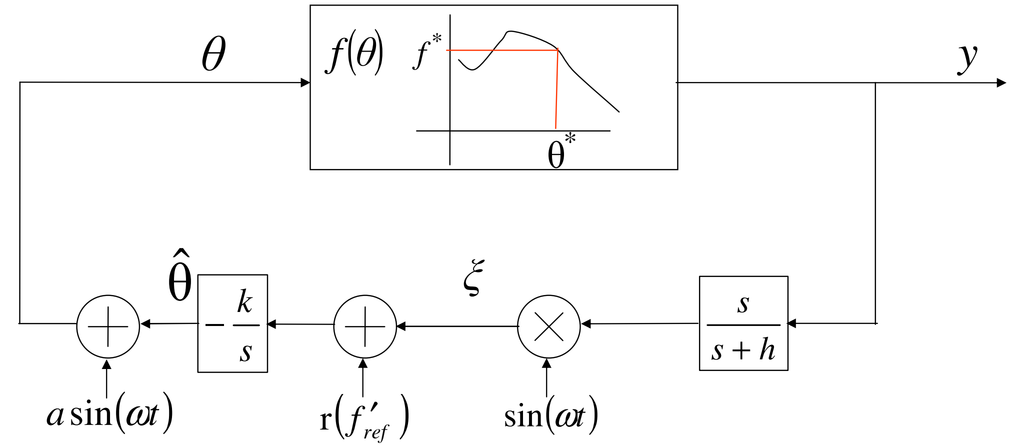
Low hysteresis: ε=0

Equilibria and Bifurcation Diagrams



Deep hysteresis: ε=0.9

Slope Seeking on a Static Map

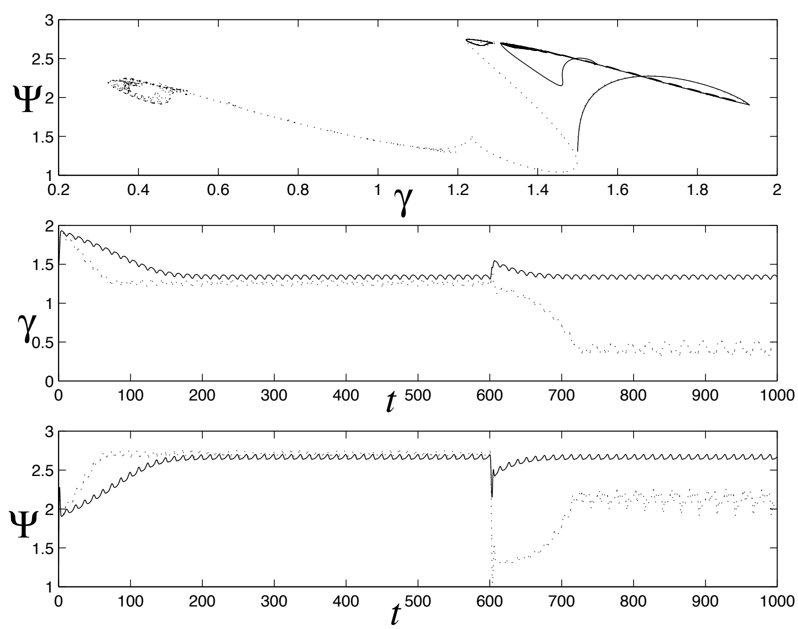


Stability Test:

y converges to an $f^* + \mathrm{O}(a + 1/\omega)$ if $\dfrac{1}{1+L(s)}$ is a.s.,

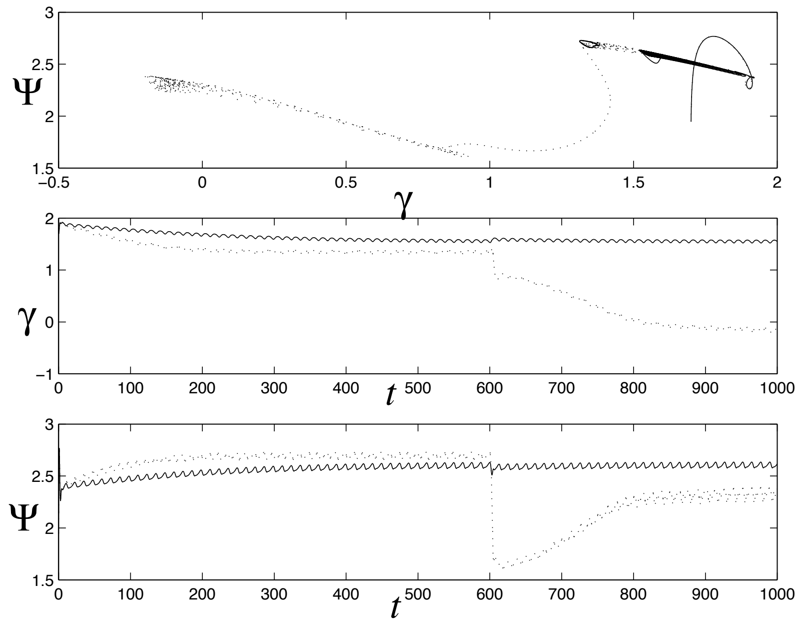
$$L(s) = \frac{kaf''}{2s}, \text{ and } \mathrm{r}(f'_{ref}) = \frac{af'_{ref}}{2}\mathrm{Re}\left\{\frac{j\omega}{j\omega + h}\right\}$$

Compressor Simulation



Low hysteresis: ε=0

Compressor Simulation



Deep hysteresis: ε=0.9