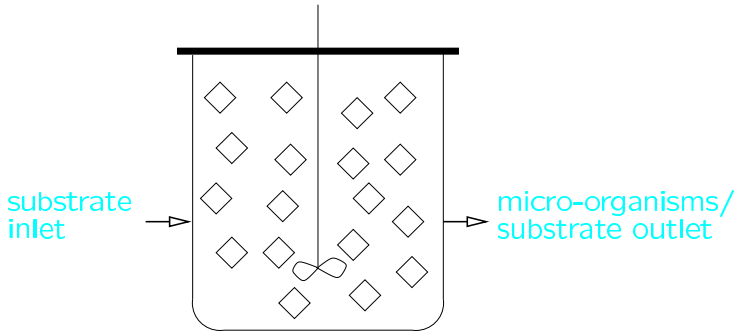
# Optimizing Bioreactors by Extremum Seeking

## Bioreactor with continuous culture






The mass balance model:

$$\begin{aligned} \dot{x} &= \mu(s)x - Dx \\ \dot{s} &= D(s_R - s) - \frac{\mu(s)x}{Y} \end{aligned}$$

where $x$ is the biomass concentration, $s$ is the substrate concentration, $\mu(s)$ is the specific growth rate function, $D$ is the dilution rate, and $Y$ is the yield coefficient.

- Monod model:

$$\mu(s) = \mu_m \left( \frac{s}{K_s + s} \right)$$

- Haldane model:

$$\mu(s) = \frac{\mu_m}{1 + \frac{K_s}{s} + \frac{s}{K_i}}$$

Normalization

$$\begin{array}{rcl} x & \sim & Y s_R \\ s & \sim & s_R \\ D & \sim & \mu_m \\ t & \sim & \dfrac{1}{\mu_m} \end{array}$$

- Monod model

$$\begin{aligned} \dot{x} &= x\left(\frac{s}{K_1+s} - D\right) \\ \dot{s} &= D(1-s) - \frac{xs}{K_1+s} \end{aligned}$$

- Haldane model

$$\begin{aligned} \dot{x} &= x\left(\frac{1}{1+\frac{K_1}{s}+\frac{s}{K_2}} - D\right) \\ \dot{s} &= D(1-s) - \frac{x}{1+\frac{K_1}{s}+\frac{s}{K_2}} \end{aligned}$$

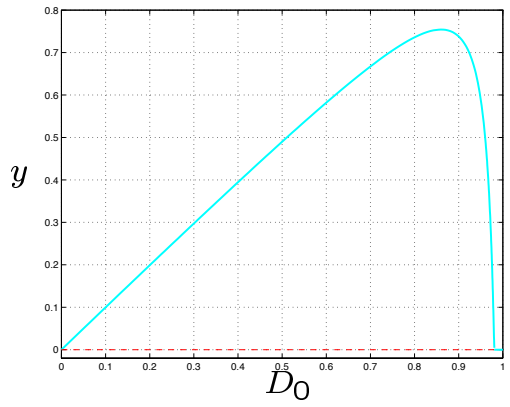


**The optimization objective**

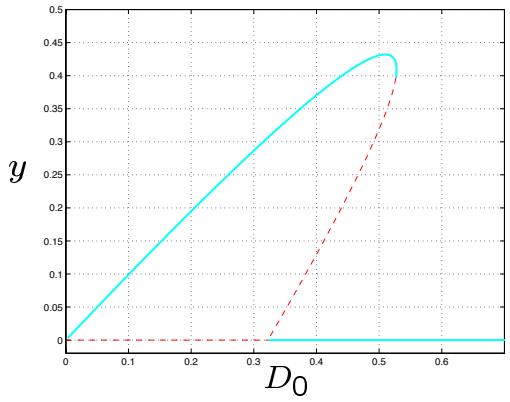
$$y = xD$$

**Bifurcation analysis of open-loop system**

Monod model:



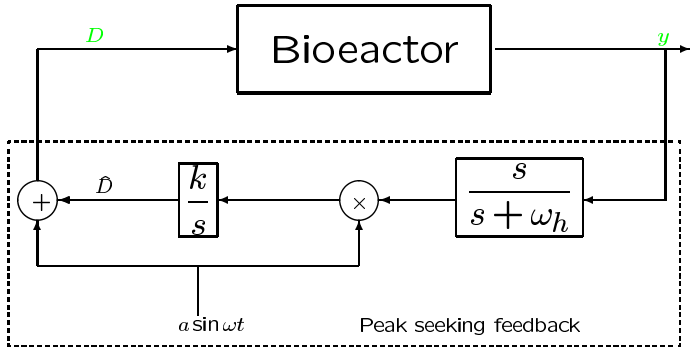
Haldane model:






## Peak seeking via the dilution rate

Peak seeking scheme for the bioreactor

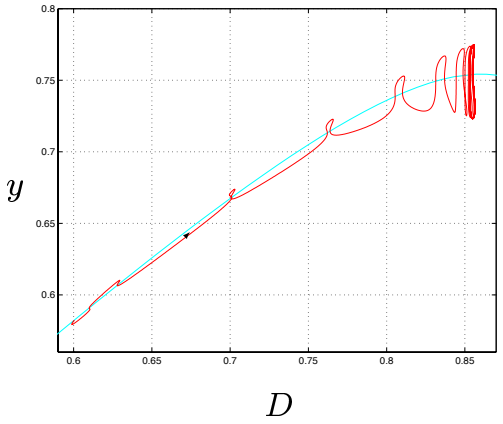



The rule for the choice of parameters

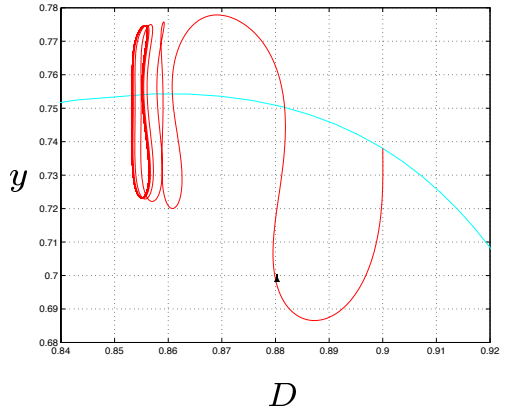
speed of nonlinear dynamics $= O(1) \gg \omega \gg \omega_h, a, k.$

Parameter design

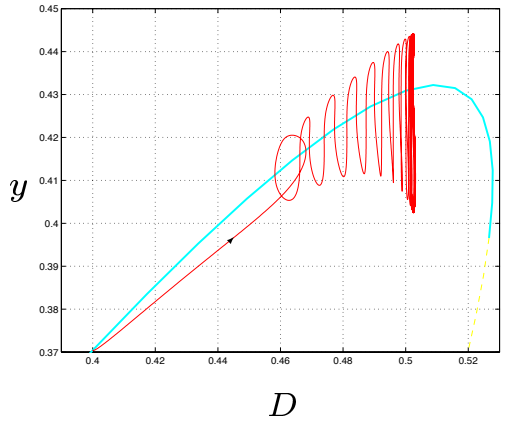
$\omega_h = 0.04\,, \omega = 0.08\,, a = 0.03\,, k = 5\,.$



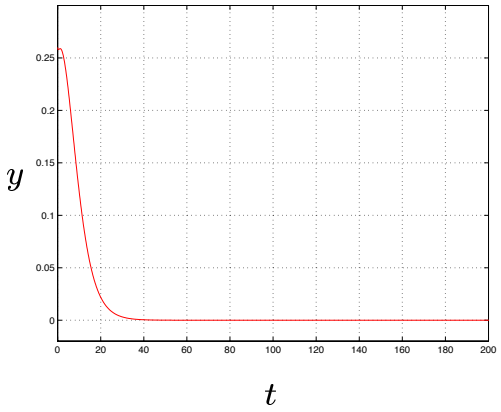



Haldane model:



For the starting point on the right side of the extremum point, the response falls to the wash-out state.



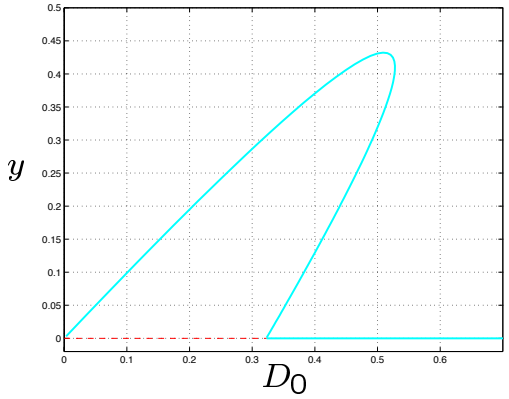


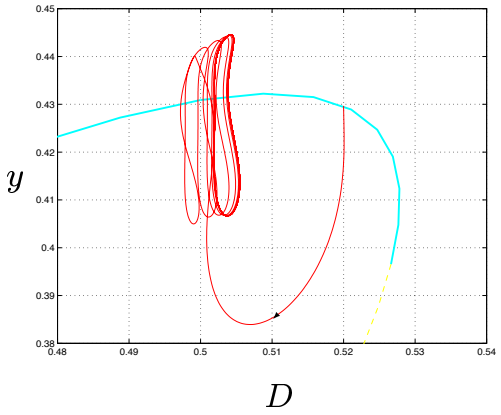

## Feedback with Washout Filters for the Haldane Model

Feedback with washout filters:

$$\begin{aligned} D &= D_0 + k_x(x - x_s) + k_s(s - s_s) \\ \dot{x}_s &= \omega_s x_s + \omega_s x \\ \dot{s}_s &= \omega_s s_s + \omega_s s\,. \end{aligned}$$

Closed loop bifurcation diagram




0.45
0.44
0.43
0.42
0.41
0.4
0.39
0.38
0.48
0.49
0.5
0.51
0.52
0.53
0.54
y
D