

# High-Fidelity Photovoltaic Array Modeling

Azad Ghaffari

## Abstract

In this note, we provide high-fidelity models of photovoltaic (PV) modules that allow, for example, for non-unimodal power characteristics (multiple local power maxima in the power-voltage curve) as a result of partial shading. The models are validated against manufacturer datasheet information.

## 1 Introduction

While several papers have discussed PV models, for example, [1–4], we believe the modeling results that we present (a) are comprehensive in that they present all the information required to build and simulate PV modules based on manufacturer datasheet information, and (b) allow for simulation of non-unimodal power characteristics, i.e., multiple local maxima in the power-voltage curve as a result of partial shading. The results we present are close to the ones reported in [5], which discusses maximum power point tracking design for the case of non-unimodal power characteristics with PV strings and [6], where the authors present a high-fidelity hardware PV module emulation system, that covers a wide range of environmental conditions and different PV modules.

## 2 PV Model Based on Manufacturer Data

Our starting point is the PV cell model from [4], with the cell modeled as an ideal current source  $I_{ph}$  in parallel with an ideal diode with voltage  $V_d$ , that both depend on the irradiance  $S$  and temperature  $T$ . A schematic of the PV cell is shown in Fig. 1. The model also contains a series resistance  $R_s$  and parallel resistance  $R_p$ , that account for the electrical losses and contactor resistance respectively. The  $I$ - $V$  characteristics of the PV cell are then described by:

$$I = I_{ph} - I_0 \left( \exp \left( \frac{(V + IR_s)}{V_t} \right) - 1 \right) - \frac{(V + IR_s)}{R_p} \quad (1)$$

$$I_{ph} := (I_{ph}^r + k_i(T - T_r)) \left( \frac{S}{1000} \right) \quad (2)$$

$$I_0 := I_0^r \left( \frac{T}{T_r} \right)^3 \exp \left( \left( \frac{T}{T_r} - 1 \right) \frac{E_g}{V_t} \right), \quad (3)$$

and  $V_t := AkT/q$  is thermal voltage of the cell,  $E_g$  is bandgap energy for the semiconductor,  $A$  is an “ideality factor”,  $I_0^r$  is the reference reverse saturation current,  $I_{ph}^r$  is the reference short-circuit current,  $T_r$  is the reference temperature,  $k_i$  is short-circuit temperature coefficient,  $k = 1.38 \times 10^{-23}$  J/°K is Boltzmann’s constant, and  $q = 1.6 \times 10^{-19}$  C is the charge on an electron. The diode voltage  $V_d$  and diode current  $I_d$  are related as  $I_d = I_0 (\exp(V_d/V_t) - 1)$  and  $V_d = V + IR_s$ .

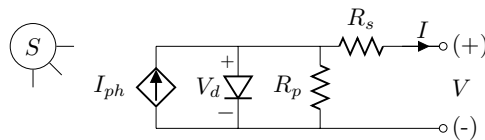


Fig. 1: Equivalent circuit of a PV cell.

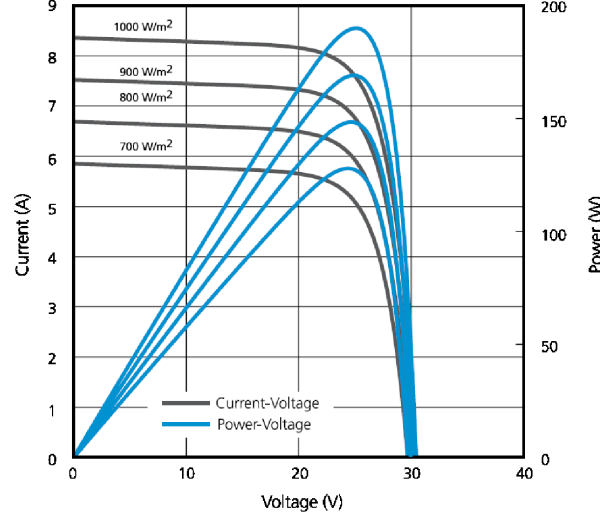


Fig. 2: Current (black) and power (blue) characteristics for PV module UD6 from Mitsubishi Electric.

Sample  $P$ - $V$  and  $I$ - $V$  characteristics for PV module model UD6 from Mitsubishi Electric with maximum power 190 W are shown in Fig. 2.

Simulation of a PV cell/module requires numerical values for  $k_i$ ,  $I_0^r$ ,  $I_{ph}^r$ ,  $A$ ,  $E_g$ ,  $R_s$ , and  $R_p$ . PV manufacturers do not provide these parameters, instead they provide other technical data measured under “Standard Test Conditions” (STC), namely  $S = 1000\text{W}/\text{m}^2$ ,  $T = 25^\circ\text{C}$ , that includes Maximum Power Point (MPP) voltage  $v_m$ , MPP current  $i_m$ , open circuit voltage  $v_{oc}$ , short circuit current  $i_{sc}$ , temperature coefficient for MPP  $k_p$ , temperature coefficient for open circuit voltage  $k_v$ , temperature coefficient for short circuit current  $k_i$ , number of parallel branches  $n_p$ , number of PV cells in each parallel branch  $n_s$ , and the number of cells in each string  $n_a$ . The relationship between a PV cell and PV module current and voltage is simply  $I = i/n_p$ , and  $V = v/n_s$ , where capital letters are used for a cell, and lowercase letters represent a module.

We briefly describe how to obtain the required numerical values for simulation from manufacturer provided data. Short-circuit quantities  $I_{sc}$  and  $V_{sc} = 0$ , and open-circuit quantities  $I_{oc} = 0$  and  $V_{oc}$  are related as

$$I_{ph} = \frac{R}{R_p} I_{sc} \quad (4)$$

$$I_0 = \left( \frac{(RI_{sc} - V_{oc})}{R_p} \right) \exp \left( -\frac{V_{oc}}{V_t^r} \right), \quad (5)$$

where  $R := R_p + R_s$ ,  $V_t^r := AkT_r/q$ , and  $T_r = 298.15$  °K is the module junction temperature under STC. Replacing  $I_m$  and  $V_m$  in (1), and using (4) and (5) results in

$$(RI_{sc} - V_{oc}) \exp \left( \frac{V_m - V_{oc} + R_s I_m}{V_t^r} \right) + R(I_m - I_{sc}) + V_m = 0. \quad (6)$$

Making use of the fact that a PV module has a unique maximum point  $(V_m, P_m)$  for its  $P$ - $V$  characteristic, at which

$$\left. \frac{dP}{dV} \right|_{V=V_m, I=I_m} = I_m + \left. \frac{dI}{dV} \right|_{V=V_m, I=I_m} V_m = 0, \quad (7)$$

along with  $I = f(I, V)$  so that

$$df(I, V) = \frac{\partial f(I, V)}{\partial I} dI + \frac{\partial f(I, V)}{\partial V} dV, \quad (8)$$

and substituting in (7) and simplifying, leads to

$$\left(R_s - \frac{V_m}{I_m}\right)(RI_{sc} - V_{oc})\exp\left(\frac{(V_m - V_{oc}R_sI_m)}{V_t^r}\right) + \left(R - \frac{V_m}{I_m}\right)V_t^r = 0. \quad (9)$$

The short-circuit  $I$ - $V$  relationship can be shown (see [1] for example) to be

$$\left.\frac{dI}{dV}\right|_{\substack{V=0 \\ I=I_{sc}}} = -\frac{1}{R_p}. \quad (10)$$

Using this, along with (8), and more steps of simplification results in

$$V_t^r R_s + (R_s - R_p)(RI_{sc} - V_{oc})\exp\left(\frac{(R_s I_{sc} - V_{oc})}{V_t^r}\right) = 0. \quad (11)$$

It is possible to solve (6), (9), and (11) using a numerical approach like Newton-Raphson to obtain the values of  $R_s$ ,  $R_p$ , and  $A$ . The procedure is described in other references (see, for example, [7]), and we use the same method here.

It is well known that the variation of short-circuit current with respect to temperature and irradiance follows

$$I_{sc}(T, S) = (I_{sc}^r + k_i(T - T_r)) \frac{S}{S_r}, \quad (12)$$

while the variation of the open-circuit voltage with respect to irradiance can be neglected, and its temperature dependence given by

$$V_{oc}(T) = V_{oc}^r + k_v(T - T_r). \quad (13)$$

Using (4), (5), (12), and (13), we get

$$I_{ph}(T, S) = \frac{R}{R_p} I_{sc}(T, S) \quad (14)$$

$$I_0(T, S) = \left(I_{ph}(T, S) - \frac{V_{oc}(T)}{R_p}\right) \exp\left(-\frac{V_{oc}(T)}{V_t^r}\right) \quad (15)$$

which along with (3) can be used to compute  $E_g$ .

The modeling for a PV cell can be extended to PV strings, where a cascade collection of cells within the module are supported by a bypass diode. Assume each PV string has  $n_a$  cells, then the following equation describes the relationship between voltage  $v_a$ , and current  $i_a$ , for each string

$$i_a = I_{ph} - I_0 \left( \exp\left(\frac{v_a + i_a R_s}{n_a V_t}\right) - 1 \right) - \left(\frac{v_a + i_a R_s}{R_p}\right). \quad (16)$$

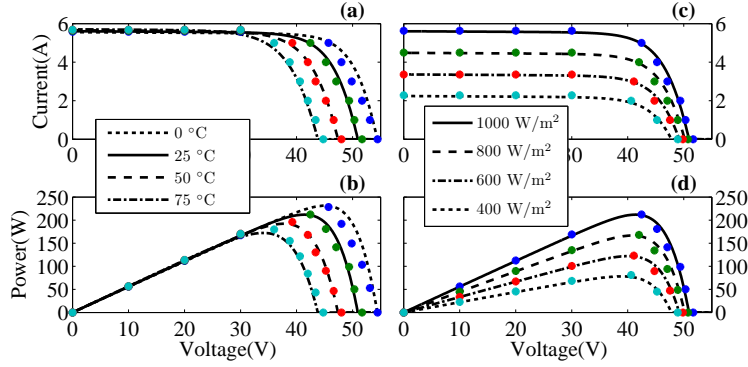


Fig. 3: Sanyo PV module 215N model verification.

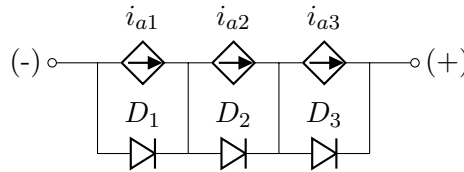


Fig. 4: Equivalent circuit for PV module 215N from Sanyo including three strings and bypass diodes.

### 3 Simulation Results and Concluding Remarks

For the numerical PV model described in Section 2, we present our validation results for the PV module model 215N from Sanyo with  $p_m = 215$  W,  $v_m = 42$  V,  $i_m = 5.13$  A,  $v_{oc} = 51.6$  V,  $i_{sc} = 5.61$  A,  $k_v = -0.143$  V/°C,  $k_i = 1.96$  mA/°C,  $n_s = 72$ ,  $n_a = 24$ , and  $n_p = 1$  under STC. The estimates of the unknown parameters using the procedure described above are:  $R_s = 2.46$  m $\Omega$ ,  $R_p = 8.7$   $\Omega$ ,  $A = 1.81$ ,  $I_0^r = 1.13 \times 10^{-6}$  A, and  $E_g = 1.16$  eV. The result of the model verification is shown in Fig. 3. The derived model fits the actual data almost everywhere. The largest error happens when temperature is 0 °C.

As previously mentioned, an important feature of this model is its ability to simulate the effects of partial shading. The schematic for this module is shown in Fig. 4, i.e., the module has 3 strings, with 24 cells in each string. A plot of the variation of current and power of the PV module under two different partial shading conditions is shown in Fig. 5. Since the selected module has three bypass diodes, the power curve shows three distinct local maxima.

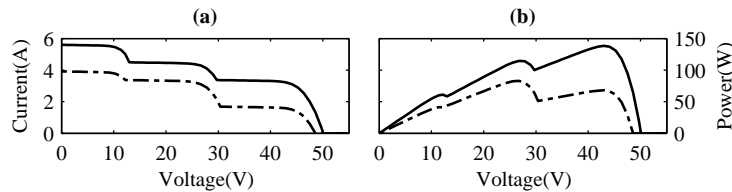


Fig. 5: Non-unimodal  $P$ - $V$  due to partial shading.

## References

- [1] Daniel S. H. Chan and Jacob C. H. Phang, “Analytical methods for the extraction of solar-cell single- and double-diode model parameters from I-V characteristics,” *IEEE Transactions on Electron Devices*, vol. ED-34, pp. 286–293, 1987.
- [2] J. C. H. Phang, D. S. H. Chan, and J. R. Phillips, “Accurate analytical method for the extraction of solar cell model parameters,” *Electronics Letters*, vol. 20, pp. 406–408, 1984.
- [3] D. Sera, R. Teodorescu, and P. Rodriguez, “PV panel model based on datasheet values,” in *IEEE International Symposium on Industrial Electronics*, 2007.
- [4] G. Vachtsevanos and K. Kalaitzakis, “A hybrid photovoltaic simulator for utility interactive studies,” *IEEE Transactions on Energy Conversion*, vol. EC-2, pp. 227–231, 1987.
- [5] Sairaj V. Dhople, Jonathan L. Ehlmann, Ali Davoudi, and Patrick L. Chapman, “Multiple-input boost converter to minimize power losses due to partial shading in photovoltaic modules,” in *Proc. of IEEE Energy Conversion Congress and Exposition (ECCE)*, 2010.
- [6] Woojoo Lee, Younghyun Kim, Yanzhi Wang, Naehyuck Chang, Massoud Pedram, and Soohye Han, “Versatile high-fidelity photovoltaic module emulation system,” in *Proc. of ISLPED, Fukuoka, Japan*, Aug 2011.
- [7] Jordana Bratt, “Grid connected PV inverters: Modeling and Simulation,” M.S. thesis, Electrical and Computer Eng., San Diego State University, 2011.