Design of Distributed Controllers for Component Swapping Modularity Using Linear Matrix Inequalities

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***Abstract*— The problem of component swapping modularity (CSM) refers to distributed control design in networks of smart components such that specific design constraints are satisfied. The CSM intends to reduce control design effort and complexity in platform-based systems. Existing CSM methods achieve promising results for low order multi-input-multi-output (MIMO) systems. However, lack of generalization, heavy computational burden, and, to a lower extent, the level of designer involvement limit the applications of the existing CSM methods. Thus, this paper presents a generalized CSM algorithm using linear matrix inequalities (LMIs) such that almost full automatic control distribution is achieved for an arbitrary linear system. The LMI-based CSM is designed to maintain both disturbance attenuation and quadratic stability. Also, it is desired to satisfy specific time response criteria. Thus, the proposed algorithm combines H_2 optimization and robust H_∞ optimization to satisfy given design constraints. The designer involvement is dramatically reduced to iterative tuning of two scalar parameters in the robust H_∞ problem. The proposed algorithm incorporates reference tracking. Also, stability measures and design criteria are checked numerically at each step. The LMI-based CSM algorithm has been numerically verified using an engine idle speed control (ISC) example.**

I. Introduction

Component swapping modularity (CSM) refers to achieving a desired level of closed-loop performance only by tuning a minimal part of the controller which may reside in a smart swappable component [1]–[5]. A swappable smart component and its local controller are shown in Fig. 1. Conventional control design methods demand complete control redesign when swapping a system's component with a dynamically different counterpart. Depending on dynamic complexity of the swapped module and given performance criteria, one can use CSM algorithms to achieve desired closed-loop performance only by tuning the low-order controller of the swapped component. The major part of the control remains unchanged for all the variants of the swappable component. The CSM algorithms dramatically simplify control design and reduce calibration time and effort, for example, in applications from the automotive industry [6] and power networks.

One may spot similarities between the CSM design methods and plug-and-play control design [7], [8]. However, two objectives separate CSM design methods from plug-and-play control: i) the control structure designed by CSM accommodates a wide variety of swappable modules by utilizing output feedback and bidirectional communication,

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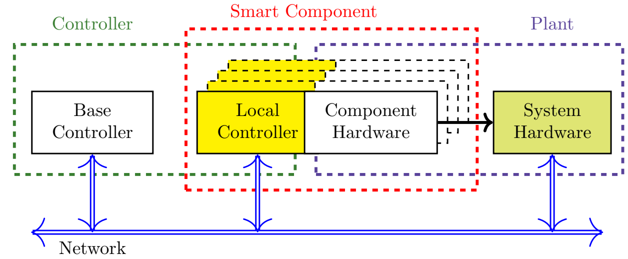

Fig. 1: Swappable smart component in a control network

and ii) CSM presents quantitative measures to specify the order of the local and base controllers to achieve the most feasible distributed control structure. As in plug-and-play control methods, one can add online tuning features to the CSM design.

Existing methods to achieve CSM in networks include: i) the 3-Step Method [1], [2] and ii) the Direct Method [3], [4] for distributed controller design. In the 3-Step Method, first, the centralized controller is designed for each system configuration with a different component variant. Then, the order and structure of the distributed controller is assumed, such that only the local controller is tuned when the smart component is swapped. Then, the CSM metric is maximized by exact (or approximate) matching of the transfer function of the distributed controller with that of the centralized controller. On the other hand, the Direct Method uses a bi-level optimization problem to calculate the distributed controller gains using a multidisciplinary design optimization. Moreover, a sensitivity analysis of the control signals with respect to the component hardware parameters is used for effective controller distribution.

The 3-Step Method [1], [2] starts with centralized control design, then the control is distributed using model order reduction based on pole locus analysis. The 3-Step Method is proven to be effective for low order multi-input-multi-output (MIMO) systems. However, it is case-sensitive and highly reliant on the designer's knowledge of the system and control. Thus, the 3-Step Method is not easy to expand to high order MIMO systems.

The Direct Method [3], [4] has reduced the designer's involvement dramatically and improved CSM, considerably. The Direct Method consists of a bi-level non-convex optimization. The convergence of the algorithm depends on the initial guess which is generated using the optimal centralized controller parameters. At both levels the Direct Method uses a constrained minimization algorithm (e.g., solver fmincon in MATLAB) which results in lengthy convergence time. Thus, the practical applications of the Direct Method is also limited to low order MIMO systems.

This paper focuses on presenting a self-contained CSM algorithm that covers high-order MIMO systems, improves the algorithm convergence time dramatically, and minimizes the designer's involvement. The backbone of the algorithm consists of H_2 and robust H_∞ optimization. Linear matrix inequalities (LMIs) have been developed to obtain the distributed controller. As shown in Fig. 1, the controller is divided into local and base controller to facilitate CSM. Since the local controller can be tuned for each variant of the smart module, then an H_2 problem is formulated to obtain the local controller. The base controller is responsible to maintaining the closed-loop stability of all variants of the local loop which includes the smart component and system hardware. The actual local loop dynamics consists of parametric uncertainty in input, output, and state. The base controller is designed using the uncertain nominal model of the local loop. The results of Khargonekar et al. [9], Xie et al. [10], and Gu [11] are used to convert the uncertain local loop to a scaled uncertainty-free system and to design the base controller which also guarantees reference tracking.

The LMIs are developed using the results by Scherer et al. [12] for continuous time systems. Discrete time LMIs can be obtained by following de Oliveira et al. [13]. The proposed CSM algorithm assumes that the combination of the base and local controller is of the order of the plant. Thus, the base and local controller are reduced order. The topic of reduced order control synthesis and design, particularly using LMIs [14]–[16], is an ongoing effort. However, numerical complexity, especially for high order systems, and lack of necessary conditions limit the applications of the existing reduced order control design methods. Moreover, reduced order control design is not the focus of this work and, instead, model order reduction is used such that at each stage the base and local controller with desired orders are obtained.

This paper takes the initial steps towards control design using LMIs to achieve CSM. The design and stability proof of the proposed algorithm is guaranteed using a combination of analytical and iterative numerical methods. This paper gathers necessary tools and presents clear guidelines to solve the CSM problem in smart control networks. The results are verified numerically on an engine idle speed control (ISC) example. Although input delay causes the ISC to become non-minimum phase the proposed LMI-based CSM successfully achieves the desired performance criteria.

The remainder of the paper is organized as follows: Section II explains the CSM problem and lays the ground work to proceed with the local controller design using H_2 optimization in Section III and to design the base controller using robust H_∞ optimization in Section IV. Numerical simulation of the ISC example [4], [5] is given in Section V. Section VI concludes the paper.

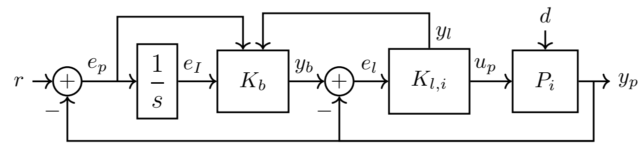

Fig. 2: Configuration of the distributed controller

II. Problem Statement

Figure 2 shows a block diagram of the distributed control system of Fig. 1. The controller is distributed into the local controller, $K_{l,i}$, and the base controller, K_b. An integrator array is added to achieve step reference tracking. The system dynamics, including the swappable component hardware, is lumped into P_i which is represented as

$$P_i : \begin{cases} \dot{x}_p = A_{p,i}x_p + B_{p,i}u_p + B_{d,i}d \\ y_p = C_{y,i}x_p + D_{d,i}d \end{cases}, \tag{1}$$

where $x_p \in \mathbb{R}^{n_p}$, $u_p \in \mathbb{R}^{r_p}$, $d \in \mathbb{R}^{r_d}$, $y_p \in \mathbb{R}^{m_p}$, the system matrices are of appropriate dimensions, and $i \in s_p$, where $s_p = \{1, 2, \cdots, n\}$ and n is the number of the swappable component variants.

It is assumed that the system dynamics of P_i are known. The base and local controllers are of dynamic output feedback type. To start with the CSM design, first the order of the base controller is selected as $n_b \in s_b$, where $s_b = \{0, 1, \cdots, n_p\}$. The order of the local controller is obtained such that $n_b + n_l = n_p$. The base and local controller can change from full order dynamic output feedback to static output feedback. The local controller offers customized tuning for each variant of the swappable component. Increasing the order of the local controller improves closed-loop performance. However, tuning a high order controller limits the practicality of the swappable distributed design. The CSM algorithm obtains the lowest order of the local control such the all design constraints are satisfied. The local controller is designed using H_2 optimization such that certain time response features are achieved.

The base controller, designed once for a given n_b, guarantees closed-loop stability and disturbance attenuation for all the local loops, which include the local controller and the plant. Since the local loop includes both disturbance and parametric uncertainty one can use the quadratic stabilization theory [9], [17] to design the base controller. The problem of quadratic stabilization is to find a feedback controller such that the closed-loop system is stable with a fixed (uncertainty-independent) Lyapunov function [10]. Khargonekar et al. [9] have shown that a certain type of quadratic stabilization problem is essentially an H_∞ control problem. Similar results for discrete-time systems can be found in the paper by Packard and Doyle [17]. The problem at hand involves both robust stabilization and H_∞ control, so it is referred to as *robust H_∞ control*. Xie et al. [10] have shown that the problem of robust H_∞ control can be cast into a scaled H_∞ control for a system without parameter uncertainty, thus allowing the designer to solve the robust H_∞ control via existing H_∞ control techniques.

An overview of the CSM design procedure is shown in Algorithm 1. The H_2 and H_∞ performance indices are selected, and modified if necessary during the design procedure, such that closed-loop stability is achieved and time response criteria are satisfied. The design steps of the local and base controllers are presented in the next two succeeding sections.

III. Local Controller Design

The CSM design starts with local controller design. So the base controller is neglected and the simplified system is shown in Fig. 3. The system dynamics are transformed into the general control configuration

$$P_i : \begin{cases} \dot{x}_p = A_{p,i}x_p + B_{p,i}u_p + \hat{B}_{p,i}w_l \\ v_p = C_{p,i}x_p + \hat{D}_{p,i}w_l \end{cases}, \tag{2}$$

where $v_p = e_l$, $w_l = [y_b^T \quad d^T]^T$, $\hat{B}_{p,i} = [0_{n_p \times r_p} \quad B_d]^T$, $C_{p,i} = -C_{y,i}$, and $\hat{D}_{p,i} = [I_{m_p} \quad -D_{p,i}]$. The local controller is of the order n_l and represented as

$$K_{l,i} : \begin{cases} \dot{x}_l = A_{l,i}x_l + B_{l,i}v_p \\ u_p = C_{l,i}x_l + D_{l,i}v_p \\ y_l = C_{l,i}x_l \end{cases}. \tag{3}$$

The local controller is designed using H_2 optimization with the following performance index

$$z_2 = C_2x_p + D_2u_p + \hat{D}_2w_l. \tag{4}$$

First, plant P_i is reduced to a model of order n_l

$$P_i^r : \begin{cases} \dot{\xi}_p = \mathcal{A}_{p,i}\xi_p + \mathcal{B}_{p,i}u_p + \hat{\mathcal{B}}_{p,i}w_l \\ v_p = \mathcal{C}_{p,i}\xi_p + \hat{D}_{p,i}w_l \\ z_2 = \mathcal{C}_2\xi_p + D_2u_p + \hat{D}_2w_l \end{cases}, \tag{5}$$

where $\xi_p \in \mathbb{R}^{n_l}$ and the input, output, and performance channel dimensions remain the same as P_i. With reduced order plant P_i^r and local controller $K_{l,i}$ defined as above, the reduced order local loop admits the realization

$$\mathcal{T}_{pl,i} : \begin{cases} \dot{\xi}_{pl} = \mathcal{A}_{pl,i}\xi_{pl} + \mathcal{B}_{pl,i}w_l \\ z_2 = \mathcal{C}_{pl,i}\xi_{pl} + \mathcal{D}_{pl,i}w_l \end{cases}, \tag{6}$$

Algorithm 1 CSM design procedure

```
procedure CSM–DESIGN
    Define H2 and H∞ perfomance indices
    for each n_b ∈ s_b do
        n_l = n_p − n_b
top:
        Reduce plant model order to n_l
        for each i ∈ s_p do
            Design K_{l,i} using H2 optimization
        end for
        Calculate the scaled model of the local loop
        Reduce the scaled model order to n_b
        Design K_b using H∞ optimization
        if Unstable or performance criteria violated then
            Modify H2 and H∞ perfomance indices
            goto top.
        end if
    end for
end procedure
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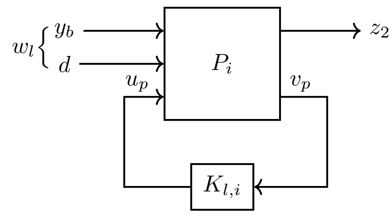

Fig. 3: Local loop transformed into general control configuration

where $\xi_{pl} = [\xi_p^T \quad x_l^T]^T$ and

$$\left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right]_{pl,i} = \left[\begin{array}{cc|c} \mathcal{A}_p + \mathcal{B}_pD_l\mathcal{C}_p & \mathcal{B}_pC_l & \hat{\mathcal{B}}_p + \mathcal{B}_pD_l\hat{D}_p \\ B_l\mathcal{C}_p & \mathcal{A}_l & B_l\hat{D}_p \\ \hline \mathcal{C}_2 + D_2D_l\mathcal{C}_p & D_2C_l & \hat{D}_2 + D_2D_l\hat{D}_p \end{array}\right]_i. \tag{7}$$

Assume that $\mathcal{A}_{pl,i}$ is stable and $\mathcal{D}_{pl,i} = 0$ for a given i. The H_2 norm of $\mathcal{T}_{pl,i}$ is defined by

$$\|\mathcal{T}_{pl,i}\|_2^2 := \frac{1}{2}\int_{-\infty}^{+\infty} \mathrm{Tr}\left(\mathcal{T}_{pl,i}(j\omega)^H\mathcal{T}_{pl,i}(j\omega)\right)\mathrm{d}\omega, \tag{8}$$

where Tr stands for trace of a marix. With an auxiliary parameter $\mathcal{Q}_i > 0$, the following analysis result is obtained: $\mathcal{A}_{pl,i}$ is stable and $\|\mathcal{T}_{pl,i}\|_2^2 < \nu$ iff there exist symmetric $\mathcal{P}_i$ and $\mathcal{Q}_i$ such that the following LMIs are satisfied.

$$\begin{bmatrix} \mathcal{A}_{pl}^T\mathcal{P} + \mathcal{P}\mathcal{A}_{pl} & \mathcal{P}\mathcal{B}_{pl} \\ \mathcal{B}_{pl}^T\mathcal{P} & -I \end{bmatrix}_i < 0 \tag{9}$$

$$\begin{bmatrix} \mathcal{P} & \mathcal{C}_{pl}^T \\ \mathcal{C}_{pl} & \mathcal{Q} \end{bmatrix}_i > 0 \tag{10}$$

$$\mathrm{Tr}(\mathcal{Q}_i) < \nu, \qquad \mathcal{D}_{pl,i} = 0. \tag{11}$$

One can refer to Scherer et al. [12] to find the appropriate transformations, (33)–(35) and (39)–(40), and the relevant LMIs, (41) and inequality set (v), to solve the H_2 problem and calculate $K_{l,i}$ for each P_i.

IV. Base Controller Design and Closed-Loop Stability

For a given order $n_b \in s_b$, the base controller will be designed such that the closed-loop system is stable for all the variants of the local loop, which includes the local controller $K_{l,i}$ and plant P_i, and external disturbance is attenuated to its minimum feasible level.

The local loop is transformed into general control configuration

$$T_{pl,i} : \begin{cases} \dot{x}_{pl} = A_{pl,i}x_{pl} + B_{pl,i}y_b + \hat{B}_{pl,i}w_b \\ v_{pl} = C_{pl,i}x_{pl} + \hat{D}_{pl,i}w_b \end{cases}, \tag{12}$$

where $x_{pl} = [x_p^T \quad x_l^T]^T$, $v_{pl} = [e_p^T \quad y_l^T]^T$, $w_b = [r^T \quad d^T]^T$, and

$$\left[\begin{array}{c|c|c} A & B & \hat{B} \\ \hline C & D & \hat{D} \end{array}\right]_{pl,i} = \left[\begin{array}{cc|c|cc} A_p + B_pD_lC_p & B_pC_l & B_pD_l & 0 & B_d - B_pD_lD_d \\ B_lC_p & A_l & B_l & 0 & -B_lD_d \\ \hline -C_y & 0 & 0 & I & -D_d \\ 0 & C_l & 0 & 0 & 0 \end{array}\right]_i. \tag{13}$$

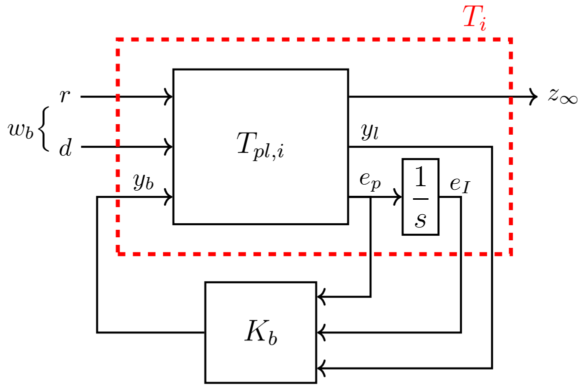

Fig. 4: Closed-loop system shown in general control configuration

The general control configuration of the closed-loop system is shown in Fig. 4. By augmenting $T_{pl,i}$ with the integrator array one obtains

$$T_i : \begin{cases} \dot{x} = A_i x + B_i y_b + \hat{B}_i w_b \\ v = C_i x + \hat{D}_i w_b \end{cases}, \tag{14}$$

where $x = [x_{pl}^T \;\; e_I^T]^T$, $v = [v_{pl}^T \;\; e_I^T]^T$ and

$$\left[\begin{array}{c|c|c} A & B & \hat{B} \\ \hline C & D & \hat{D} \end{array}\right]_i = \left[\begin{array}{cc|c|c} A_{pl} & 0 & B_{pl} & \hat{B}_{pl} \\ [C_p \;\; 0] & 0 & 0 & \hat{D}_p \\ \hline C_{pl} & 0 & 0 & \hat{D}_{pl} \\ 0 & I & 0 & 0 \end{array}\right]_i . \tag{15}$$

The base controller is designed using the following performance index

$$z_\infty = \alpha C_\infty x + \beta D_\infty y_b + \hat{D}_\infty w_b, \tag{16}$$

where α and β are design parameters and are used by the designer to achieve desired closed-loop performance criteria.

Assuming a fixed order for the base controller, one can reduce T_i and (16) to

$$\mathcal{T}_i : \begin{cases} \dot{\xi} = \mathcal{A}_i \xi + \mathcal{B}_i y_b + \hat{\mathcal{B}}_i w_b \\ v = \mathcal{C}_i \xi + \hat{\mathcal{D}}_i w_b \\ z_\infty = \mathcal{C}_{\infty,i} \xi + \mathcal{D}_{\infty,i} y_b + \hat{\mathcal{D}}_{\infty,i} w_b \end{cases}, \tag{17}$$

where $\xi \in \mathbb{R}^{n_b}$ and the rest of $\mathcal{T}_i$ dimensions are in agreement with those of T_i and (16). It is assumed that one can model (17) as a nominal model plus variable perturbation

$$\mathcal{T} : \begin{cases} \dot{\xi} = (\mathcal{A} + \Delta\mathcal{A}(t))\, \xi + (\mathcal{B} + \Delta\mathcal{B}(t))\, y_b + \\ \qquad + \left(\hat{\mathcal{B}} + \Delta\hat{\mathcal{B}}(t)\right) w_b \\ v = (\mathcal{C} + \Delta\mathcal{C}(t))\, \xi + \left(\hat{\mathcal{D}} + \Delta\hat{\mathcal{D}}(t)\right) w_b \\ z_\infty = (\mathcal{C}_\infty + \Delta\mathcal{C}_\infty(t))\, \xi + (\mathcal{D}_\infty + \Delta\mathcal{D}_\infty(t))\, y_b + \\ \qquad + \left(\hat{\mathcal{D}}_\infty + \Delta\hat{\mathcal{D}}_\infty(t)\right) w_b \end{cases} \tag{18}$$

where

$$\begin{bmatrix} \Delta\mathcal{A} & \Delta\mathcal{B} & \Delta\hat{\mathcal{B}} \\ \Delta\mathcal{C} & 0 & \Delta\hat{\mathcal{D}} \\ \Delta\mathcal{C}_\infty & \Delta\mathcal{D}_\infty & \Delta\hat{\mathcal{D}}_\infty \end{bmatrix} = \begin{bmatrix} H_\xi \\ H_v \\ H_z \end{bmatrix} F(t) \left[E_\xi \quad E_{y_b} \quad E_{w_b}\right], \tag{19}$$

where $\sigma_{\max}(F(t)) \leq 1$. System $\mathcal{T}$ includes both external disturbances and parametric uncertainties. Thus, the conventional H_∞ problem cannot be used to design a robust controller for $\mathcal{T}$. So, the results presented by Gu [11] are adopted to design the base controller using robust H_∞ control.

Corollary 1: (Gu [11]) The system $\mathcal{T}$ where y_b is the control input, w_b the exogenous input, v measured output, and z_∞ regulated output with uncertainties (19) can be made quadratically stable with an H_∞-norm bound γ by a strictly proper linear output feedback control iff there exists a $\lambda > 0$ such that the scaled (uncertainty-free) system

$$\mathcal{T}_s : \begin{cases} \dot{\xi} = \mathcal{A}\xi + \mathcal{B} y_b + \begin{bmatrix} \hat{\mathcal{B}} & \gamma\lambda H_\xi \end{bmatrix} \begin{bmatrix} w_b \\ \hat{w}_b \end{bmatrix} \\ v = \mathcal{C}\xi + \begin{bmatrix} \hat{\mathcal{D}} & \gamma\lambda H_v \end{bmatrix} \begin{bmatrix} w_b \\ \hat{w}_b \end{bmatrix} \\ \begin{bmatrix} z_\infty \\ \hat{z}_\infty \end{bmatrix} = \begin{bmatrix} \mathcal{C}_\infty \\ \frac{1}{\lambda} E_\xi \end{bmatrix} \xi + \begin{bmatrix} \mathcal{D}_\infty \\ \frac{1}{\lambda} E_{y_b} \end{bmatrix} y_b + \begin{bmatrix} \hat{\mathcal{D}}_\infty & \gamma\lambda H_z \\ \frac{1}{\lambda} E_{w_b} & 0 \end{bmatrix} \begin{bmatrix} w_b \\ \hat{w}_b \end{bmatrix} \end{cases} \tag{20}$$

with y_b the control input, $[w_b^T \;\; \hat{w}_b^T]^T$ the exogenous input, v measured output and $[z_\infty^T \;\; \hat{z}_\infty^T]^T$ regulated output, can be stabilized with its H_∞-norm less than γ by an output feedback control. □

The formulation involves a parameter λ which is used to tune the scaled system $\mathcal{T}_s$ such that the uncertain system $\mathcal{T}$ achieves H_∞-norm bound γ using a strictly proper controller defined as

$$K_b : \begin{cases} \dot{x}_b = A_b x_b + B_b v \\ y_b = C_b x_b \end{cases}. \tag{21}$$

The combination of $\mathcal{T}_s$ and K_b gives

$$\mathcal{T}_{cl} : \begin{cases} \dot{\xi}_{cl} = \mathcal{A}_{cl} \xi_{pl} + \mathcal{B}_{cl} w \\ Z_\infty = \mathcal{C}_{cl} \xi_{cl} + \mathcal{D}_{cl} w \end{cases}, \tag{22}$$

where $\xi_{cl} = [\xi^T \;\; x_b^T]^T$, $w = [w_b^T \;\; \hat{w}_b^T]^T$, $Z_\infty = [z_\infty^T \;\; \hat{z}_\infty^T]^T$ and

$$\left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right]_{cl} = \left[\begin{array}{cc|cc} \mathcal{A} & \mathcal{B} C_b & \hat{\mathcal{B}} & \gamma\lambda H_\xi \\ B_b \mathcal{C} & A_b & B_b \hat{\mathcal{D}} & \gamma\lambda B_b H_v \\ \hline \mathcal{C}_\infty & \mathcal{D}_\infty C_b & \hat{\mathcal{D}}_\infty & \gamma\lambda H_z \\ \frac{1}{\lambda} E_\xi & \frac{1}{\lambda} E_{y_b} C_b & \frac{1}{\lambda} E_{w_b} & 0 \end{array}\right]. \tag{23}$$

By virtue of the Bounded Real Lemma [18], $\mathcal{A}_{cl}$ is stable and the H_∞-norm of $\mathcal{T}_{cl}$ is smaller than ρ iff there exists a symmetric $\mathcal{P}$ which satisfies the following LMIs:

$$\begin{bmatrix} \mathcal{A}^T\mathcal{P} + \mathcal{P}\mathcal{A} & \mathcal{P}\mathcal{B} & \mathcal{C}^T \\ \mathcal{B}^T\mathcal{P} & -\rho I & \mathcal{D}^T \\ \mathcal{C} & \mathcal{D} & -\rho I \end{bmatrix} < 0, \qquad \mathcal{P} > 0. \tag{24}$$

One can refer to Scherer et al. [12] to find the appropriate transformations, (33)–(35) and (39)–(40), and the relevant LMIs, (41) and (42), to solve the H_∞ problem and calculate K_b.

For each $n_b \in s_b$, the base controller is designed by iteratively solving the H_∞ LMIs by updating the values of α and β in (16) such that K_b satisfies stability requirements and performance criteria of all variants of the closed loop composed of T_i and K_b.

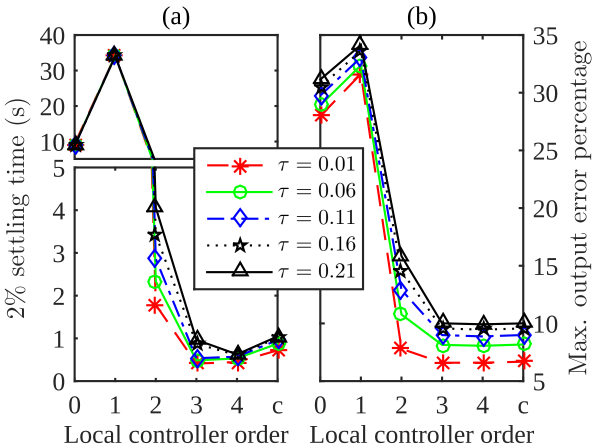

Fig. 5: (a) Settling time and (b) maximum output error percentage versus local controller order. The centralized controller is shown by c.

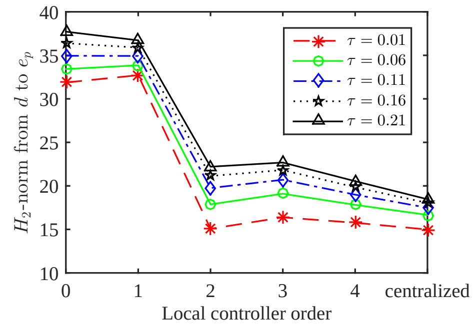

Fig. 6: H_2-norm from disturbance to output error versus local controller order

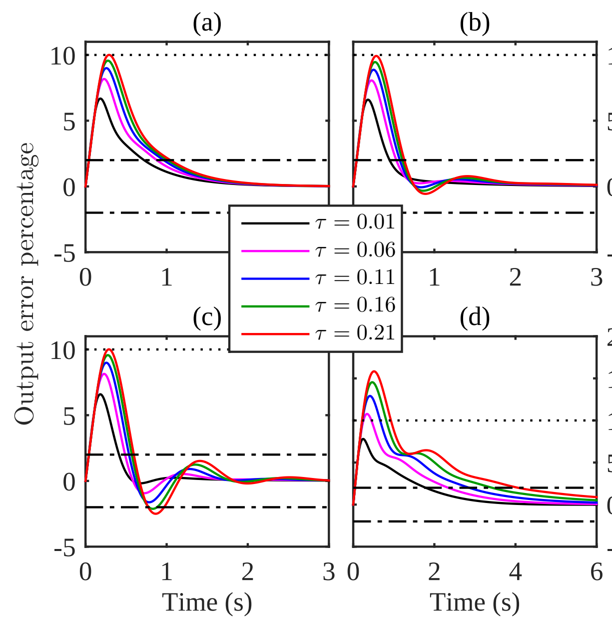

Fig. 7: Output percentage error versus time for (a) centralized controller, (b) $(\boldsymbol{n_b, n_l})$=$(\mathbf{0, 4})$, (c) $(\boldsymbol{n_b, n_l})$=$(\mathbf{1, 3})$, and (d) $(\boldsymbol{n_b, n_l})$=$(\mathbf{2, 2})$. Dotted and dash-dotted lines represent 10% and 2% error bands, respectively.

V. Simulations: Engine Idle Speed Control

To verify the LMI-based CSM design method the example of engine idle speed control (ISC) is selected. The primary objective of the ISC system is to regulate the engine speed to a set-point despite torque disturbances due to accessory loads (e.g., air conditioning, power steering, alternator, etc.) or due to engagement of the transmission. The throttle actuator is considered as a swappable smart component to achieve component swapping modularity. The throttle actuator is modeled as a first-order linear system with time constant τ, with nominal value of 0.05 s, and unity dc gain [4].

The model is linearized around an idle speed of 800 rev/min, a nominal throttle position of $3.15°$, and a load torque of 31.15 Nm. A minimal state space representation of the ISC module is obtained as (1) with

$$A_{p,i}=\begin{bmatrix} -\frac{1}{\tau_i} & 0 & 0 & 0 \\ 27+\frac{1}{\tau_i} & -27 & 0 & 0 \\ 0 & 17.9 & 0 & -2.2 \\ 0 & 0 & 1 & -1.5 \end{bmatrix},\ B_{p,i}=\begin{bmatrix} \frac{1}{\tau_i} \\ -\frac{1}{\tau_i} \\ 0 \\ 0 \end{bmatrix}$$

$$B_{d,i}=[0\ 0\ -1.8\ -1.2]^T, C_{y,i}=[0\ 0\ 0\ 32], D_{d,i}=0,$$

where the control input is throttle position (°), the output is engine speed (rev/min), and disturbance is load torque (Nm). Since the engine has time delay, represented via a Pade approximation, the system is non-minimum phase [5].

A step torque disturbance of 10 Nm is applied at time 0, while the speed deviation is maintained at $r(t) = 0$ rev/min. Design criteria are introduced as $u(t) \in [-3, 13]$, maximum engine speed deviation less than 10% of the speed set-point value, settling time less than 1.5 s. Also five actuators with different time constants $\tau \in \{0.01, 0.06, 0.011, 0.16, 0.21\}$ are used in the simulations.

As shown in Fig. 2, an integrator is included in the controller to achieve reference tracking. The full order controller designed for each variant of the throttle actuator is termed the centralized controller which results in the best closed-loop performance as evidenced in Fig 5–7. This case involves full redesign when the throttle actuator is swapped. The proposed CSM design algorithm is verified for all possible combinations of the local and base controllers, i.e., $(n_b, n_l) \in \{(0,4),(1,3),(2,2),(3,1),(4,0)\}$.

$(\boldsymbol{n_b, n_l}) = (\mathbf{0, 4})$**:** In this case the base controller is a static gain and the local controller is full order. As expected, one can observe from the conducted simulations that the distributed controller achieves the same level of performance as the centralized controller. The settling time has improved slightly. However, as Fig. 6 shows H_2-norm starts to increase gradually.

$(\boldsymbol{n_b, n_l})$=$(\mathbf{1, 3})$**:** The best performance has been achieved with $(\alpha, \beta) = (24, 20)$. In this case the design performance criteria are satisfied for all the variants of the throttle actuator.

(n_b, n_l)=$(2,2)$: The best performance has been achieved with $(\alpha, \beta) = (0.4, 1.8)$. As Fig. 5 and 7 show in this case the distributed controller fails to satisfy the design criteria. Nevertheless, if the settling time and maximum output percentage specifications are slightly relaxed, then the case of $(n_b, n_l) = (2, 2)$ also guarantees CSM.

(n_b, n_l)=$(3,1)$: The best performance has been achieved with $(\alpha, \beta) = (0.1, 2.0)$. Except the bounds on the control input, other design criteria are not met. Since in this case the base controller order is higher than the local controller, one can conclude that the H_∞ performance prevails which causes a dramatic increase in the H_2-norm as shown in Fig. 6.

(n_b, n_l)=$(4,0)$: The best performance has been achieved with $(\alpha, \beta) = (0.4, 5.0)$. This case corresponds in fact to one H_∞ robust controller for all the variants of the throttle actuator. The possibility of using the local controller to compensate the effect of variable time constant of the throttle actuator is very limited which is shown in the simulation results.

The local controller is designed using an H_2 optimization algorithm such that the disturbance effect in the output is minimized. So, the H_2-norm between the disturbance and the output error can be used to roughly evaluate CSM. As shown in Fig. 6 there is a considerable gap between the case of $(n_b, n_l) = (2, 2)$ and $(n_b, n_l) = (3, 1)$ which marks the boundary for CSM. However, since time response criteria cannot be directly converted to the H_2-norm measure, the CSM should be evaluated by analyzing the given design constraints. Moreover, because the control signal always remains inside the design limits, i.e., $u(t) \in [-3, 13]$, the relevant plot is not shown to save space.

The results of the LMI-based CSM design fall between the results of the Direct and 3-Step Methods [4], [5]. These results show that CSM can be achieved with $n_b \in \{1, 2, 3, 4\}$, $n_b \in \{3, 4\}$, and $n_b = 4$ for the Direct, LMI-based, and 3-Step Methods, respectively. The Direct Method treats the optimization problem as a whole and it does not provide an organized framework to formulate the performance index and system constraints into manageable mathematical relationships. So, the Direct Method demands high processing power, converges very slowly, and relies on the designer's knowledge of the system. On the other hand, the LMI-based Method proposes a structurally viable formulation methodology which is numerically efficient and converges rapidly. Also, the LMI-based Method presents a generalized and semi-automatic CSM algorithm with applications to a wide range of arbitrary linear systems.

VI. Conclusions

The proposed LMI-based CSM algorithm automatically distributes controllers in control networks which include smart components. To achieve specified time response criteria H_2 optimization is used to design the local controller. On the other hand, the base controller is designed using a robust H_∞ method such that the closed-loop system remains stable for all the variants of the swappable component and also the disturbance is attenuated to its minimum feasible level. The proposed algorithm used the flexibility of LMIs to generalize the applications and to reduce the computational complexity associated with distributed control design. The simulation results verified the practicality of the proposed algorithm to achieve CSM for local controller order $n_l \in \{2, 3, 4\}$ in engine idle speed control. The throttle actuator was assumed as a smart component to accommodate the local controller. Although the proposed LMI-based CSM requires minimal designer involvement, future research work will focus on incorporating adaptivity for α and β in the proposed algorithm to obtain a fully automatic CSM design.

References

[1] M. Cakmakci and A. G. Ulsoy, "Improving component swapping modularity using bidirectional communication in networked control system," *IEEE/ASME Transactions on Mechatronics*, vol. 14, pp. 307–316, 2009.

[2] ——, "Swappable distributed MIMO controller for a VCT engine," *IEEE Transactions on Control Systems Technology*, vol. 19, pp. 1168–1177, 2011.

[3] S. Li, I. V. Kolmanovsky, and A. G. Ulsoy, "Distributed supervisory controller design for battery swapping modularity in plug-in hybrid electric vehicles," *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 134, article no. 041013, 2012.

[4] ——, "Direct optimal design for component swapping modularity in control systems," *IEEE/ASME Transactions on Mechatronics*, vol. 18, pp. 297–306, 2013.

[5] S. Li, M. Cakmakci, I. V. Kolmanovsky, and A. G. Ulsoy, "Throttle actuator swapping modularity design for idle speed control," in *Proc. American Control Conference*, 2009, pp. 2702–2707.

[6] A. G. Ulsoy, H. Peng, and M. Cakmakci, *Automotive Control Systems*. NY: New York: Cambridge University Press, 2012.

[7] S. Riverso, M. Farina, and G. Ferrari-Trecate, "Plug-and-play decentralized model predictive control for linear systems," *IEEE Transactions on Automatic Control*, vol. 58, pp. 2608–2614, 2013.

[8] J. Bendtsen, K. Trangbaek, and J. Stoustrup, "Plug-and-play control—modifying control systems online," *IEEE Transactions on Control Systems Technology*, vol. 21, pp. 79–93, 2013.

[9] P. P. Khargonekar, I. R. Petersen, and K. Zhou, "Robust stabilization of uncertain linear systems: Quadratic stabilizability and H^∞ control theory," *IEEE Transactions on Automatic Control*, vol. 35, pp. 356–361, 1990.

[10] L. Xie, M. Fu, and C. E. de Souza, "H_∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback," *IEEE Transactions on Automatic Control*, vol. 37, pp. 1253–1256, 1992.

[11] K. Gu, "H_∞ control of systems under norm bounded uncertainties in all system matrices," *IEEE Transactions on Automatic Control*, vol. 39, pp. 1320–1322, 1994.

[12] C. Scherer, P. Gahinet, and M. Chilali, "Multiobjective output-feedback control via LMI optimization," *IEEE Transactions on Automatic Control*, vol. 42, pp. 896–911, 1997.

[13] M. C. de Oliveira, J. C. Geromel, and J. Bernussou, "Extended H_2 and H_∞ norm characterizations and controller parameterizations for discrete-time systems," *International Journal of Control*, vol. 75, pp. 666–679, 2002.

[14] G. Hilhorst, G. Pipeleers, W. Michiels, and J. Swevers, "Sufficient LMI conditions for reduced-order multi-objective H_2/H_∞ control of LTI systems," *European Journal of Control*, vol. 23, pp. 17–25, 2015.

[15] C. M. Agulhari, R. C. L. F. Oliveira, and P. L. D. Peres, "LMI relaxations for reduced-order robust H_∞ control of continuous-time uncertain linear systems," *IEEE Transactions on Automatic Control*, vol. 57, pp. 1532–1537, 2012.

[16] S. Datta, D. Chakraborty, and M. N. Belur, "Reduced-order controller synthesis with regional pole constraint," *International Journal of Control*, vol. 89, pp. 221–234, 2016.

[17] A. Packard and J. Doyle, "Quadratic stability with real and complex perturbations," *IEEE Transactions on Automatic Control*, vol. 35, pp. 198–201, 1990.

[18] B. D. O. Anderson and S. Vongpanitlerd, *Network Analysis*. NJ: Englewood Cliffs: Prentice Hall, 1973.