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Predictor Feedback for Delay Systems: Implementations and Approximations

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Preface

Time delays abound in dynamical systems. And long values of such delays often induce instability. The idea of using “predictors” of the future state or output, initiated with the “Smith predictor” in the late 1950s for compensating long delays in actuation, sensing, computation, or communication, prevents such instabilities—for *linear* systems. Next to the PID control, the Smith predictor is arguably the most frequently used feedback strategy.

About half a century after the Smith predictor—in the late 2000s—the predictor idea was extended to nonlinear systems. In principle, any stabilizable nonlinear system can be stabilized in the presence of an arbitrarily long input delay.

The reason why it took half a century to make the advance from linear to nonlinear systems has largely to do with the distinctions between the solvability of linear and nonlinear systems. Predictors employ model-based solutions of differential equations to produce a value of the future state, which the feedback law employs to compensate the delay. Model-based and *explicit* solutions are available for all linear systems. Nonlinear systems, in turn, are generically unsolvable in closed form.

The nonlinear predictor results of the late 2000s, ushered in by the second author of this book, do not tackle the question of solvability of nonlinear systems. They only employ the properties of such solutions, without presenting ways in which those solutions would be generated in real time, to prove stabilization.

This book bridges the chasm between the nonlinear predictor as a concept and the nonlinear predictor as a practical tool.

Since nonlinear predictors are not solutions of nonlinear differential equations that have to be performed just once—off-line—but have to be performed continuously, in real time, developing methods to generate nonlinear predictors is not merely about solving differential equations but about ensuring stability by applying feedback generated by *imperfect* solutions to differential equations in real time.

This book supplies several methods for generating such solutions. We refer to them as *approximate predictors*. The solution methods we offer are diverse and include both numerical methods and methods that employ iterative analytical integrations.

Since this book deals with bringing the nonlinear predictor methodology closer to reality, we address, in addition to delays on the input and output, a number of other "reality-induced" inconveniences:

- partial state measurement,
- sampled measurement,
- uncertainty in the sampling schedule,
- input application via Zero Order Hold,
- the presence of measurement noise and modeling errors.

What Does the Book Cover?

The book covers all aspects of the solution of robust, global stabilization problems by means of predictor feedback for systems with constant delays.

The book is divided into three parts. Part I of the book is devoted to Linear Time-Invariant (LTI) systems. The reason for the study of the application of predictor feedback to LTI systems in a book that is mostly "nonlinear" is mainly pedagogical. All concepts and novelties introduced in the book are clearly shown in the context of LTI systems, and the reader is taught the following notions without the technical difficulties that are present for nonlinear systems:

- the concept of an approximate predictor
- closed-loop robustness to measurement noise and modeling errors under predictor feedback.

Chapter 2 deals with predictor feedback for LTI systems with state measurement. The reader is introduced to three ways of implementing the predictor feedback:

1) the direct implementation,
2) the dynamic implementation, and
3) the hybrid implementation.

All advantages and disadvantages of each way of implementation are discussed, and it is emphasized that each way of implementing predictor feedback leads to a different kind of closed-loop system:

a) the direct implementation leads to a system described by Integral Delay Equations (IDEs),
b) the dynamic implementation leads to a standard time-delayed system with distributed state delays, and
c) the hybrid implementation leads to a complicated hybrid system with delays.

Chapter 2 shows the robustness properties of each implementation but also emphasizes the fact that there are severe disturbance attenuation limitations due to delays. More specifically, novel results are provided for the specification of the disturbance attenuation limitation for every linear and nonlinear system under any kind of controller. Chapter 2 discusses the robustness of predictor feedback with respect to delay perturbations, and simple (but conservative) explicit formulas are provided. Finally, Chapter 2 discusses the concept of the approximate predictor (i.e., a mapping that approximates the future value of the state vector) and its use in predictor feedback.

Chapter 3 is dedicated to the predictor feedback for LTI systems with partial state measurement. The reader is introduced to the Inter-Sample Predictor-Observer-Predictor-Delay Free Controller (ISP-O-P-DFC) control scheme in the context of LTI systems. Indeed, when delayed output measurement is available, we can no longer use the (approximate) predictor mapping directly. First, we have to use an observer (and an inter-sample predictor if the measurement is sampled) in order to obtain a continuous estimation of the past value of the state vector. Only then we are in a position to use the (approximate) predictor mapping, which can provide us an estimation of the future value of the state.

Part II of the book is devoted to nonlinear time-invariant systems. All notions introduced in a pedagogical way for LTI systems are now used in full extent. The use of approximate predictors is almost always necessary because the predictor mapping is available explicitly only for limited classes of nonlinear systems. Chapters 4 and 5 show that predictor feedback can be applied successfully with approximate predictors to wide classes of nonlinear systems.

Part III of the book is devoted to extensions of predictor feedback either in a conceptual level or to different classes of systems. The conceptual extension of predictor feedback leads us to the notion of a system described by IDEs, which are analyzed in detail in Chapter 7. The extension of predictor feedback to other classes of systems leads us to the application of predictor feedback methodologies to discrete-time systems: this is the topic of Chapter 8.

Who Is the Book for?

This book should be of interest to researchers working on control of time-delay systems. Many engineers, mathematicians, and students are working on important control-theoretic aspects of time-delay systems, and a significant number of them have become users of predictor feedback methodologies. A Ph.D. student can find the current state of the art in predictor feedback design in this book.

Time-Delay systems are abundant in many sciences: physics, biology, engineering and economics. Stabilization problems arise naturally in many cases, and the implementation of globally stabilizing predictor feedback laws can solve such problems elegantly and robustly.

Mathematicians may also be interested in the material of Chapter 7 on IDEs. It has been shown that IDEs are closely related to systems described by first-order hyperbolic partial differential equations arising in mathematical physics, mathematical biology, and engineering.

The book starts from the already fairly advanced state of the art in stability theory and feedback design for nonlinear ordinary differential equations. We assume that the reader is familiar with stability and feedback stabilization theory for uncertain nonlinear finite-dimensional systems at least at a moderately advanced level. The mathematical background needed for the complete comprehension of the results in the present book is provided in Chapter 1, where many useful notions in nonlinear mathematical control theory are reviewed for the reader's convenience. The reader who is not familiar with these notions can return to Chapter 1 when needed.

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