Preface

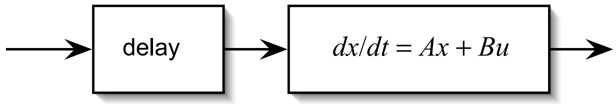
This year is the 50th anniversary of Otto J. Smith's 1959 publication of a control design idea commonly referred as the *Smith predictor* for the compensation of actuator delays. Actuator and sensor delays are among the most common dynamic phenomena that arise in engineering practice but fall outside the scope of the standard finite-dimensional systems.

Predictor-based feedbacks and other controllers for systems with input and output delays have been (and continue to be) an active area of research during the last five decades. Several books exist that focus on the mathematical and engineering problems in this area. The goals of this book are not to duplicate the material in those books nor to present a comprehensive account about control of systems with input and output delays. Instead, the book's goal is to shed light on new opportunities for predictor feedback, through extensions to nonlinear systems, delay-adaptive control, and actuator dynamics modeled by PDEs more complex than transport (pure delay) PDEs.

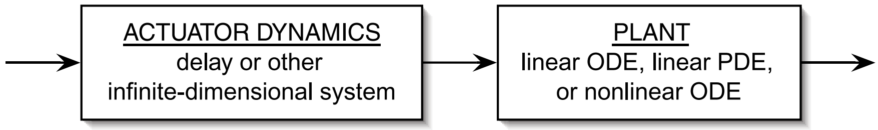
* * *

What Does the Book Cover? This book is a research monograph that introduces the treatment of systems with input delays as PDE-ODE cascade systems with *boundary control*. The PDE-based approach yields Lyapunov–Krasovskii functionals that make the control design constructive and enables stability analysis with quantitative estimates, which leads to the resolution of several long-standing problems in predictor feedback for linear time-invariant (LTI) systems. More importantly, the PDE-based approach enables the extension of predictor feedback design to nonlinear systems and to adaptive control for systems with unknown delays.

However, the book's treatment of input and output delays as transport PDEs allows it to aim even further, in expanding the predictor feedback ideas to systems with other types of infinite-dimensional actuator dynamics and sensor dynamics. We develop methods for compensating heat PDE and wave PDE dynamics at the input of an arbitrary, possibly unstable, LTI-ODE plant. Similarly, we develop observers for LTI-ODE systems with similar types of sensor dynamics. Finally, we introduce problems for PDE-PDE cascades, such as, for example, the notoriously



The standard problem of linear ODE plant with input delay, leading to conventional predictor feedback control.



The problems considered in this book.

difficult problem of a wave PDE with input delay where, if the delay is left uncompensated, an arbitrarily short delay destroys the closed-loop stability (as shown by Datko in 1988).

* * *

Who Is the Book For? The book should be of interest to all researchers working on control of delay systems—engineers, graduate students, and delay systems specialists in academia. The latter group will especially benefit from this book, as it opens several new paradigms for delay research. Many opportunities present themselves to extend the present results to systems that contain state delays (discrete and/or distributed) in addition to input delays.

Mathematicians with interest in the broad area of control of distributed parameter systems, and PDEs in particular, will find the book stimulating because it tackles nonlinear ODEs simultaneously with linear PDEs, as well as PDEs from different classes. These problems present many stimulating challenges for further research on the stabilization of ever-expanding classes of unstable infinite-dimensional systems.

Chemical engineers and process dynamics researchers, who have traditionally been users of the Smith predictor and related approaches, should find the various extensions of this methodology that the book presents (adaptive, nonlinear, other PDEs) to be useful and exciting. Engineers from other areas—electrical and computer engineering (telecommunication systems and networks), mechanical and aerospace engineering (combustion systems and machining), and civil/structural engineering—have no doubt faced problems with actuator delays and other distributed parameter input dynamics and will appreciate the advances introduced by this book.

This book is not meant to be a standalone textbook for any individual graduate course. However, its parts can be used as supplemental material in lectures or projects in many graduate courses:

- general distributed parameter systems (Chapters 2, 3, 6, 14–20),
- linear delay systems (Chapters 2, 3, 6, 18, and 19),
- partial differential equations (Chapters 14–20),
- nonlinear control (Chapters 10–13),

- state estimators/observers (Chapters 3 and 17),
- adaptive control (Chapters 7–9), and
- robust control (Chapters 4 and 5),
- linear time-varying (LTV) systems (Chapter 6).

The background required to read this book includes little beyond the basics of function spaces and Lyapunov theory for ODEs. However, the basics of the Poincaré and Agmon inequalities, Lyapunov and input-to-state stability, parameter projection for adaptive control, and Bessell functions are summarized in appendices for the reader's convenience.

I hope that the reader will not view the book as a collection of problems that have been solved, but will focus on it as a collection of tools and techniques that are applicable in open problems, many more of which exist than have been solved in this book, particularly in the areas of interconnected systems of ODEs and PDEs, systems with simultaneous input and state delays, nonlinear delay systems, and systems with unknown delays.

In no book are all chapters equal in value for the reader. My personal recommendations to a reader on a time budget are Chapters 7, 10, 16, and 18 if the reader is interested mainly in feedback design problems and tools. A reader primarily interested in analysis and robustness problems for delay systems might also enjoy Chapter 5.

Acknowledgments. I would like to thank Delphine Bresch-Pietri, Andrey Smyshlyaev, and Rafael Vazquez for their contributions in Chapters 8, 9, 11, 14, and elsewhere.

I am also grateful to Mrdjan Jankovic for exchanges of ideas and his guidance through the area of control of delay systems. If it were not for Mrdjan's superb and innovative papers on control of nonlinear delay systems, I would never have been enticed to start to work on these problems. I am also pleased to express special gratitude to Iasson Karafyllys for some helpful and inspiring discussions.

Many thanks to Manfred Morari, Silviu Niculescu, Galip Ulsoy, and Qing-Chang Zhong for discussions on delay systems and on the Smith predictor. I would also like to thank Anu Annaswammy for getting me intrigued with her papers on adaptive control of delay systems.

Finally, Petar Kokotovic's encouragement and interest in new research results are priceless—often a key difference between deciding whether or not to spend time on writing a new book.

Over the course of writing this book, I had the pleasure to meet Otto J. M. Smith on the occasion of my visit to the University of California at Berkeley in October 2008. "Predictably," I chose the results on predictor feedback as the topic of my Nokia Distinguished Lecture. Otto Smith was a professor at Berkeley from 1947 until his retirement in 1988. I have never met a 91-year-old person with as sharp a mind as Otto Smith's. Truth be told, I have met few 30-year-olds who would be worth a comparison. Even at this age, Otto Smith was every bit the inventor and creative engineer as his list of patents indicates. I had the pleasure of hearing about his favorite designs, from the HP function generator to his current interest in solar

energy turbine power plants with controlled focusing via heliostats. We never got to discuss the "Smith predictor"—there was so much else worth hearing about from Otto Smith's bank of engineering knowledge. I am grateful to Alex Bayen and Dean Shankar Sastry for arranging for Otto Smith to come to campus that day. I also thank Masayoshi Tomizuka for sharing many thoughts on Otto Smith during my Springer Professor sabbatical stay at Berkeley in the fall of 2007.

Sadly, Otto Smith passed away on May 10, 2009 as a result of a fall at his home. In the five decades since the publication of his influential paper on compensation of dead time, he had seen his idea become one of the most commonly used tools in control practice.

I am grateful to Cymer (Bob Akins, Danny Brown, and other friends) and General Atomics (Mike Reed, Sam Gurol, Bogdan Borowy, Dick Thome, Linden Blue, and other friends) for their support through the *Cymer Center for Control Systems and Dynamics* at UC San Diego. I also very much appreciate the support by Bosch (Nalin Chaturvedi and Aleksandar Kojic) and the National Science Foundation (Kishan Baheti and Suhada Jayasuriya).

Finally, for all the hundreds of evening and weekend hours that were spent on this book and not with my family, for all the mathematics homework that I was excused from helping with, my gratitude and love go to Alexandra, Victoria, and Angela.

La Jolla, California *Miroslav Krstic*
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Contents

Preface v

1 Introduction 1
- 1.1 Delay Systems 1
- 1.2 How Does the Difficulty of Delay Systems Compare with PDEs? . 2
- 1.3 A Short History of Backstepping 3
- 1.4 From Predictor Feedbacks for LTI-ODE Systems to the Results in This Book 4
- 1.5 Organization of the Book 4
- 1.6 Use of Examples 5
- 1.7 Krasovskii Theorem or Direct Stability Estimates? 7
- 1.8 DDE or Transport PDE Representation of the Actuator/Sensor State? 9
- 1.9 Notation, Spaces, Norms, and Solutions 9
- 1.10 Beyond This Book 11

Part I Linear Delay-ODE Cascades

2 Basic Predictor Feedback 17
- 2.1 Basic Idea of Predictor Feedback Design for ODE Systems with Actuator Delay 18
- 2.2 Backstepping Design Via the Transport PDE 19
- 2.3 On the Relation Among the Backstepping Design, the FSA/Reduction Design, and the Original Smith Controller 22
- 2.4 Stability of Predictor Feedback 23
- 2.5 Examples of Predictor Feedback Design 27
- 2.6 Stability Proof Without a Lyapunov Function 30
- 2.7 Backstepping Transformation in the Standard Delay Notation 36
- 2.8 Notes and References 39

3 Predictor Observers . . . 41
3.1 Observers for ODE Systems with Sensor Delay . . . 41
3.2 Example: Predictor Observer . . . 44
3.3 On Observers That Do Not Estimate the Sensor State . . . 46
3.4 Observer-Based Predictor Feedback for Systems with Input Delay 48
3.5 The Relation with the Original Smith Controller . . . 48
3.6 Separation Principle: Stability Under Observer-Based Predictor Feedback . . . 49
3.7 Notes and References . . . 52

4 Inverse Optimal Redesign . . . 53
4.1 Inverse Optimal Redesign . . . 54
4.2 Is Direct Optimality Possible Without Solving Operator Riccati Equations? . . . 59
4.3 Disturbance Attenuation . . . 60
4.4 Notes and References . . . 63

5 Robustness to Delay Mismatch . . . 65
5.1 Robustness in the L_2 Norm . . . 65
5.2 Aside: Robustness to Predictor for Systems That Do Not Need It . . 72
5.3 Robustness in the H_1 Norm . . . 73
5.4 Notes and References . . . 83

6 Time-Varying Delay . . . 85
6.1 Predictor Feedback Design with Time-Varying Actuator Delay . . . 85
6.2 Stability Analysis . . . 88
6.3 Observer Design with Time-Varying Sensor Delay . . . 96
6.4 Examples . . . 97
6.5 Notes and References . . . 101

Part II Adaptive Control

7 Delay-Adaptive Full-State Predictor Feedback . . . 107
7.1 Categorization of Adaptive Control Problems with Actuator Delay 109
7.2 Delay-Adaptive Predictor Feedback with Full-State Measurement . 110
7.3 Proof of Stability for Full-State Feedback . . . 112
7.4 Simulations . . . 117
7.5 Notes and References . . . 119

8 Delay-Adaptive Predictor with Estimation of Actuator State . . . 121
8.1 Adaptive Control with Estimation of the Transport PDE State . . . 121
8.2 Local Stability and Regulation . . . 123
8.3 Simulations . . . 131
8.4 Notes and References . . . 131

9 Trajectory Tracking Under Unknown Delay and ODE Parameters . . . 135
9.1 Problem Formulation 135
9.2 Control Design 137
9.3 Simulations 140
9.4 Proof of Global Stability and Tracking 140
9.5 Notes and References 149

Part III Nonlinear Systems

10 Nonlinear Predictor Feedback 153
10.1 Predictor Feedback Design for a Scalar Plant with a Quadratic Nonlinearity 155
10.2 Nonlinear Infinite-Dimensional "Backstepping Transformation" and Its Inverse 157
10.3 Stability 159
10.4 Failure of the Uncompensated Controller 165
10.5 What Would the Nonlinear Version of the Standard "Smith Predictor" Be? 168
10.6 Notes and References 169

11 Forward-Complete Systems 171
11.1 Predictor Feedback for General Nonlinear Systems 171
11.2 A Categorization of Systems That Are Globally Stabilizable Under Predictor Feedback 173
11.3 The Nonlinear Backstepping Transformation of the Actuator State 176
11.4 Lyapunov Functions for the Autonomous Transport PDE 178
11.5 Lyapunov-Based Stability Analysis for Forward-Complete Nonlinear Systems 181
11.6 Stability Proof Without a Lyapunov Function 187
11.7 Notes and References 190

12 Strict-Feedforward Systems 191
12.1 Example: A Second-Order Strict-Feedforward Nonlinear System . 192
12.2 General Strict-Feedforward Nonlinear Systems: Integrator Forwarding 197
12.3 Predictor for Strict-Feedforward Systems 199
12.4 General Strict-Feedforward Nonlinear Systems: Stability Analysis 201
12.5 Example of Predictor Design for a Third-Order System That Is Not Linearizable 207
12.6 An Alternative: A Design with Nested Saturations 211
12.7 Extension to Nonlinear Systems with Time-Varying Input Delay . . 212
12.8 Notes and References 214

13 Linearizable Strict-Feedforward Systems 217
13.1 Linearizable Strict-Feedforward Systems 218
13.2 Integrator Forwarding (SJK) Algorithm Applied to Linearizable Strict-Feedforward Systems 218
13.3 Two Sets of Linearizing Coordinates 219
13.4 Predictor Feedback for Linearizable Strict-Feedforward Systems . . 220
13.5 Explicit Closed-Loop Solutions for Linearizable Strict-Feedforward Systems 223
13.6 Examples with Linearizable Strict-Feedforward Systems 227
13.7 Notes and References 230

Part IV PDE-ODE Cascades

14 ODEs with General Transport-Like Actuator Dynamics 235
14.1 First-Order Hyperbolic Partial Integro-Differential Equations 235
14.2 Examples of Explicit Design 242
14.3 Korteweg–de Vries-like Equation 243
14.4 Simulation Example 246
14.5 ODE with Actuator Dynamics Given by a General First-Order Hyperbolic PIDE 246
14.6 An ODE with Pure Advection-Reaction Actuator Dynamics 250
14.7 Notes and References 251

15 ODEs with Heat PDE Actuator Dynamics 253
15.1 Stabilization with Full-State Feedback 254
15.2 Example: Heat PDE Actuator Dynamics 261
15.3 Robustness to Diffusion Coefficient Uncertainty 262
15.4 Expressing the Compensator in Terms of Input Signal Rather Than Heat Equation State 264
15.5 On Differences Between Compensation of Delay Dynamics and Diffusion Dynamics 264
15.6 Notes and References 266

16 ODEs with Wave PDE Actuator Dynamics 269
16.1 Control Design for Wave PDE Compensation with Neumann Actuation 270
16.2 Stability of the Closed-Loop System 277
16.3 Robustness to Uncertainty in the Wave Propagation Speed 283
16.4 An Alternative Design with Dirichlet Actuation 290
16.5 Expressing the Compensator in Terms of Input Signal Rather Than Wave Equation State 294
16.6 Examples: Wave PDE Actuator Dynamics 297
16.7 On the Stabilization of the Wave PDE Alone by Neumann and Dirichlet Actuation 302
16.8 Notes and References 304

17 Observers for ODEs Involving PDE Sensor and Actuator Dynamics . 305
17.1 Observer for ODE with Heat PDE Sensor Dynamics 306
17.2 Example: Heat PDE Sensor Dynamics 309
17.3 Observer-Based Controller for ODEs with Heat PDE Actuator Dynamics 310
17.4 Observer for ODE with Wave PDE Sensor Dynamics 316
17.5 Example: Wave PDE Sensor Dynamics 320
17.6 Observer-Based Controller for ODEs with Wave PDE Actuator Dynamics 322
17.7 Notes and References 327

Part V Delay-PDE and PDE-PDE Cascades

18 Unstable Reaction-Diffusion PDE with Input Delay 331
18.1 Control Design for the Unstable Reaction-Diffusion PDE with Input Delay 331
18.2 The Baseline Design ($D = 0$) for the Unstable Reaction-Diffusion PDE 334
18.3 Inverse Backstepping Transformations 335
18.4 Stability of the Target System (w,z) 336
18.5 Stability of the System in the Original Variables (u,v) 339
18.6 Estimates for the Transformation Kernels 341
18.7 Explicit Solutions for the Control Gains 349
18.8 Explicit Solutions of the Closed-Loop System 350
18.9 Notes and References 354

19 Antistable Wave PDE with Input Delay 357
19.1 Control Design for Antistable Wave PDE with Input Delay 357
19.2 The Baseline Design ($D = 0$) for the Antistable Wave PDE 363
19.3 Explicit Gain Functions 365
19.4 Stability of the Target System (w,z) 370
19.5 Stability in the Original Plant Variables (u,v) 377
19.6 Notes and References 383

20 Other PDE-PDE Cascades 385
20.1 Antistable Wave Equation with Heat Equation at Its Input 385
20.2 Unstable Reaction-Diffusion Equation with a Wave Equation at Its Input 388
20.3 Notes and References 391

A Poincaré, Agmon, and Other Basic Inequalities 393

B Input–Output Lemmas for LTI and LTV Systems 397

C Lyapunov Stability and ISS for Nonlinear ODEs 403
C.1 Lyapunov Stability and Class-$\mathcal{K}$ Functions 403
C.2 Input-to-State Stability 406

D Bessel Functions 413
D.1 Bessel Function J_n 413
D.2 Modified Bessel Function I_n 414

E Parameter Projection 417

F Strict-Feedforward Systems: A General Design 421
F.1 The Class of Systems 421
F.2 The Sepulchre–Jankovic–Kokotovic Algorithm 422

G Strict-Feedforward Systems: A Linearizable Class 425
G.1 Linearizability of Feedforward Systems 425
G.2 Algorithms for Linearizable Feedforward Systems 428

H Strict-Feedforward Systems: Not Linearizable 441
H.1 Algorithms for Nonlinearizable Feedforward Systems 441
H.2 Block-Forwarding 444
H.3 Interlaced Feedforward-Feedback Systems 448

References 453

Index 465