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Model-Free Stabilization by Extremum Seeking

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Preface

Originating in 1922, in its 95-year history, extremum seeking has served as a tool for model-free real-time optimization of stable dynamic systems. We introduce a paradigm in which not only is the system being optimized allowed to be time varying and open-loop **unstable**, but also the very goal of extremum seeking is to stabilize the system. The cost function and the control Lyapunov function (CLF) play interchangeable roles, with the unknown optimal set point being implicitly defined through the cost/CLF and coinciding with the equilibrium to be stabilized.

Our "extremum seeking for stabilization" (ESS) consists of employing the CLF as the cost function in a slightly modified extremum seeking algorithm. The goal is to minimize the CLF, i.e., to drive the CLF value to zero over time, which amounts to asymptotic stabilization. Unlike conventional CLF-based stabilization approaches, which employ the knowledge of the system model in the feedback law (Sontag's formula being a "universal" and a particularly clear example of such a feedback law), our ESS approach does not rely on the system model and does not require its knowledge. Instead, ESS employs periodic perturbation signals, along with the CLF. The same effect as that of CLF-based feedback laws that imply the modeling knowledge is achieved, but in a time-average sense.

Averaging is an important tool in the analysis of ESS controllers. Rather than standard averaging, which utilizes integrals of the system's vector field, we employ Lie bracket-based (i.e., derivative-based) averaging, based on weak limits of combinations of dithering terms and their integrals. As results based on averaging are of "approximate" nature, so are the stability properties that we achieve. For instance, in contrast to global stability properties that are achieved by CLF-based control laws that employ the full modeling knowledge, our model-free ESS controllers achieve stability that is semiglobal and "practical" asymptotic (or exponential). This is an acceptable price we pay for achieving model-free stabilization with very simple control algorithms.

In addition to developing simple robust/adaptive model-free stabilizing controllers, we develop new extremum seeking algorithms, which employ bounded updates. One of the corollaries of our effort is also that we provide alternative and more generally applicable solutions to the problem of controlling systems with unknown signs of high-frequency gains (the Morse–Nussbaum problem). While standard adaptive solutions require the high-frequency gains (and their signs) to be constant, our perturbation-based extremum seeking solution allows the high-frequency gain to vary with time and even its sign to change.

Our exposition is mathematically self-contained. We present many illustrative examples and even several experimental applications. The intended audience for this brief ranges from theoretical control engineers and mathematicians to practicing engineers in various industrial areas and in robotics. Chapter 1 motivates the problems considered and gives the overview of the topics. Chapter 2 presents the mathematical foundations on which the rest of the brief is built. Chapters 3–8 present the control designs and their mathematical properties established through theorems. In particular, Chap. 8 demonstrates the generality of our weak-limit averaging approach in utilizing discontinuous and non-differentiable dithers. Chapter 9 presents experimental applications and provides design guidelines.

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