Preface

Inspiration for the book

This book was inspired by a seemingly non-mathematical question of understanding the biological phenomenon of bacterial chemotaxis, where it is conjectured that a simple extremum seeking-like algorithm, employing stochastic perturbations instead of the conventional sinusoidal probing, enables bacteria to move in space towards areas with higher food concentration by estimating the gradient of the unknown concentration distribution.

While constructing stochastic algorithms that both mimic bacterial motions and are biologically plausible in their simplicity is easy, developing a mathematical theory that supports such algorithms was far from straightforward. The algorithms that perform stochastic extremum seeking violate one or more assumptions of any of the available theorems on stochastic averaging. As a result, we were compelled to develop, from the ground up, stochastic averaging and stability theorems that constitute significant generalizations of the existing stochastic averaging theory developed since the 1960s. This book presents the new theorems on stochastic averaging and then develops the theory and several applications of stochastic extremum seeking, including applications to non-cooperative/Nash games and to robotic vehicles. The new stochastic extremum seeking theory constitutes an alternative to established, sinusoid-based, deterministic extremum seeking.

Stochastic averaging

The averaging method is a powerful and elegant asymptotic analysis technique for nonlinear time-varying dynamical systems. Its basic idea can be dated back to the late 18th century, when in 1788, Lagrange formulated the gravitational three-body problem as a perturbation of the two-body problem. No rigorous proof of its validity was given until Fatou provided the first proof of the asymptotic validity of the method in 1928. After the systematic research conducted by Krylov, Bogoliubov, and Mitropolsky, in the 1930s, the averaging method gradually became one of the classical methods in analyzing nonlinear oscillations. In the past three decades, the averaging method has been extensively applied to theoretical research and engineering applications on nonlinear random vibrations.

Stochastic averaging method was first proposed in 1963 by Stratonovich based on physical consideration and later proved mathematically by Khasminskii in 1966. Since then, extensive research interest has developed in stochastic averaging in the fields of mathematics and mechanical engineering.

Stochastic extremum seeking

Extremum seeking is a real-time optimization tool and also a method of adaptive control, although it is different from the classical adaptive control in two aspects: (i) extremum seeking does not fit into the classical paradigm or model reference and related schemes, which deal with the problem of stabilization of a known reference trajectory or set point; (ii) extremum seeking is not model based. Extremum seeking is applicable in situations where there is a nonlinearity in the control problem, and the nonlinearity has a local minimum or a maximum. The nonlinearity may be in the plant, as a physical nonlinearity, possibly manifesting itself through an equilibrium map, or it may be in the control objective, added to the system through a cost functional of an optimization problem. Hence, one can use extremum seeking both for tuning a set point to achieve an optimal value of the output, or for tuning parameters of a feedback law.

With many applications of extremum seeking involving mechanical systems and vehicles, which are naturally modeled by nonlinear continuous-time systems, much need exists for continuous-time extremum seeking algorithms and stability theory. Unfortunately, existing stochastic averaging theorems in continuous time are too restrictive to be applicable to extremum seeking algorithms. Such algorithms violate the global Lipschitz assumptions, do not possess an equilibrium at the extremum, the average system is only locally exponentially stable, and the user's interest is in infinite-time behavior (stability) rather than merely in finite-time approximation.

This book develops the framework of stochastic extremum seeking and its applications. In the first part of the book we develop the theoretical analysis tools of stochastic averaging for general nonlinear systems (Chapters 3 and 4). In the second part of the book, we develop stochastic extremum seeking algorithms for static maps or dynamical nonlinear systems (Chapters 5, 8 and 11). In the third part, we investigate the applications of stochastic extremum seeking (Chapters 6, 7, 9 and 10).

Organization of the book

Chapter 1 is a basic introduction to the deterministic/stochastic averaging theory. Chapter 2 provides a brief review of developments in extremum seeking in the last 15 years and presents a basic idea of stochastic extremum seeking. Chapter

3 presents stochastic averaging theory for locally Lipschitz systems that maintain an equilibrium in the presence of a stochastic perturbation. Chapter 4 presents stochastic averaging theory developed to analyze the algorithms where equilibrium is not preserved and practical stability is achieved. Chapter 5 presents single-input stochastic extremum seeking algorithm and its convergence analysis. Chapter 6 presents an application of single-parameter stochastic extremum seeking to stochastic source seeking by nonholonomic vehicles with tuning angular velocity. Chapter 7 presents stochastic source seeking with tuning forward velocity. Chapter 8 presents multi-parameter stochastic extremum seeking and slope seeking. Chapter 9 presents the application of multi-parameter stochastic extremum seeking to Nash equilibrium seeking for games with general nonlinear payoffs. Chapter 10 presents some special cases of Chapter 9: seeking of Nash equilibria for games with quadratic payoffs and applications to oligopoly economic markets and to planar multi-vehicle deployment. 11 introduces a Newton-based stochastic extremum seeking algorithm, which allows the user to achieve an arbitrary convergence rate, even in multivariable problems, despite the unknown Hessian of the cost function.

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