

Preface

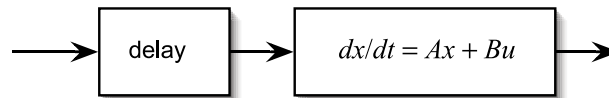
This year is the 50th anniversary of Otto J. Smith's 1959 publication of a control design idea commonly referred as the *Smith predictor* for the compensation of actuator delays. Actuator and sensor delays are among the most common dynamic phenomena that arise in engineering practice but fall outside the scope of the standard finite-dimensional systems.

Predictor-based feedbacks and other controllers for systems with input and output delays have been (and continue to be) an active area of research during the last five decades. Several books exist that focus on the mathematical and engineering problems in this area. The goals of this book are not to duplicate the material in those books nor to present a comprehensive account about control of systems with input and output delays. Instead, the book's goal is to shed light on new opportunities for predictor feedback, through extensions to nonlinear systems, delay-adaptive control, and actuator dynamics modeled by PDEs more complex than transport (pure delay) PDEs.

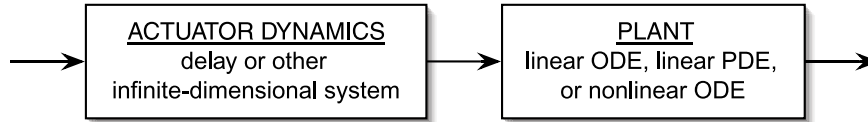
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What Does the Book Cover? This book is a research monograph that introduces the treatment of systems with input delays as PDE-ODE cascade systems with *boundary control*. The PDE-based approach yields Lyapunov–Krasovskii functionals that make the control design constructive and enables stability analysis with quantitative estimates, which leads to the resolution of several long-standing problems in predictor feedback for linear time-invariant (LTI) systems. More importantly, the PDE-based approach enables the extension of predictor feedback design to nonlinear systems and to adaptive control for systems with unknown delays.

However, the book's treatment of input and output delays as transport PDEs allows it to aim even further, in expanding the predictor feedback ideas to systems with other types of infinite-dimensional actuator dynamics and sensor dynamics. We develop methods for compensating heat PDE and wave PDE dynamics at the input of an arbitrary, possibly unstable, LTI-ODE plant. Similarly, we develop observers for LTI-ODE systems with similar types of sensor dynamics. Finally, we introduce problems for PDE-PDE cascades, such as, for example, the notoriously



The standard problem of linear ODE plant with input delay, leading to conventional predictor feedback control.



The problems considered in this book.

difficult problem of a wave PDE with input delay where, if the delay is left uncompensated, an arbitrarily short delay destroys the closed-loop stability (as shown by Datko in 1988).

* * *

Who Is the Book For? The book should be of interest to all researchers working on control of delay systems—engineers, graduate students, and delay systems specialists in academia. The latter group will especially benefit from this book, as it opens several new paradigms for delay research. Many opportunities present themselves to extend the present results to systems that contain state delays (discrete and/or distributed) in addition to input delays.

Mathematicians with interest in the broad area of control of distributed parameter systems, and PDEs in particular, will find the book stimulating because it tackles nonlinear ODEs simultaneously with linear PDEs, as well as PDEs from different classes. These problems present many stimulating challenges for further research on the stabilization of ever-expanding classes of unstable infinite-dimensional systems.

Chemical engineers and process dynamics researchers, who have traditionally been users of the Smith predictor and related approaches, should find the various extensions of this methodology that the book presents (adaptive, nonlinear, other PDEs) to be useful and exciting. Engineers from other areas—electrical and computer engineering (telecommunication systems and networks), mechanical and aerospace engineering (combustion systems and machining), and civil/structural engineering—have no doubt faced problems with actuator delays and other distributed parameter input dynamics and will appreciate the advances introduced by this book.

This book is not meant to be a standalone textbook for any individual graduate course. However, its parts can be used as supplemental material in lectures or projects in many graduate courses:

- general distributed parameter systems (Chapters 2, 3, 6, 14–20),
- linear delay systems (Chapters 2, 3, 6, 18, and 19),
- partial differential equations (Chapters 14–20),
- nonlinear control (Chapters 10–13),

- state estimators/observers (Chapters 3 and 17),
- adaptive control (Chapters 7–9), and
- robust control (Chapters 4 and 5),
- linear time-varying (LTV) systems (Chapter 6).

The background required to read this book includes little beyond the basics of function spaces and Lyapunov theory for ODEs. However, the basics of the Poincaré and Agmon inequalities, Lyapunov and input-to-state stability, parameter projection for adaptive control, and Bessel functions are summarized in appendices for the reader's convenience.

I hope that the reader will not view the book as a collection of problems that have been solved, but will focus on it as a collection of tools and techniques that are applicable in open problems, many more of which exist than have been solved in this book, particularly in the areas of interconnected systems of ODEs and PDEs, systems with simultaneous input and state delays, nonlinear delay systems, and systems with unknown delays.

In no book are all chapters equal in value for the reader. My personal recommendations to a reader on a time budget are Chapters 7, 10, 16, and 18 if the reader is interested mainly in feedback design problems and tools. A reader primarily interested in analysis and robustness problems for delay systems might also enjoy Chapter 5.

Acknowledgments. I would like to thank Delphine Bresch-Pietri, Andrey Smyshlyaev, and Rafael Vazquez for their contributions in Chapters 8, 9, 11, 14, and elsewhere.

I am also grateful to Mrdjan Jankovic for exchanges of ideas and his guidance through the area of control of delay systems. If it were not for Mrdjan's superb and innovative papers on control of nonlinear delay systems, I would never have been enticed to start to work on these problems. I am also pleased to express special gratitude to Iasson Karafyllis for some helpful and inspiring discussions.

Many thanks to Manfred Morari, Silviu Niculescu, Galip Ulsoy, and Qing-Chang Zhong for discussions on delay systems and on the Smith predictor. I would also like to thank Anu Annaswamy for getting me intrigued with her papers on adaptive control of delay systems.

Finally, Petar Kokotovic's encouragement and interest in new research results are priceless—often a key difference between deciding whether or not to spend time on writing a new book.

Over the course of writing this book, I had the pleasure to meet Otto J. M. Smith on the occasion of my visit to the University of California at Berkeley in October 2008. "Predictably," I chose the results on predictor feedback as the topic of my Nokia Distinguished Lecture. Otto Smith was a professor at Berkeley from 1947 until his retirement in 1988. I have never met a 91-year-old person with as sharp a mind as Otto Smith's. Truth be told, I have met few 30-year-olds who would be worth a comparison. Even at this age, Otto Smith was every bit the inventor and creative engineer as his list of patents indicates. I had the pleasure of hearing about his favorite designs, from the HP function generator to his current interest in solar

energy turbine power plants with controlled focusing via heliostats. We never got to discuss the “Smith predictor”—there was so much else worth hearing about from Otto Smith’s bank of engineering knowledge. I am grateful to Alex Bayen and Dean Shankar Sastry for arranging for Otto Smith to come to campus that day. I also thank Masayoshi Tomizuka for sharing many thoughts on Otto Smith during my Springer Professor sabbatical stay at Berkeley in the fall of 2007.

Sadly, Otto Smith passed away on May 10, 2009 as a result of a fall at his home. In the five decades since the publication of his influential paper on compensation of dead time, he had seen his idea become one of the most commonly used tools in control practice.

I am grateful to Cymer (Bob Akins, Danny Brown, and other friends) and General Atomics (Mike Reed, Sam Gurol, Bogdan Borowy, Dick Thome, Linden Blue, and other friends) for their support through the *Cymer Center for Control Systems and Dynamics* at UC San Diego. I also very much appreciate the support by Bosch (Nalin Chaturvedi and Aleksandar Kojic) and the National Science Foundation (Kishan Baheti and Suhada Jayasuriya).

Finally, for all the hundreds of evening and weekend hours that were spent on this book and not with my family, for all the mathematics homework that I was excused from helping with, my gratitude and love go to Alexandra, Victoria, and Angela.

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Miroslav Krstic

Contents

Preface	v
1 Introduction	1
1.1 Delay Systems	1
1.2 How Does the Difficulty of Delay Systems Compare with PDEs? ..	2
1.3 A Short History of Backstepping	3
1.4 From Predictor Feedbacks for LTI-ODE Systems to the Results in This Book	4
1.5 Organization of the Book	4
1.6 Use of Examples	5
1.7 Krasovskii Theorem or Direct Stability Estimates?	7
1.8 DDE or Transport PDE Representation of the Actuator/Sensor State?	9
1.9 Notation, Spaces, Norms, and Solutions	9
1.10 Beyond This Book	11
Part I Linear Delay-ODE Cascades	
2 Basic Predictor Feedback	17
2.1 Basic Idea of Predictor Feedback Design for ODE Systems with Actuator Delay	18
2.2 Backstepping Design Via the Transport PDE	19
2.3 On the Relation Among the Backstepping Design, the FSA/Reduction Design, and the Original Smith Controller	22
2.4 Stability of Predictor Feedback	23
2.5 Examples of Predictor Feedback Design	27
2.6 Stability Proof Without a Lyapunov Function	30
2.7 Backstepping Transformation in the Standard Delay Notation	36
2.8 Notes and References	39

3	Predictor Observers	41
3.1	Observers for ODE Systems with Sensor Delay	41
3.2	Example: Predictor Observer	44
3.3	On Observers That Do Not Estimate the Sensor State	46
3.4	Observer-Based Predictor Feedback for Systems with Input Delay	48
3.5	The Relation with the Original Smith Controller	48
3.6	Separation Principle: Stability Under Observer-Based Predictor Feedback	49
3.7	Notes and References	52
4	Inverse Optimal Redesign	53
4.1	Inverse Optimal Redesign	54
4.2	Is Direct Optimality Possible Without Solving Operator Riccati Equations?	59
4.3	Disturbance Attenuation	60
4.4	Notes and References	63
5	Robustness to Delay Mismatch	65
5.1	Robustness in the L_2 Norm	65
5.2	Aside: Robustness to Predictor for Systems That Do Not Need It ..	72
5.3	Robustness in the H_1 Norm	73
5.4	Notes and References	83
6	Time-Varying Delay	85
6.1	Predictor Feedback Design with Time-Varying Actuator Delay ...	85
6.2	Stability Analysis	88
6.3	Observer Design with Time-Varying Sensor Delay	96
6.4	Examples	97
6.5	Notes and References	101
Part II Adaptive Control		
7	Delay-Adaptive Full-State Predictor Feedback	107
7.1	Categorization of Adaptive Control Problems with Actuator Delay	109
7.2	Delay-Adaptive Predictor Feedback with Full-State Measurement .	110
7.3	Proof of Stability for Full-State Feedback	112
7.4	Simulations	117
7.5	Notes and References	119
8	Delay-Adaptive Predictor with Estimation of Actuator State	121
8.1	Adaptive Control with Estimation of the Transport PDE State ...	121
8.2	Local Stability and Regulation	123
8.3	Simulations	131
8.4	Notes and References	131

9	Trajectory Tracking Under Unknown Delay and ODE Parameters . . .	135
9.1	Problem Formulation	135
9.2	Control Design	137
9.3	Simulations	140
9.4	Proof of Global Stability and Tracking	140
9.5	Notes and References	149
Part III Nonlinear Systems		
10	Nonlinear Predictor Feedback	153
10.1	Predictor Feedback Design for a Scalar Plant with a Quadratic Nonlinearity	155
10.2	Nonlinear Infinite-Dimensional “Backstepping Transformation” and Its Inverse	157
10.3	Stability	159
10.4	Failure of the Uncompensated Controller	165
10.5	What Would the Nonlinear Version of the Standard “Smith Predictor” Be?	168
10.6	Notes and References	169
11	Forward-Complete Systems	171
11.1	Predictor Feedback for General Nonlinear Systems	171
11.2	A Categorization of Systems That Are Globally Stabilizable Under Predictor Feedback	173
11.3	The Nonlinear Backstepping Transformation of the Actuator State	176
11.4	Lyapunov Functions for the Autonomous Transport PDE	178
11.5	Lyapunov-Based Stability Analysis for Forward-Complete Nonlinear Systems	181
11.6	Stability Proof Without a Lyapunov Function	187
11.7	Notes and References	190
12	Strict-Feedforward Systems	191
12.1	Example: A Second-Order Strict-Feedforward Nonlinear System	192
12.2	General Strict-Feedforward Nonlinear Systems: Integrator Forwarding	197
12.3	Predictor for Strict-Feedforward Systems	199
12.4	General Strict-Feedforward Nonlinear Systems: Stability Analysis	201
12.5	Example of Predictor Design for a Third-Order System That Is Not Linearizable	207
12.6	An Alternative: A Design with Nested Saturations	211
12.7	Extension to Nonlinear Systems with Time-Varying Input Delay	212
12.8	Notes and References	214

13	Linearizable Strict-Feedforward Systems	217
13.1	Linearizable Strict-Feedforward Systems	218
13.2	Integrator Forwarding (SJK) Algorithm Applied to Linearizable Strict-Feedforward Systems	218
13.3	Two Sets of Linearizing Coordinates	219
13.4	Predictor Feedback for Linearizable Strict-Feedforward Systems ..	220
13.5	Explicit Closed-Loop Solutions for Linearizable Strict- Feedforward Systems	223
13.6	Examples with Linearizable Strict-Feedforward Systems	227
13.7	Notes and References	230
 Part IV PDE-ODE Cascades		
14	ODEs with General Transport-Like Actuator Dynamics	235
14.1	First-Order Hyperbolic Partial Integro-Differential Equations	235
14.2	Examples of Explicit Design	242
14.3	Korteweg–de Vries-like Equation	243
14.4	Simulation Example	246
14.5	ODE with Actuator Dynamics Given by a General First-Order Hyperbolic PIDE	246
14.6	An ODE with Pure Advection-Reaction Actuator Dynamics	250
14.7	Notes and References	251
15	ODEs with Heat PDE Actuator Dynamics	253
15.1	Stabilization with Full-State Feedback	254
15.2	Example: Heat PDE Actuator Dynamics	261
15.3	Robustness to Diffusion Coefficient Uncertainty	262
15.4	Expressing the Compensator in Terms of Input Signal Rather Than Heat Equation State	264
15.5	On Differences Between Compensation of Delay Dynamics and Diffusion Dynamics	264
15.6	Notes and References	266
16	ODEs with Wave PDE Actuator Dynamics	269
16.1	Control Design for Wave PDE Compensation with Neumann Actuation	270
16.2	Stability of the Closed-Loop System	277
16.3	Robustness to Uncertainty in the Wave Propagation Speed	283
16.4	An Alternative Design with Dirichlet Actuation	290
16.5	Expressing the Compensator in Terms of Input Signal Rather Than Wave Equation State	294
16.6	Examples: Wave PDE Actuator Dynamics	297
16.7	On the Stabilization of the Wave PDE Alone by Neumann and Dirichlet Actuation	302
16.8	Notes and References	304

17	Observers for ODEs Involving PDE Sensor and Actuator Dynamics	305
17.1	Observer for ODE with Heat PDE Sensor Dynamics	306
17.2	Example: Heat PDE Sensor Dynamics	309
17.3	Observer-Based Controller for ODEs with Heat PDE Actuator Dynamics	310
17.4	Observer for ODE with Wave PDE Sensor Dynamics	316
17.5	Example: Wave PDE Sensor Dynamics	320
17.6	Observer-Based Controller for ODEs with Wave PDE Actuator Dynamics	322
17.7	Notes and References	327
Part V Delay-PDE and PDE-PDE Cascades		
18	Unstable Reaction-Diffusion PDE with Input Delay	331
18.1	Control Design for the Unstable Reaction-Diffusion PDE with Input Delay	331
18.2	The Baseline Design ($D = 0$) for the Unstable Reaction-Diffusion PDE	334
18.3	Inverse Backstepping Transformations	335
18.4	Stability of the Target System (w, z)	336
18.5	Stability of the System in the Original Variables (u, v)	339
18.6	Estimates for the Transformation Kernels	341
18.7	Explicit Solutions for the Control Gains	349
18.8	Explicit Solutions of the Closed-Loop System	350
18.9	Notes and References	354
19	Antistable Wave PDE with Input Delay	357
19.1	Control Design for Antistable Wave PDE with Input Delay	357
19.2	The Baseline Design ($D = 0$) for the Antistable Wave PDE	363
19.3	Explicit Gain Functions	365
19.4	Stability of the Target System (w, z)	370
19.5	Stability in the Original Plant Variables (u, v)	377
19.6	Notes and References	383
20	Other PDE-PDE Cascades	385
20.1	Antistable Wave Equation with Heat Equation at Its Input	385
20.2	Unstable Reaction-Diffusion Equation with a Wave Equation at Its Input	388
20.3	Notes and References	391
A	Poincaré, Agmon, and Other Basic Inequalities	393
B	Input–Output Lemmas for LTI and LTV Systems	397

C	Lyapunov Stability and ISS for Nonlinear ODEs	403
C.1	Lyapunov Stability and Class- \mathcal{K} Functions	403
C.2	Input-to-State Stability	406
D	Bessel Functions	413
D.1	Bessel Function J_n	413
D.2	Modified Bessel Function I_n	414
E	Parameter Projection	417
F	Strict-Feedforward Systems: A General Design	421
F.1	The Class of Systems	421
F.2	The Sepulchre–Jankovic–Kokotovic Algorithm	422
G	Strict-Feedforward Systems: A Linearizable Class	425
G.1	Linearizability of Feedforward Systems	425
G.2	Algorithms for Linearizable Feedforward Systems	428
H	Strict-Feedforward Systems: Not Linearizable	441
H.1	Algorithms for Nonlinearizable Feedforward Systems	441
H.2	Block-Forwarding	444
H.3	Interlaced Feedforward-Feedback Systems	448
	References	453
	Index	465